Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2017

Manitoba

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Disponible en français.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, extension 7590
Email: youyi.sun@gov.mb.ca

Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error). **Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).**				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines

Booklet 1 Questions

Question 1 P3

There are 24 different movies Kiandra can download to her computer. Determine the number of ways she can select 15 movies.

Solution

${}_{24}C_{15} = 1\ 307\ 504$ combinations **1 mark**

Exemplar 1

$_{24}C_{15}$

$$\frac{24!}{(24-15)!\,15!}$$

$$\frac{24!}{9!\,15!}$$

1 out of 1

award full marks
E1 (final answer not stated)

Given $\theta = 40^\circ$,

a) convert θ to radians.

b) determine the coterminal angles of θ where $\theta \in \mathbb{R}$.

Solution

a)

$$\theta = 40\left(\frac{\pi}{180}\right)$$

$$\theta = \frac{2\pi}{9}$$

or

$$\theta = 0.698$$

1 mark

b)

$$\theta = \frac{2\pi}{9} + 2\pi k \in \mathbb{Z}$$

or

$$\theta = 0.698 + 2\pi k \in \mathbb{Z}$$

or

$$\theta = 40^\circ + 360^\circ k,\ k \in \mathbb{Z}$$

1 mark

Exemplar 1

a)

$$40^\circ \times \frac{\pi}{180} = \frac{40\pi}{180} = \frac{2\pi}{9}$$

1 out of 1

b)

$\frac{2\pi}{9} + 2\pi k$ where $k \in \mathbb{R}$

½ out of 1

award full marks
− ½ mark for procedural error ($k \in \mathbb{R}$ instead of $k \in \mathbb{Z}$)

Exemplar 2

a)

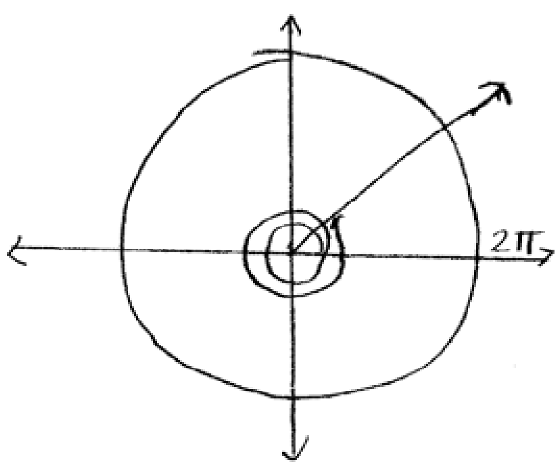
$\theta = 40^\circ$

$40^\circ = \frac{\pi}{180}$

$\theta = 4,5$

0 out of 1

b)



$\boxed{4,5 + 2\pi n, n \in Z}$

1 out of 1

award full marks
(consistent with concept error in a))

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Peter invests \$560 per month at an annual interest rate of 4.2%, compounded monthly. Determine how many monthly investments he will need to make to obtain at least \$500 000.
Express your answer as a whole number.

Use the formula:

$$\mathrm{FV} = \frac{\mathrm{R}\left[(1+i)^n - 1\right]}{i}$$

where FV = the future value
R = the investment amount each period
$i = \dfrac{\text{the annual interest rate}}{\text{the number of compounding periods per year}}$
n = the number of investments

Solution

$$500\ 000 = \frac{560\left[\left(1+\frac{0.042}{12}\right)^n - 1\right]}{\frac{0.042}{12}}$$ ½ mark for substitution

$$500\ 000 = \frac{560\left[(1+0.0035)^n - 1\right]}{0.0035}$$

$$500\ 000 = 160\ 000\left(1.0035^n - 1\right)$$

$$3.125 = 1.0035^n - 1$$

$$4.125 = 1.0035^n$$ ½ mark for simplification

$$\log 4.125 = \log 1.0035^n$$ ½ mark for applying logarithms

$$\log 4.125 = n \log 1.0035$$ 1 mark for power law

$$n = \frac{\log 4.125}{\log 1.0035}$$

$$n = 405.584$$ ½ mark for solving for n

$\therefore$ 406 monthly investments are needed.

3 marks

Exemplar 1

$$500\,000 = \frac{560\left[(1+0.042)^n - 1\right]}{0.042}$$

$$\frac{21\,000}{560} = \frac{560\left[(1+0.042)^n - 1\right]}{560}$$

$$37.5 = (1.042)^n - 1$$

$$38.5 = (1.042)^n$$

$$\frac{\log 38.5}{\log 1.042} = \frac{n \log 1.042}{\log 1.042}$$

$$n = 88.733$$

$\therefore$ 88 monthly investments

2½ out of 3

+ ½ mark for simplification
+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for solving for *n*
E6 (rounding error)

Exemplar 2

$$500{,}000 = \frac{560\left[\cancel{1}+(.042)^n - \cancel{1}\right]}{.042}$$

$$21{,}000 = 560\,(.042)^n$$

$$37.5 = .042^n$$

$$\log 37.5 = n \log .042$$

$$n = 114.3294321$$

1½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law

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Question 4 P2

Ishmael has 4 dogs, 5 cats, and 3 horses.

If he arranges all of them in a row, determine how many ways they can be arranged if each type of animal must be grouped together.

Solution

$$\underset{\text{types of animals}}{\underline{3!}} \bullet \underset{\text{dogs}}{\underline{4!}} \bullet \underset{\text{cats}}{\underline{5!}} \bullet \underset{\text{horses}}{\underline{3!}} = 103\,680 \text{ ways}$$

1 mark for arrangement of types of animals
1 mark for arrangement of dogs, cats, and horses

2 marks

Exemplar 1

DDDD CCCCC HHH

$\underline{4!}\ \underline{5!}\ \underline{3!} = 4320$

½ out of 2

+ 1 mark for arrangement of dogs, cats, and horses
− ½ mark for arithmetic error

Exemplar 2

$3!(4!+5!+3!)$

$= 6(24+120+6)$

$= 6(150)$

$= 900$ ways

1 out of 2

award full marks
− 1 mark for concept error (adding instead of multiplying)

Exemplar 3

$\boxed{6 \text{ ways}}$

1 out of 2

+ 1 mark for arrangement of types of animals

Solve the following equation algebraically over the interval $0 \le \theta \le 2\pi$.

$$2\cos^2\theta + 9\cos\theta - 5 = 0$$

Solution

$2\cos^2\theta + 9\cos\theta - 5 = 0$

$(2\cos\theta - 1)(\cos\theta + 5) = 0$

$\cos\theta = \dfrac{1}{2}$	$\cos\theta = -5$	1 mark for solving for $\cos\theta$
$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$	no solution	2 marks for solving for θ (½ mark for each value, 1 mark for indicating no solution)

3 marks

Exemplar 1

$(2\cos\theta - 1)(\cos\theta + 5)$

$2\cos\theta - 1 = 0$ $\qquad$ $\cos\theta \ -5$

$2\cos\theta \ \ 1$

$\cos\theta \ \ \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

1½ out of 3

+ 1 mark for solving for $\cos\theta$
+ ½ mark for solving for θ (value of angle is consistent with reference angle)
E2 (changing an equation to an expression in line 1)

Exemplar 2

$(2\cos - 1)(\cos + 10)$

$2\cos\theta - 1 = 0$ $\qquad$ $\cos\theta + 10 = 0$

$\frac{2\cos\theta}{2} = \frac{1}{2}$ $\qquad$ $\cancel{\cos\theta = -10}$

$\cos\theta = \frac{1}{2}$

$\theta = 60^\circ$

2 out of 3

+ 1 mark for solving for $\cos\theta$
+ ½ mark for solving for one value of θ
+ 1 mark for indicating no solution
– ½ mark for arithmetic error in line 1
E2 (changing an equation to an expression in line 1)
E3 (variable omitted in line 1)
E5 (answer stated in degrees instead of radians)

Exemplar 3

$2x^2 + 9x - 5 = 0$

$(2x - 1)(x + 5)$

$x = \frac{1}{2}$	$x = -5$
$\cos\theta = \frac{1}{2}$	$\cos\theta = -5$
$\boxed{\pi/3, \; 5\pi/3}$	$\varnothing$

3 out of 3

award full marks
E3 (variable introduced without being defined in line 1)
E2 (changing an equation to an expression in line 2)
E7 (notation error in line 5)

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Determine which term contains $\frac{1}{x^6}$ in the binomial expansion of $\left(\frac{2}{x^3} + 3x^2\right)^7$.

Solution

Method 1

$$\left(x^{-3}\right)^{7-k}\left(x^2\right)^k = x^{-6}$$
$$x^{-21+3k}x^{2k} = x^{-6}$$
$$x^{-21+5k} = x^{-6}$$
$$-21 + 5k = -6$$
$$5k = 15$$
$$k = 3$$

$\therefore$ the fourth term contains $\frac{1}{x^6}$

½ mark for substitution

½ mark for solving for k

1 mark for the 4th term (or a term consistent with the value of k)

2 marks

Method 2

$$\left(\frac{1}{x^3}\right)^7, \left(\frac{1}{x^3}\right)^6\left(x^2\right), \left(\frac{1}{x^3}\right)^5\left(x^2\right)^2, \ldots$$

$$\frac{1}{x^{21}}, \frac{1}{x^{16}}, \frac{1}{x^{11}}, \ldots$$

$\therefore$ the fourth term contains $\frac{1}{x^6}$

1 mark for determining a pattern

1 mark for the 4th term (or a term consistent with the pattern)

2 marks

Exemplar 1

$t_{k+1} = {}_nC_k\, a^{n-k} b^k$

$x^{-6} = {}_7C_k (x^{-3})^{7-k} (x^2)^k$

$x^{-6} = x^{-21+3k}\; x^{2k}$

$-6 = -21 + 3k + 2k$

$\frac{15}{5} = \frac{5k}{5}$

$3 = k$

$\frac{1}{x^6}$ is in the Second (2^{nd}) Term.

1 out of 2

+ ½ mark for substitution
+ ½ mark for solving for k

Exemplar 2

$$t_1 = \cancel{{}_7C_0}^{1} \left(\frac{2}{x^3}\right)^7 \cancel{(3x^2)}^{1} = \frac{128}{x^{21}}$$

$$t_2 = {}_7C_1 \left(\frac{2}{x^3}\right)^6 (3x^2)^1 = 7\left(\frac{64}{x^{18}}\right)(3x^2)$$

$$t_3 = {}_7C_2 \left(\frac{2}{x^3}\right)^5 (3x^2)^2 = 21\left(\frac{32}{x^{15}}\right)(9x^4)$$

$$t_4 = {}_7C_3 \left(\frac{2}{x^3}\right)^4 (3x^2)^3 = 35\left(\frac{16}{x^{12}}\right)(27x^6)$$

$$t_5 = {}_7C_4 \left(\frac{2}{x^3}\right)^3 (3x^2)^4 = 35\left(\frac{8}{x^9}\right)(81x^8)$$

$$t_6 = {}_7C_5 \left(\frac{2}{x^3}\right)^2 (3x^2)^5 = 21\left(\frac{4}{x^6}\right)(243x^{10})$$

1 out of 2

+ 1 mark for determining a pattern

Exemplar 3

$$(x^{-3})^{7-k}(x^2)^k = x^{-6}$$

$$x^{-21-k}x^{2k} = x^{-6}$$

$$x^{-21+k} = x^{-6}$$

$$-21+k = -6$$

$$k = 15$$

the 16th term

1½ out of 2

+ ½ mark for substitution
+ 1 mark for term consistent with the value of k

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Question 7 T1

Determine the radius of a circle which has an arc length of 5 cm with a central angle of 3 radians.

Solution

$s = \theta r$

$5 = 3r$

$r = \dfrac{5}{3}$ cm

1 mark

Exemplar 1

$$r = \frac{s}{\theta}$$

$$r = \frac{5}{3}$$

$$r = 1.67$$

1 out of 1

award full marks
E5 (units of measure omitted in final answer)
E6 (rounding error)

Exemplar 2

$$s = \theta r$$
$$5 = 3r$$

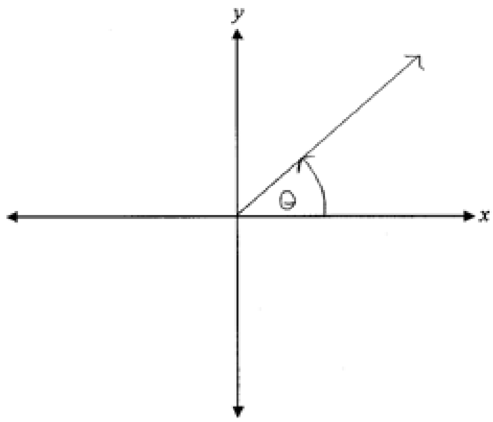
1 out of 1

award full marks
E1 (final answer not stated)

Question 8 T1

Tyler incorrectly sketched the angle $\theta = -\dfrac{7\pi}{4}$ in standard position.

Describe his error.



Solution

Tyler incorrectly indicated the direction of the angle as positive.

or

Tyler sketched the reference angle, not $\theta = -\dfrac{7\pi}{4}$.

1 mark

Exemplar 1

He didn't show the arrow going in the correct direction

1 out of 1

Exemplar 2

Wrong direction

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 3

His arrow is wrong

0 out of 1

Given the identity $\sec\theta + \cos\theta = \dfrac{2 - \sin^2\theta}{\cos\theta}$,

a) determine the non-permissible values of θ, over the interval $0 \leq \theta \leq 2\pi$.

b) prove the identity for all permissible values of θ.

Solution

a) $\cos\theta = 0$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

1 mark for non-permissible values
(½ mark for each value)

1 mark

Solution

Method 1

b)

Left-Hand Side	Right-Hand Side
$\sec\theta + \cos\theta$	$\dfrac{2-\sin^2\theta}{\cos\theta}$
$\dfrac{1}{\cos\theta} + \cos\theta$	
$\dfrac{1+\cos^2\theta}{\cos\theta}$	
$\dfrac{1+1-\sin^2\theta}{\cos\theta}$	
$\dfrac{2-\sin^2\theta}{\cos\theta}$	

1 mark for correct substitution of appropriate identities

1 mark for algebraic strategies

1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\sec\theta + \cos\theta$	$\dfrac{2-\sin^2\theta}{\cos\theta}$
	$\dfrac{2-\left(1-\cos^2\theta\right)}{\cos\theta}$
	$\dfrac{1+\cos^2\theta}{\cos\theta}$
	$\dfrac{1}{\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta}$
	$\sec\theta + \cos\theta$

1 mark for correct substitution of appropriate identities

1 mark for algebraic strategies

1 mark for logical process to prove the identity

3 marks

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Exemplar 1

a)

$\cos\theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$

1 out of 1

b)

Left-Hand Side	Right-Hand Side
$\sec\theta + \cos\theta$ $\frac{1}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$ $\frac{1+\cos^2\theta}{\cos\theta}$	$\frac{2-\sin^2\theta}{\cos\theta}$

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies

Exemplar 2

a)

NPV: $\cos\theta \neq 0$

$\cos\theta \neq \pi, \ \frac{3\pi}{2}$

0 out of 1

+ ½ mark for $\theta = \frac{3\pi}{2}$

– ½ mark for procedural error $\left(\cos\theta = \frac{3\pi}{2}\right)$

b)

Left-Hand Side	Right-Hand Side
$\sec\theta + \cos\theta$	$\frac{2 - \sin^2\theta}{\cos\theta}$
	$\frac{2 - 1 - \cos^2\theta}{\cos\theta}$
	$\frac{1 + \cos^2\theta}{\cos\theta}$
	$\frac{1}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$
	$\sec\theta + \frac{\cos\theta(\cancel{\cos\theta})}{\cancel{\cos\theta}}$
	$\sec\theta + \cos\theta$

LHS = RHS

3 out of 3

award full marks
E4 (missing brackets but still implied in line 2)

Exemplar 3

a)

$$\theta \neq \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\boxed{\theta \neq \frac{\pi}{2} + k\pi, \; k \in I}$$

1 out of 1

award full marks
E8 (answer outside the given domain)

b)

Left-Hand Side	Right-Hand Side
$\frac{1}{\cos\theta} + \left(\frac{\cos\theta}{1}\right)\frac{\cos\theta}{\cos\theta}$	$\frac{2}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$
$\frac{1}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$	$\frac{2}{\cos\theta} - \frac{\cos^2\theta - 1}{\cos\theta}$
$\frac{\cos^2\theta}{\cos\theta}$	$\frac{2 + \cos^2\theta - 1}{\cancel{\cos\theta}}$
$\cos\theta$	$\cos\theta$

LHS = RHS

0 out of 3

Question 10 R8

Expand using the laws of logarithms.

$$\log\left(\frac{a}{b^4}\right)$$

Solution

$\log a - \log b^4$	1 mark for quotient law
$\log a - 4\log b$	1 mark for power law
	2 marks

Exemplar 1

$$= \log 2 - \log b^4$$
$$= \log (2 - b^4)$$

0 out of 2

+ 1 mark for quotient law
– 1 mark for concept error (subtracting the arguments)

Exemplar 2

$$M = a$$
$$N = b^4$$

$$\log\left(\frac{a}{b^4}\right) = \log_a a - \log_a b^4$$

½ out of 2

+ 1 mark for quotient law
– ½ mark for procedural error (changing base of logarithms)

Exemplar 3

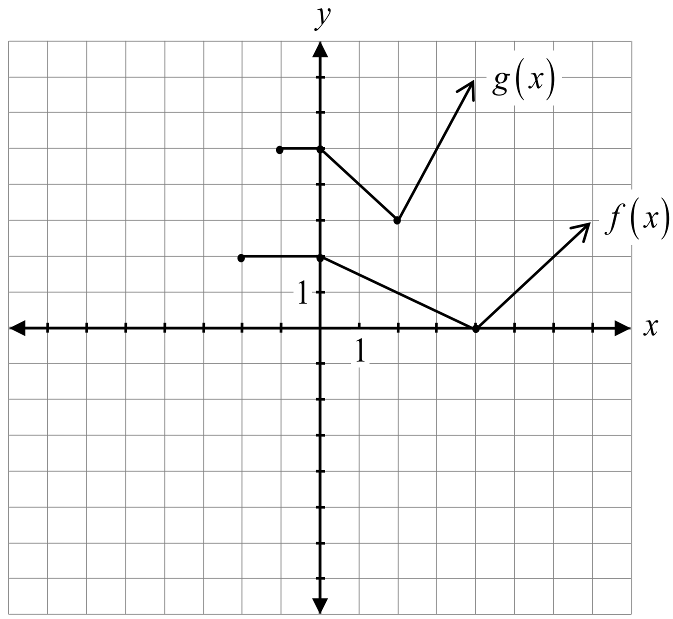
$$= \log a - \log b^4$$
$$= \log a - \log b + 4\log$$

1 out of 2

+ 1 mark for quotient law

Question 11 R4

State the equation of $g(x)$ in terms of $f(x)$.



Solution

$g(x) = f(2x) + 3$

1 mark for horizontal compression
1 mark for vertical translation

2 marks

Exemplar 1

$(-2,2) \rightarrow (-1,5)$

$(0,2) \rightarrow (0,5)$

$(4,0) \rightarrow (2,3)$

$a=1$

$b=2$

$d=3$

$g(x) = 2(x)+3$

1 out of 2

award full marks
– 1 mark for concept error (not writing an equation in terms of $f(x)$)

Exemplar 2

$g(x) = f(2x+2)+3$

1 out of 2

award full marks
– 1 mark for concept error (introducing an incorrect horizontal transformation)

Exemplar 3

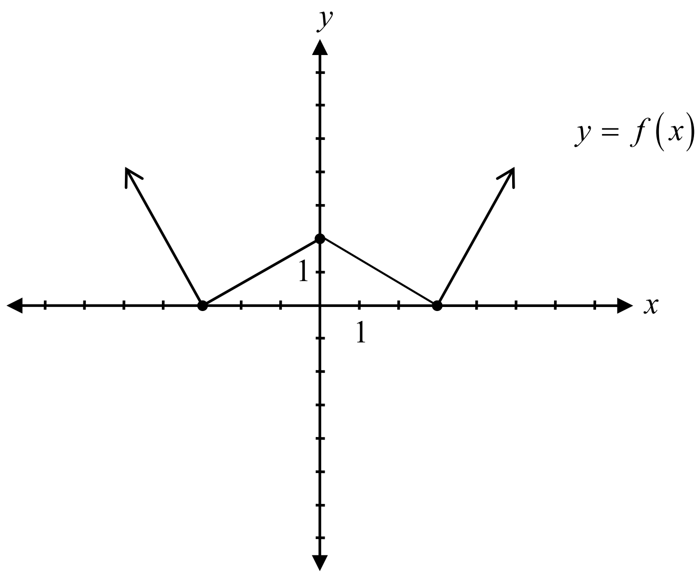
$g(x) = 2f(2x)+1$

1 out of 2

+ 1 mark for horizontal compression

Question 12 R6

Explain why the inverse of the graph of $y = f(x)$ is not a function.



Solution

The domain of $f(x)$ was not restricted to ensure there is only one value of y for each x and one value of x for each y.

or

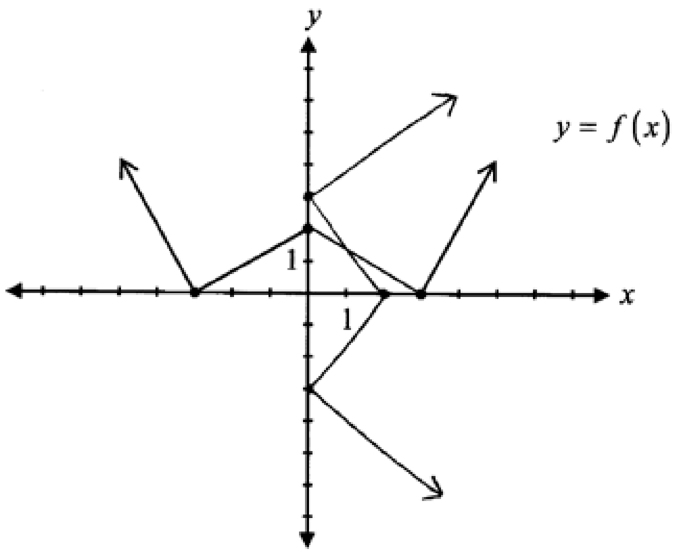
The graph of the inverse will not pass the vertical line test.

or

The graph of $f(x)$ does not pass the horizontal line test.

1 mark

Exemplar 1



because there is more than one value of y for each value of x

1 out of 1

Exemplar 2

Becacse it does not pass the horizontal line test

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

It doesn't pass the vertical line test

1 out of 1

Question 13 R10

Solve the following equation algebraically:

$$\log\left(x^2+5\right)-\log\left(x^2+1\right)=\log 3$$

Solution

$$\log\left(\frac{x^2+5}{x^2+1}\right)=\log 3$$

1 mark for quotient law

$$\frac{x^2+5}{x^2+1}=3$$

½ mark for equating arguments

$$x^2+5=3\left(x^2+1\right)$$

$$x^2+5=3x^2+3$$

$$2=2x^2$$

$$1=x^2$$

$$\pm 1=x$$

½ mark for solving for x

2 marks

Exemplar 1

$$\frac{\log(x^2+5)}{\log(x^2+1)} = \log 3$$

$$\cancel{(x^2+1)}\frac{x^2+5}{\cancel{x^2+1}} = 3\ (x^2+1)$$

$$x^2+5 = 3\ (x^2+1)$$

$$x^2+5 = 3x^2+3$$

$$-x^2 \qquad -x^2$$

$$5 = 2x^2+3$$

$$-3 \qquad -3$$

$$\frac{2}{2} = \frac{2x^2}{2}$$

$$\pm 1 = x$$

1 out of 2

+ ½ mark for equating arguments
+ ½ mark for solving for x

Exemplar 2

$$\text{Log}\left(\frac{x^2+5}{x^2+1}\right) = \text{Log}\,3$$

$$\boxed{x = 1}$$

1 out of 2

+ 1 mark for quotient law

Exemplar 3

$$\log\left(\frac{x^2+5}{x^2+1}\right) = \log 3$$

$$\frac{x^2+5}{x^2+1} = 3$$

$$x^2+5 = 3(x^2+1)$$

$$x^2+5 = 3x^2+3$$

$$-2x^2 = -2$$

$$x^2 = 1$$

$$x = 1$$

1½ out of 2

+ 1 mark for quotient law
+ ½ mark for equating arguments

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Question 14 R4

Describe the transformations used to obtain the graph of the function $y = 5f\left(x+1\right)$ from the graph of $y = f\left(x\right)$.

Solution

Vertically stretch the graph by a factor of 5 and translate the graph one unit left.

1 mark for vertical stretch
1 mark for horizontal translation

2 marks

Exemplar 1

$y \cdot 5$

$x - 1$

0 out of 2

Exemplar 2

- horizontal translation by a factor of 1 to the left
- vertical stretch by a factor of 5 $(y \cdot 5)$

1½ out of 2

award full marks
– ½ mark for terminology error in description

Solve algebraically:

$${}_{n}C_{2} = 3n + 4$$

Solution

$$\frac{n!}{(n-2)!\,2!} = 3n + 4$$

½ mark for substitution into equation

$$\frac{n(n-1)\cancel{(n-2)}!}{\cancel{(n-2)}!} = 2!(3n+4)$$

1 mark for factorial expansion
½ mark for simplification of factorials

$$n(n-1) = 2(3n+4)$$

$$n^2 - n = 6n + 8$$

$$n^2 - 7n - 8 = 0$$

$$(n-8)(n+1) = 0$$

$$n = 8 \quad \cancel{n = -1}$$

½ mark for rejecting the extraneous root
½ mark for the value of n

3 marks

Exemplar 1

$$\frac{n!}{(n-2)!2!} = 3n+4$$

$$\frac{(n)(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}2!} = 3n+4$$

$$\cancel{2!}\,\frac{n^2-n}{\cancel{2!}} = 3n+4 \cdot 2!$$

$$n^2-n = 3n+8$$

$$n^2-2n-8=0$$

$$(n-4)(n+2)$$

$$n=4 \qquad n=-2$$

1½ out of 3

+ ½ mark for substitution into equation
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the value of *n*
– ½ mark for arithmetic error in line 4
– ½ mark for arithmetic error in line 5
E2 (changing an equation to an expression in line 6)

Exemplar 2

$$_nC_2 = \frac{n!}{r!(n-r)!}$$

$$= \frac{n!}{2!(n-2)!}$$

0 out of 3

Exemplar 3

$$\frac{n!}{2!(n-2)!} = 3n+4$$

$$\frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = 3n+4$$

$$\frac{n(n-1)}{2!} = 3n+4$$

$$\frac{n(n-1)}{2} = 3n+4$$

$$2(n(n-1)) = 2(3n+4)$$

$$2(n^2-n) = 10n$$

$$2n^2 - 2n = 10n$$

$$2n^2 - 12n = 0$$

$$(2n)(n-6) = 0$$

$$n = 0 \text{ or } 6$$

$$n \neq 0,\ n = 6$$

2 out of 3

award full marks
– ½ mark for arithmetic error in line 5
– ½ mark for arithmetic error in line 6

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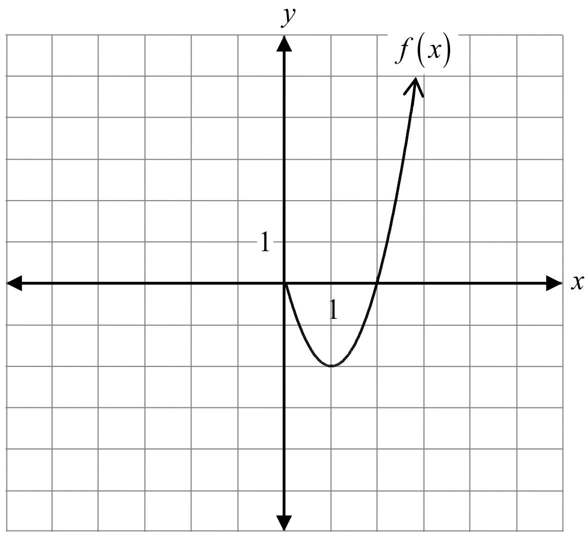
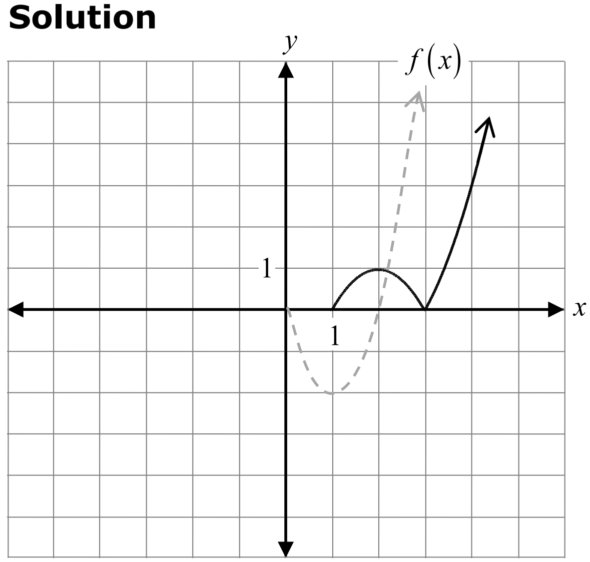
Given the graph of $f(x)$, sketch the graph of $y = \left|\frac{1}{2} f(x-1)\right|$.



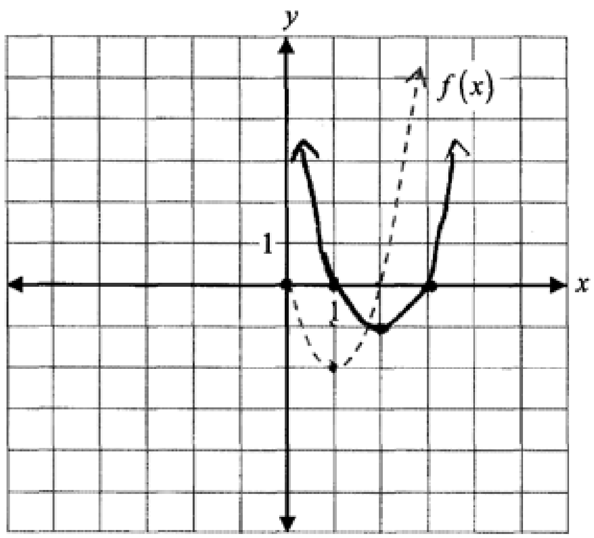
Solution



1 mark for vertical stretch
1 mark for horizontal translation
1 mark for absolute value

3 marks

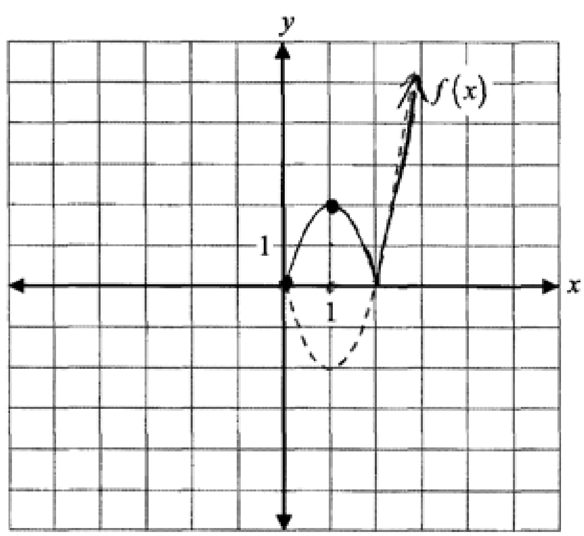
Exemplar 1



2 out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal translation
E8 (answer included outside the given domain)

Exemplar 2



1 out of 3

+ 1 mark for absolute value

Question 17 R12

Explain why $f(x) = (x+2)^3 (x-1)^{\frac{1}{2}}$ is not a polynomial function.

Solution

All factors in a polynomial function must have exponents that are whole numbers.

1 mark

Exemplar 1

There cannot be an exponent to the a half in polynomial functions.

1 out of 1

Exemplar 2

because $(x-1)^{1/2} = \sqrt{(x-1)}$ and this is a radical

1 out of 1

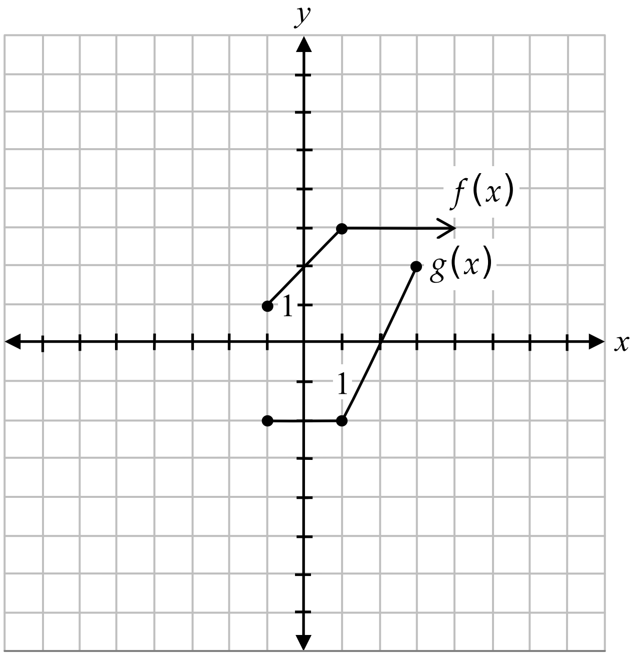
Exemplar 3

$(x-1)^{\frac{1}{2}}$ is the same as $\sqrt{(x-1)}$

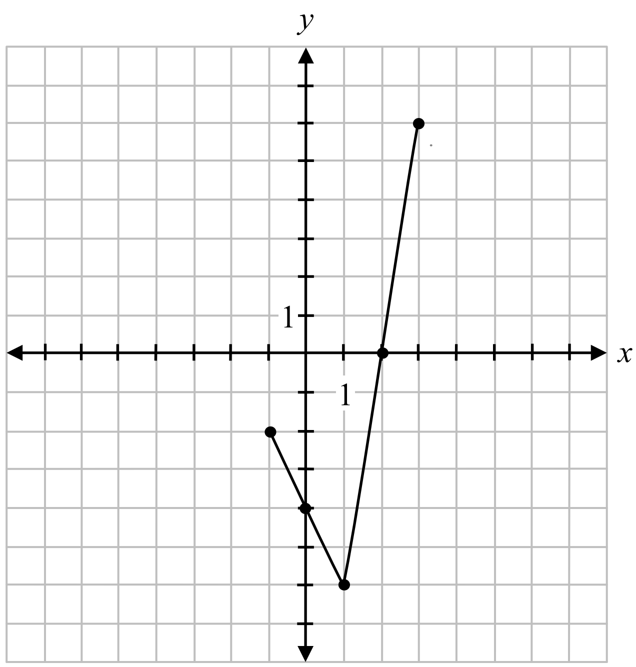
½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = (f \bullet g)(x)$.



Solution

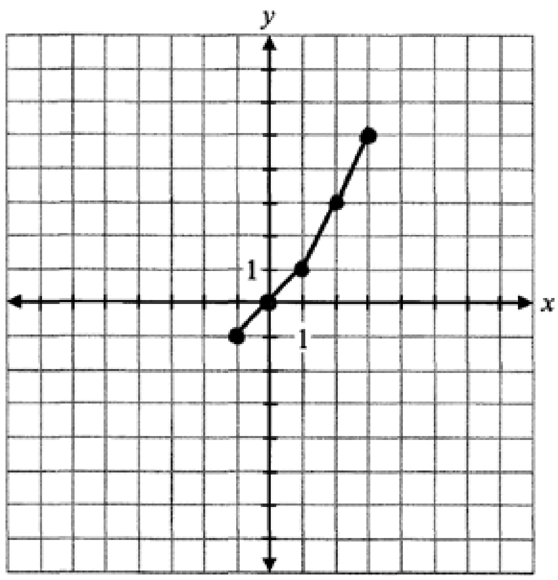


x	$f(x)$	$g(x)$	$(f \bullet g)(x)$
–1	1	–2	–2
1	3	–2	–6
3	3	2	6

1 mark for operation of multiplication
1 mark for restricted domain

2 marks

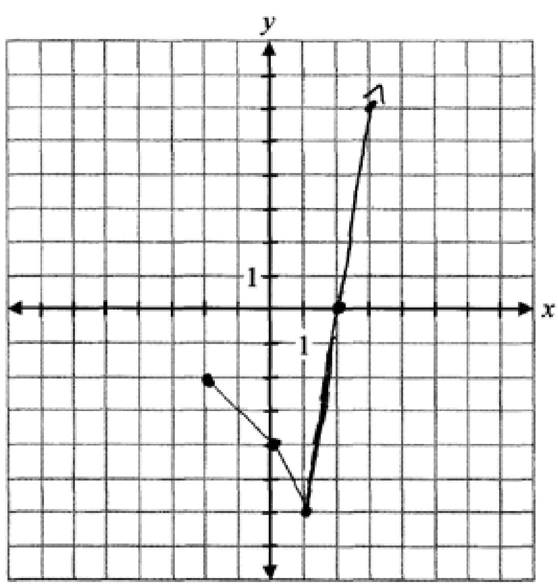
Exemplar 1



1 out of 2

+ 1 mark for restricted domain

Exemplar 2



½ out of 2

+ 1 mark for operation of multiplication
– ½ mark for procedural error (one incorrect point)

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Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
19	B	T6
20	B	R7
21	D	T4
22	C	P4
23	B	T5
24	D	R6
25	C	R11
26	A	T3
27	C, B, A, D	R12

Question 19 T6

Identify the trigonometric function that is equivalent to $\sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3}$.

a) $\sin\frac{2\pi}{7}$

b) $\sin\frac{7\pi}{12}$

c) $\cos\frac{2\pi}{7}$

d) $\cos\frac{7\pi}{12}$

Question 20 R7

Identify the logarithmic form of $5^x = 6$.

a) $\log_5 x = 6$

b) $\log_5 6 = x$

c) $\log_6 x = 5$

d) $\log_6 5 = x$

Question 21 T4

Given $f(\theta) = 3\cos 2\theta - 1$ and $g(\theta) = \sin\theta + 1$, identify which statement is true.

a) Both functions have the same period.

b) Both functions have the same amplitude.

c) Both functions have the same minimum value.

d) Both functions have the same maximum value.

Question 22 P4

Identify the fourth term in the expansion of $(x + y)^5$.

a) $10x^4y$

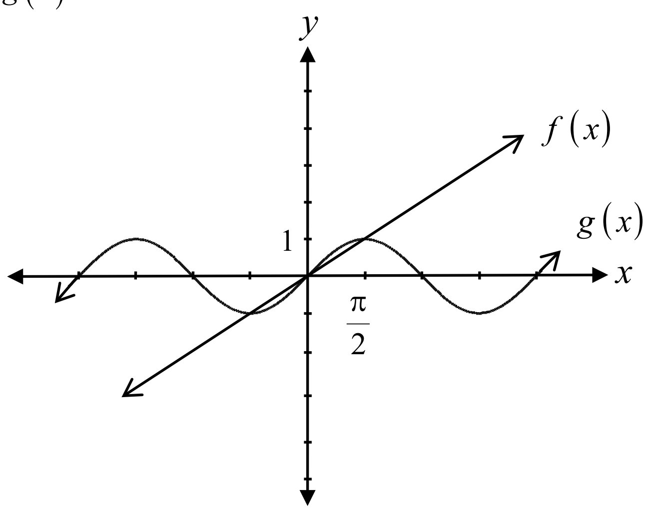
b) $10x^3y^2$

c) $10x^2y^3$

d) $10xy^4$

Question 23 T5

Given the graphs of $f(x)$ and $g(x)$, identify the choice with all of the solutions of the equation $f(x) = g(x)$.



a) $x = -2\pi, -\pi, 0, \pi, 2\pi$

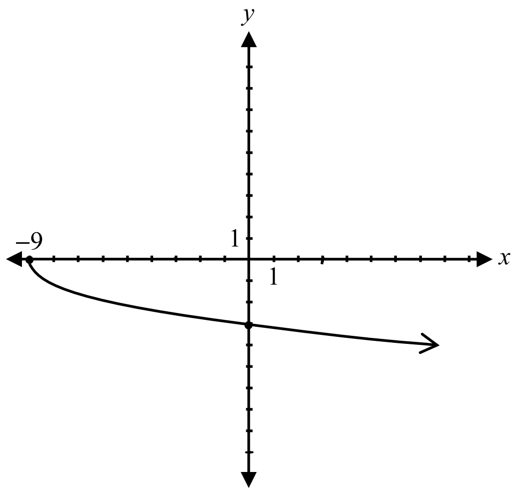
b) $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$

c) $x = \frac{\pi}{2}$

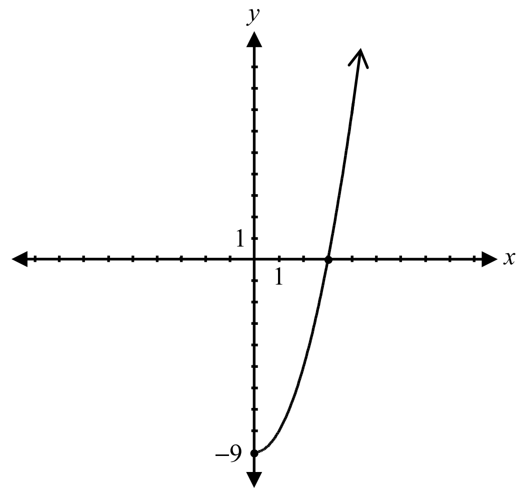
d) $x = -1, 0, 1$

Identify the graph of $f^{-1}(x)$ if $f(x) = x^2 - 9, x \geq 0$.

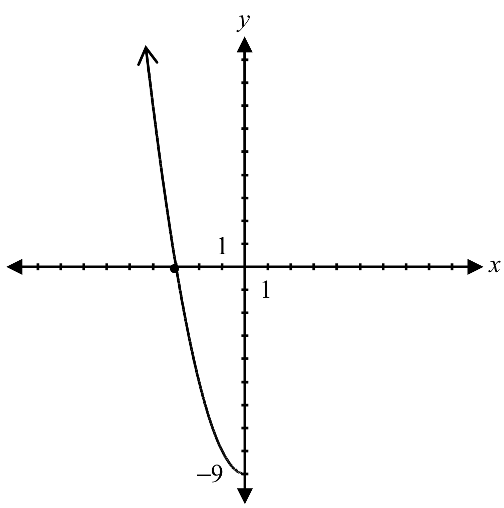
a)



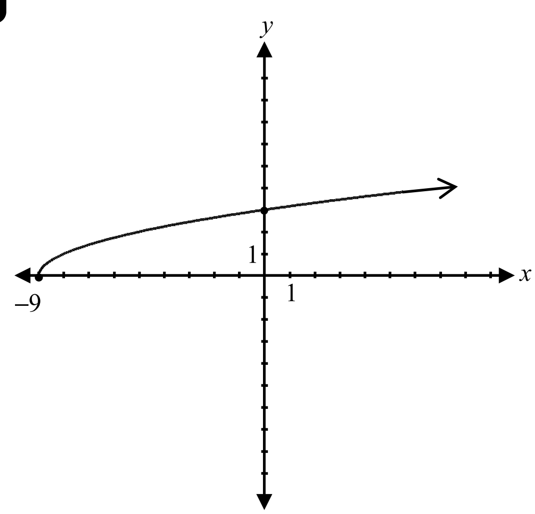
b)



c)



d)



Question 25 R11

Using the remainder theorem, identify which value of x results in a remainder of zero given $p(x) = x^3 + 7x^2 + 14x + 8$.

a) 1

b) 0

c) -1

d) -3

Question 26 T3

Evaluate $\cos\left(\cos\left(\frac{3\pi}{2}\right)\right)$.

a) 1

b) $\frac{1}{2}$

c) 0

d) -1

Match the following equations with their graphs:

Solution

Place the appropriate letter in this column.

$f(x) = (x-1)^3(x+1)(x-3)$ C

$g(x) = (x+1)^2(x-1)(x+3)$ B

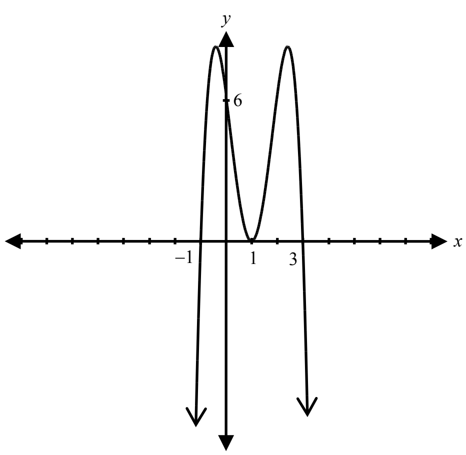
$h(x) = -2(x-1)^2(x+1)(x-3)$ A

$k(x) = 2(x+1)^2(x-1)(x+3)$ D

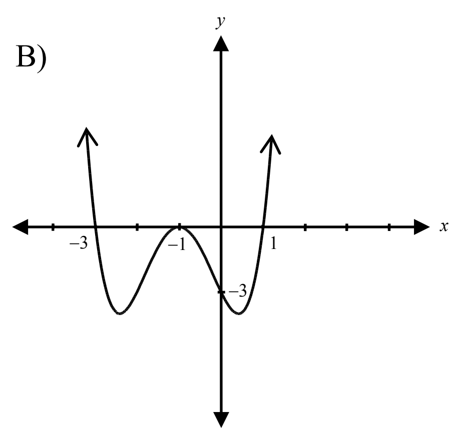
½ mark for each correct answer

2 marks

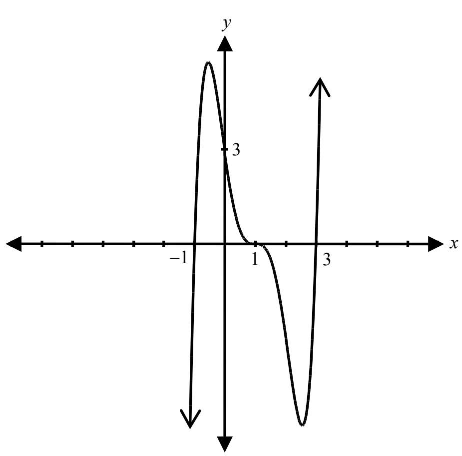
A)



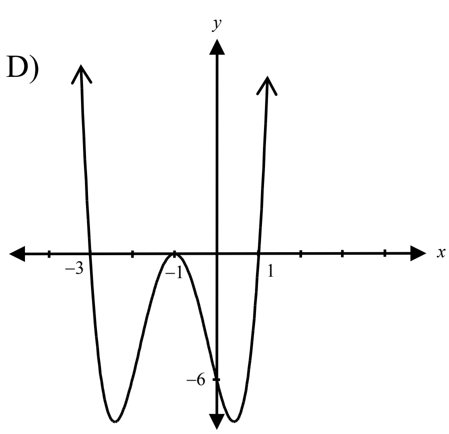
B)



C)



D)



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Question 28 R8

If $\log 6 = p$, $\log 5 = r$ and $\log 2 = q$, express $\log 60$ in terms of p, q and r.

Solution

$$\begin{aligned} \log 60 &= \log(6 \bullet 5 \bullet 2) \\ &= \log 6 + \log 5 + \log 2 \\ &= p + r + q \end{aligned}$$

½ mark for combination

1 mark for product law

½ mark for substitution

2 marks

Exemplar 1

$$\log 60 = \log 6 \cdot \log 5 \cdot \log 2$$

$$= p \cdot r \cdot q$$

$$= prq$$

½ out of 2

+ ½ mark for substitution

Exemplar 2

$$\log 60 = \log 6 + \log 5 + \log 2$$

$$= p + r + q$$

$$\log 60 = p \cdot r \cdot q$$

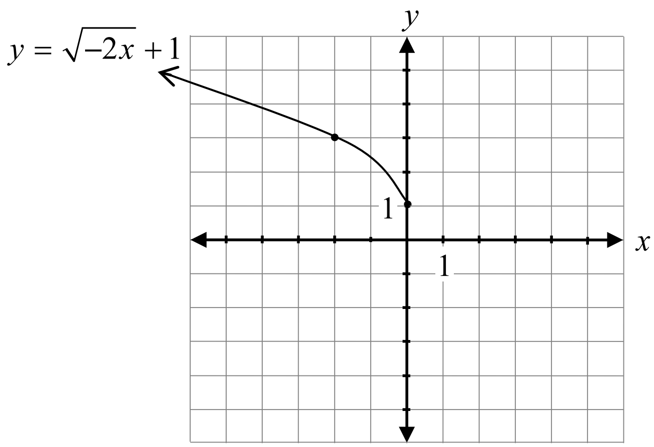
1 out of 2

award full marks
– 1 mark for concept error in line 3

Sketch the graph of $y = \sqrt{-2x} + 1$.

Solution

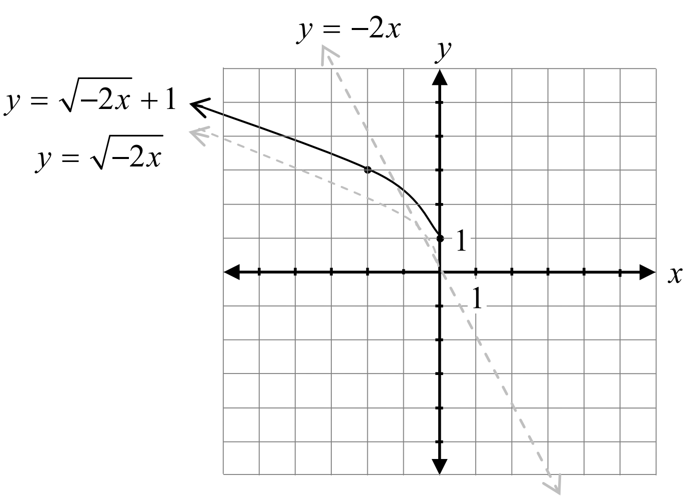
Method 1



1 mark for shape of a radical function
1 mark for horizontal compression
1 mark for vertical translation
1 mark for horizontal reflection

4 marks

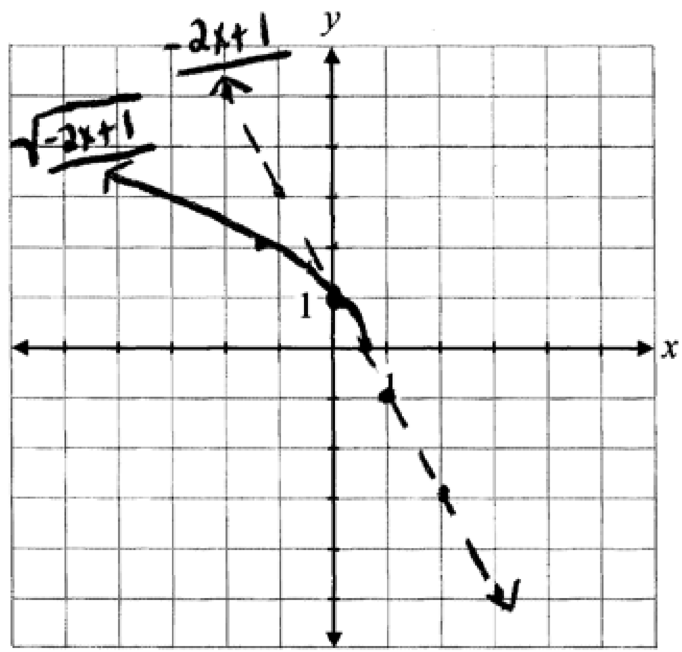
Method 2



½ mark for shape between invariant points
½ mark for shape to the left of the invariant points
1 mark for invariant points where
$y = 0$ and $y = 1$ (½ mark for each point)
1 mark for domain of $(-\infty, 0]$
1 mark for vertical translation

4 marks

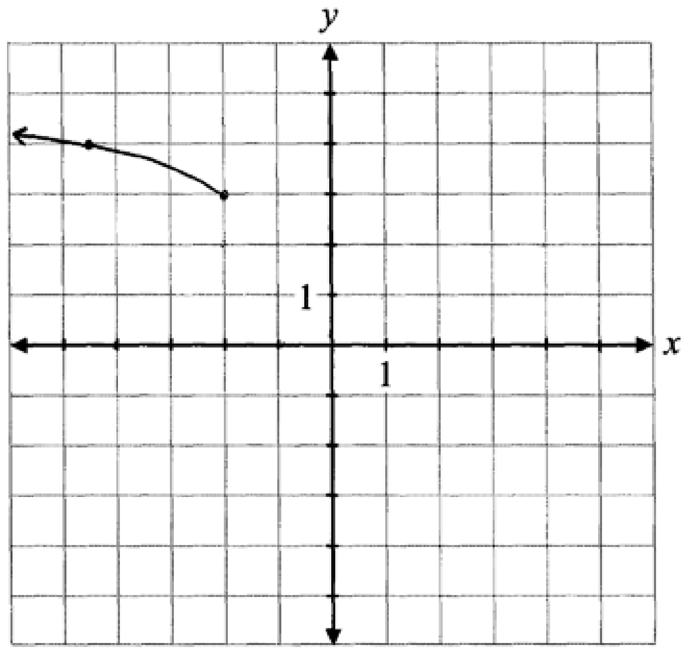
Exemplar 1



3 out of 4

Method 2
+ ½ mark for shape between invariant points
+ ½ mark for shape to the left of the invariant points
+ 1 mark for invariant points where $y = 0$ and $y = 1$
+ 1 mark for domain (consistent with graph)

Exemplar 2



x	y
-4.5	4
-4	
-3	
-2	3
-1	
0	
1	
2	

3 out of 4

award full marks
– 1 mark for concept error (incomplete domain)

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Question 30 P2

Justify why the letters of the word FRANCE have a greater number of possible arrangements than the letters of the word CANADA.

Solution

CANADA has repeated letters that, when placed in different orders, do not change the arrangements of the letters.

FRANCE: $6!$ CANADA: $\dfrac{6!}{3!}$

1 mark

$\therefore$ France has a greater number of arrangements.

Exemplar 1

Because the word Canada has repetition of letters

France is 6!

Canada is 4!

0 out of 1

award full marks

– 1 mark for concept error (incorrect permutations with repeated letters)

Exemplar 2

France = 6!

Canada = 6! ÷ 3!

1 out of 1

Exemplar 3

France has no repeated letters

Canada has 3 repeated letters

1 out of 1

Question 31 R1, R14

Given $f(x) = \dfrac{1}{x-2}$ and $g(x) = x + 5$,

a) determine the equation for $f(g(x))$.

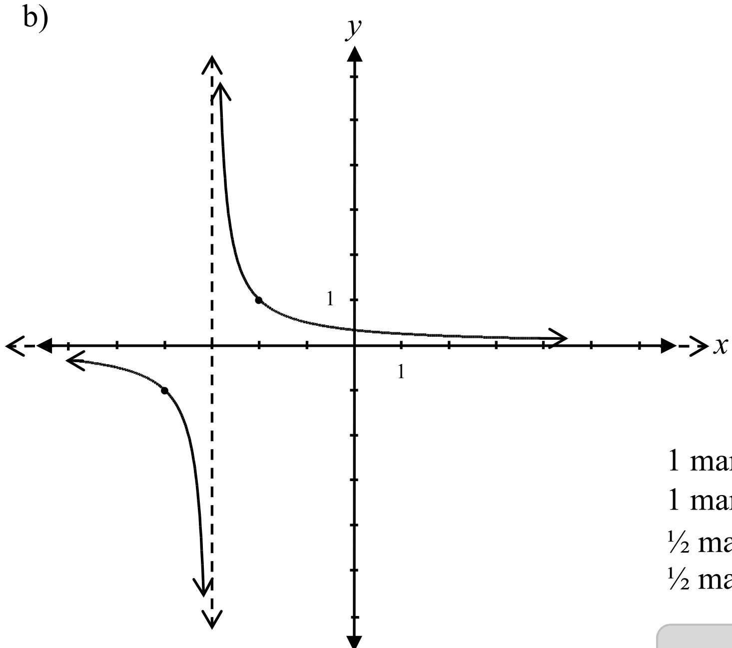
b) sketch the graph of $f(g(x))$.

Solution

a) $$f(g(x)) = \frac{1}{(x+5)-2} = \frac{1}{x+3}$$

1 mark

b)



1 mark for asymptotic behaviour approaching $x = -3$
1 mark for asymptotic behaviour approaching $y = 0$
½ mark for branch left of vertical asymptote
½ mark for branch right of vertical asymptote

3 marks

Exemplar 1

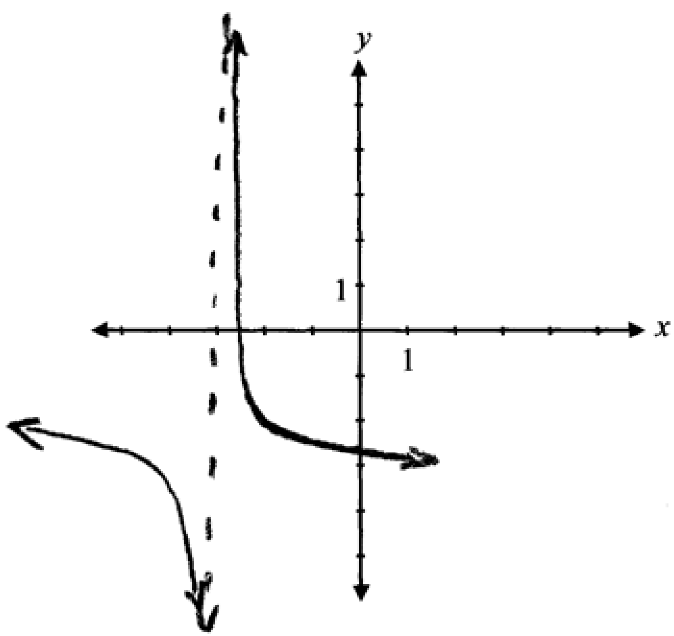
a)

$$\frac{1}{x+5-2}$$

$$f(g(x)) = \frac{1}{x+3}$$

1 out of 1

b)



1 out of 3

+ 1 mark for asymptotic behaviour approaching $x = -3$

Exemplar 2

a)

$g(x) = x+5$

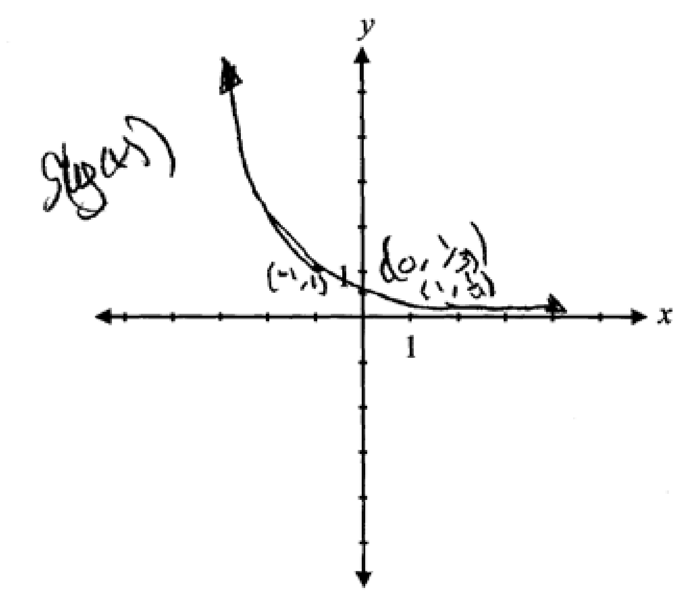
$f(g(x+5)) = \frac{1}{(x+5)-2}$

$\boxed{f(g(x)) = \frac{1}{x+3}}$

1 out of 1

award full marks
E7 (notation error in line 2)

b)



2 out of 3

+ 1 mark for asymptotic behaviour approaching $x = -3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
E10 (asymptotes omitted but still implied)

Exemplar 3

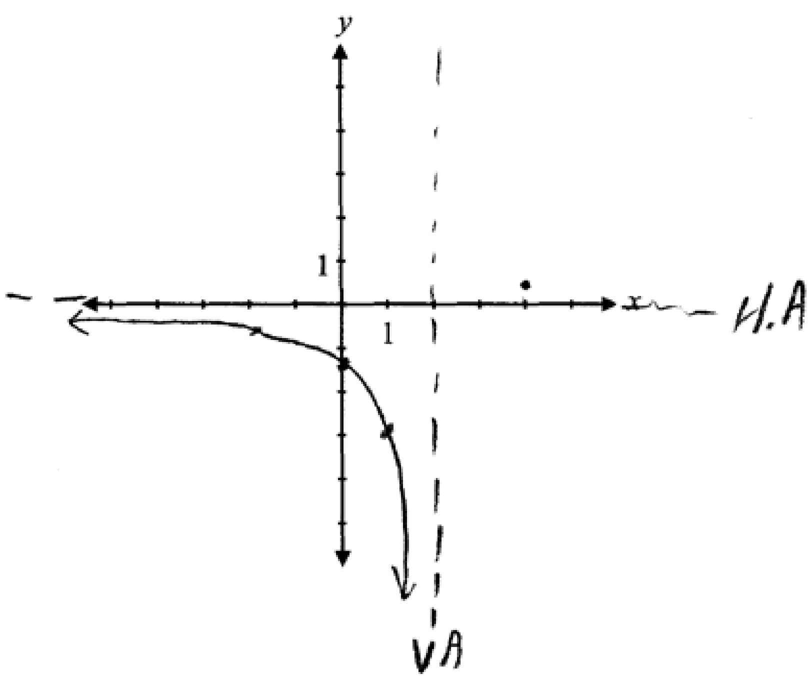
a)

$$\frac{1}{x-2} \cdot x+5$$

$$f(g(x)) = \frac{x+5}{x-2}$$

0 out of 1

b)



x	y
1	$\frac{6}{-1}$ - $\frac{3}{1}$
2	$\frac{7}{0}$ VA
3	$\frac{8}{1}$
0	$\frac{5}{-2}$
4	$\frac{1}{2}$
5	$\frac{10}{3}$
-2	$\frac{-3}{4}$

1 out of 3

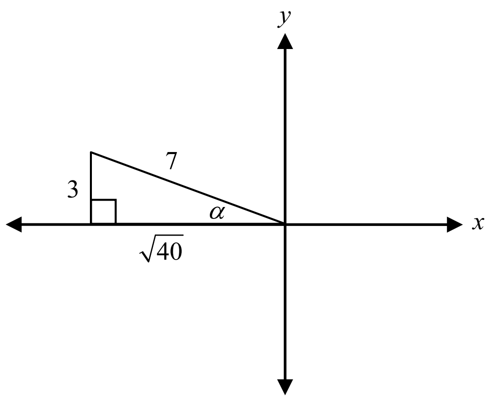
+ 1 mark for asymptotic behaviour approaching $x = 2$ (consistent with the equation in a))

Question 32 T6

Given that $\sin\alpha = \frac{3}{7}$, where α is in Quadrant II, and $\cos\beta = \frac{4}{5}$, where β is in Quadrant IV, determine the exact value of:

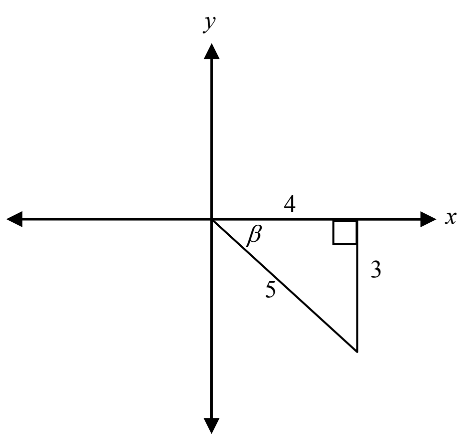
a) $\sin(\alpha - \beta)$

Solution



$$x^2 + y^2 = r^2$$
$$x^2 + 9 = 49$$
$$x^2 = 40$$
$$x = \pm\sqrt{40}$$

½ mark for value of x



$$x^2 + y^2 = r^2$$
$$16 + y^2 = 25$$
$$y^2 = 9$$
$$y = \pm 3$$

½ mark for value of y

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \left(\frac{3}{7}\right)\left(\frac{4}{5}\right) - \left(\frac{-\sqrt{40}}{7}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{12}{35} - \frac{3\sqrt{40}}{35}$$
$$= \frac{12 - 3\sqrt{40}}{35} \text{ or } \frac{12 - 6\sqrt{10}}{35}$$

½ mark for $\cos\alpha$
½ mark for $\sin\beta$
1 mark for substitution into correct identity

3 marks

Note(s):

- accept any of the following values for x: $x = \pm\sqrt{40}$, $x = -\sqrt{40}$ or $x = \sqrt{40}$
- accept any of the following values for y: $y = \pm 3$, $y = -3$ or $y = 3$

b) $\cos 2\alpha$

Solution

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$
$$= \left(\frac{-\sqrt{40}}{7}\right)^2 - \left(\frac{3}{7}\right)^2$$
$$= \frac{40}{49} - \frac{9}{49}$$
$$= \frac{31}{49}$$

or

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$
$$= 2\left(\frac{-\sqrt{40}}{7}\right)^2 - 1$$
$$= 2\left(\frac{40}{49}\right) - 1$$
$$= \frac{80}{49} - 1$$
$$= \frac{31}{49}$$

or

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$
$$= 1 - 2\left(\frac{3}{7}\right)^2$$

1 mark for substitution of an appropriate identity

1 mark

$$= 1 - 2\left(\frac{9}{49}\right)$$
$$= 1 - \frac{18}{49}$$
$$= \frac{31}{49}$$

Exemplar 1

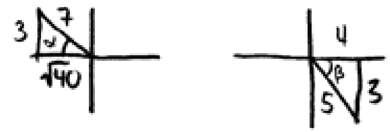
a)

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\left(\frac{3}{7}\right)\left(\frac{4}{5}\right) - \left(\frac{\sqrt{40}}{7}\right)\left(\frac{3}{5}\right)$

$\frac{12}{35} - \frac{3\sqrt{40}}{35}$

$\boxed{\frac{12 - 3\sqrt{40}}{35}}$



$3^2 + b^2 = 7^2$

$9 + b^2 = 49$

$b = \sqrt{40}$

SOH CAH TOA

2 out of 3

+ ½ mark for value of x
+ ½ mark for value of y
+ 1 mark for substitution into correct identity

b)

$= 1 - 2\sin^2 a$

$= 1 - 2\left(\frac{3}{7}\right)^2$

$= 1 - 2\left(\frac{9}{49}\right)$

$= 1 - \frac{18}{98}$

$= \frac{98}{98} - \frac{18}{98}$

$= \frac{80}{98}$

$= \frac{40}{49}$

½ out of 1

award full marks
– ½ mark for arithmetic error in line 4

Exemplar 2

a)

$$\sin(\alpha-\beta) = \overset{\left(\frac{3}{7}\right)}{\sin\alpha}\,\overset{\left(\frac{4}{5}\right)}{\cos\beta} - \overset{\left(\frac{\sqrt{40}}{7}\right)}{\cos\alpha}\,\overset{\left(\frac{3}{5}\right)}{\sin\beta}$$

$\underline{\cos\alpha}$

$a^2+b^2=c^2$

$3^2+b^2=7^2$

$b^2=49-9$

$\sqrt{b^2}=\sqrt{40}$

$b=\sqrt{40}$

$\left[\cos\alpha=-\frac{\sqrt{40}}{7}\right]$

$\underline{\sin\beta}$

$a^2+b^2=c^2$

$4^2+b^2=5^2$

$b^2=25-16$

$\sqrt{b^2}=\sqrt{9}$

$b=3$

$\left[\sin\beta=\frac{3}{5}\right]$

$$\sin(\alpha-\beta)=\left(\frac{3}{7}\right)\left(\frac{4}{5}\right)-\left(\frac{\sqrt{40}}{7}\right)\left(\frac{3}{5}\right)$$

$$=\left(\frac{15\cdot 28}{35}\right)-\left(\frac{5\sqrt{40}\cdot 21}{35}\right)$$

---

2 out of 3

+ ½ mark for value of x
+ ½ mark for value of y
+ ½ mark for $\cos\alpha$
+ 1 mark for substitution into correct identity
– ½ mark for arithmetic error in line 9
E1 (final answer not stated)
E7 (transcription error in line 8)

---

b)

$$\cos 2a = \cos^2\alpha - \sin^2\alpha$$

$$=\left(\frac{\sqrt{40}}{7}\right)^2-\left(\frac{3}{7}\right)^2$$

$$=\left(\frac{40}{49}\right)-\left(\frac{9}{49}\right)$$

$$\cos 2\alpha = \left(\frac{31}{49}\right)$$

---

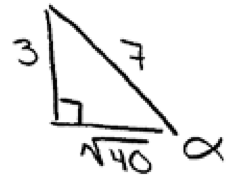
1 out of 1

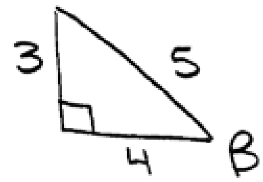
E7 (transcription error in line 2)

Exemplar 3

a)

$$\sin\left(\frac{3}{7}\right)\cos\left(\frac{4}{5}\right) - \cos\left(\frac{\sqrt{40}}{7}\right)\sin\left(\frac{3}{5}\right)$$





1½ out of 3

+ ½ mark for value of x
+ ½ mark for value of y
+ ½ mark for $\cos\alpha$

b)

$$1 - 2\sin^2\left(\frac{3}{7}\right)$$

0 out of 1

Exemplar 4

a)

$$\sin\left(\frac{3}{7}\right)\cos\left(\frac{4}{5}\right) - \cos\left(-\frac{\sqrt{40}}{7}\right)\sin\left(-\frac{3}{5}\right)$$

$$\left(\frac{3}{7}\right)\left(\frac{4}{5}\right) - \left(-\frac{\sqrt{40}}{7}\right)\left(-\frac{3}{5}\right)$$

$$\frac{12}{35} - \frac{3\sqrt{40}}{35}$$

$$\frac{12-3\sqrt{40}}{35}$$

2½ out of 3

award full marks

– ½ mark for procedural error in line 1

b)

$$1-2\sin^2\left(\frac{3}{7}\right)$$

$$1-2\left(\frac{3}{7}\right)^2$$

$$1-2\left(\frac{9}{49}\right)$$

$$1-\frac{18}{49}$$

1 out of 1

award full marks (consistent with procedural error in a))

E1 (final answer not stated)

Question 33 R13

Determine the domain and range of $f\left(x\right) = \sqrt{x-5} - 1$.

Solution

Domain: $\left\{x \in \mathbb{R} \mid x \geq 5\right\}$ **or** $\left[5, \infty\right)$ 1 mark for domain

Range: $\left\{y \in \mathbb{R} \mid y \geq -1\right\}$ **or** $\left[-1, \infty\right)$ 1 mark for range

2 marks

Exemplar 1

Domain: $(5, \infty)$

Range: $(-1, \infty)$

2 out of 2

award full marks
E8 (bracket error made when stating domain or range)

Exemplar 2

Domain: $x > 5$

Range: $y \in R$

1 out of 2

+ 1 mark for domain
E8 (bracket error made when stating domain)

Exemplar 3

Domain: $(\infty, -5]$

Range: $(\infty, -1]$

1 out of 2

+ 1 mark for range
E8 (range written in incorrect order)

Question 34 R7

Justify why 4.7 is a better estimate than 4.3 for the value of $\log_2 26$.

Solution

$2^4 = 16 \qquad 2^5 = 32$

or

$\log_2 16 = 4 \qquad \log_2 32 = 5$

26 is closer to 32 than 16; therefore $\log_2 26$ is closer to 5 than 4. **1 mark**

Exemplar 1

because the answer to $\log_2 26$ is going to be closer to 5 than it will be to 4

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 2

$\log_2 26$

$4^2 = 16$

$5^2 = 25$

4,7 is closer to 25 than 4,3.

0 out of 1

Exemplar 3

4.7 is a closer and more accurate number value for $\log_2 26$.

$\log_2 26$ is closer to 2^5 than 2^4

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 4

$2^4 = 16$ $2^{4.3} = 22$

$2^{4.5} = 24$
$2^{4.7} = 26$

$2^5 = 32$

1 out of 1

award full marks
E7 (notation error of using "=" instead of "≈")

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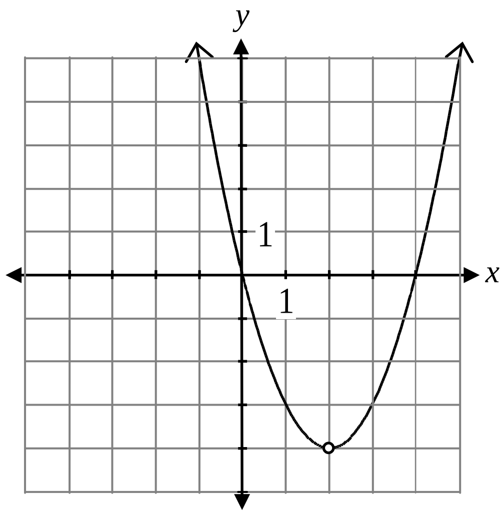
Question 35 R14

Sketch the graph of the function:

$$f(x) = \frac{x(x-2)(x-4)}{(x-2)}$$

Solution



1 mark for point of discontinuity (hole) at $(2, -4)$
(½ mark for x value, ½ mark for y value)

½ mark for shape of a parabola

½ mark for end behaviour

2 marks

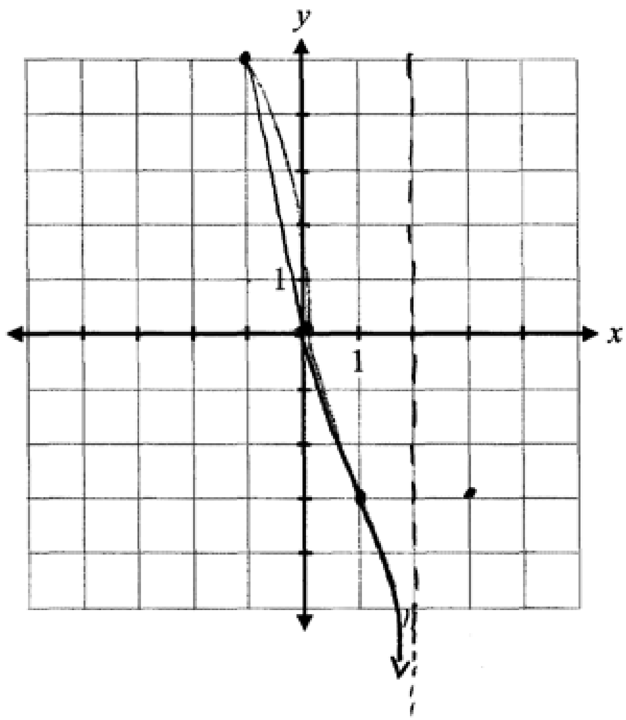
Exemplar 1

- - - : asymptote

1	-3
3	
-1	

$\frac{-1(-3)(-5)}{-3}$

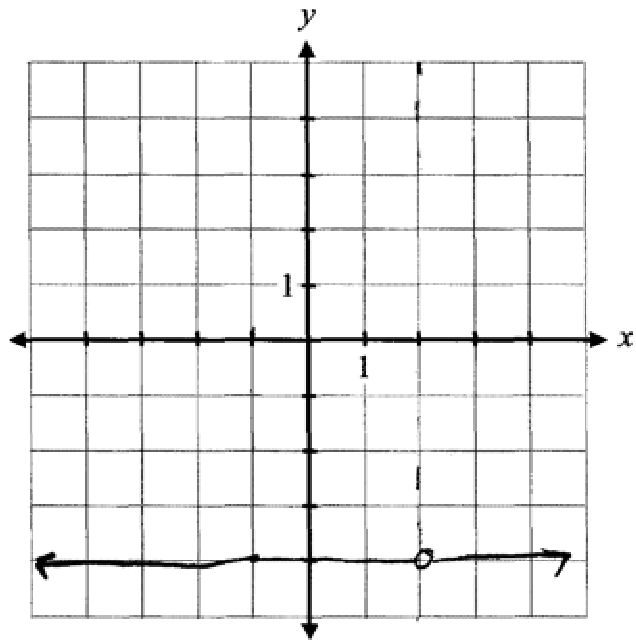
$\frac{-15}{-3}$



0 out of 2

Exemplar 2

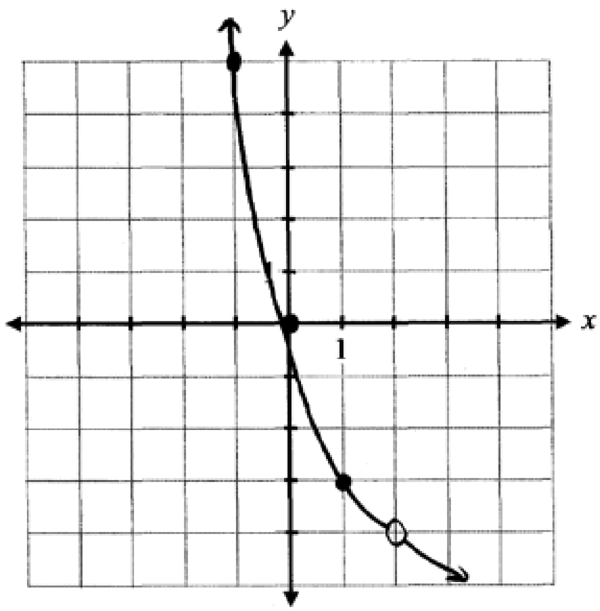
$x(x-4)$



1 out of 2

+ 1 mark for point of discontinuity (hole) at $(2, -4)$

Exemplar 3



$x^2 - 4x$

$4 - 8 = -4$

$16+$

$x^2 - 4x$

$2^2 - 4(2)$

$4 - 8 = -4$

1. $1 - 4$

0.

$-1.$ $1 + 4 = 5$

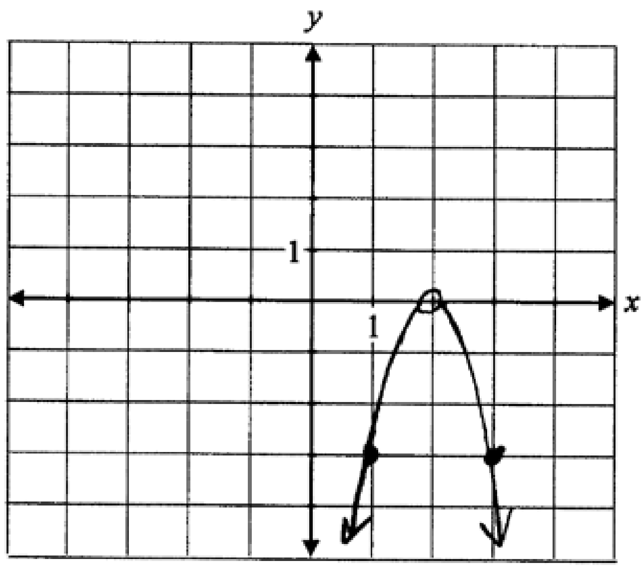
$-2.$ $4 + 8 = 12$

2. $4 - 8 = -4$

1 out of 2

+ 1 mark for point of discontinuity (hole) at $(2, -4)$

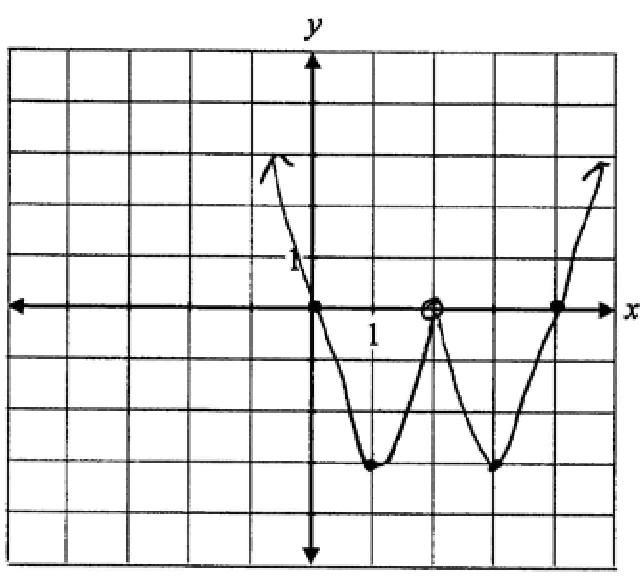
Exemplar 4



1 out of 2

+ ½ mark for point of discontinuity (hole) at $x = 2$
+ ½ mark for shape of a parabola

Exemplar 5



1 out of 2

+ ½ mark for point of discontinuity (hole) at $x = 2$
+ ½ mark for end behaviour

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Question 36 T3

Evaluate:

$$\sec^2\left(\frac{\pi}{6}\right)+\tan\left(\frac{7\pi}{6}\right)\csc\left(-\frac{2\pi}{3}\right)$$

Solution

$$\left(\frac{2}{\sqrt{3}}\right)^2+\left(\frac{\sqrt{3}}{3}\right)\left(-\frac{2}{\sqrt{3}}\right)$$

$$\left(\frac{4}{3}\right)-\left(\frac{2}{3}\right)$$

$$\frac{2}{3}$$

1 mark for $\sec\left(\frac{\pi}{6}\right)$ (½ mark for value, ½ mark for quadrant)

1 mark for $\tan\left(\frac{7\pi}{6}\right)$ (½ mark for value, ½ mark for quadrant)

1 mark for $\csc\left(-\frac{2\pi}{3}\right)$ (½ mark for value, ½ mark quadrant)

3 marks

Exemplar 1

$$= \left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{4}{3} + \frac{2}{3\sqrt{3}}$$

$$= \frac{6}{3\sqrt{3}}$$

$$\boxed{= 2\sqrt{3}}$$

1½ out of 3

+ 1 mark for $\sec\left(\frac{\pi}{6}\right)$

+ ½ mark for quadrant of $\tan\left(\frac{7\pi}{6}\right)$

+ ½ mark for value of $\csc\left(-\frac{2\pi}{3}\right)$

– ½ mark for arithmetic errors in lines 3 and 4

Exemplar 2

$$\left(\frac{2}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)\left(-\frac{2}{\sqrt{3}}\right)$$

$$-\frac{2}{3}\pi + \frac{6\pi}{3} = \frac{4\pi}{3}$$

$$\left(\frac{4}{3}\right) + \left(\frac{2}{3}\right)$$

$$\frac{6}{3}$$

$$\boxed{2}$$

2½ out of 3

+ 1 mark for $\sec\left(\frac{\pi}{6}\right)$

+ ½ mark for value of $\tan\left(\frac{7\pi}{6}\right)$

+ 1 mark for $\csc\left(-\frac{2\pi}{3}\right)$

Exemplar 3

$$\frac{1}{\frac{\sqrt{3}}{2}} + \frac{\sqrt{3}}{3} \times \left(-\frac{2\sqrt{3}}{3}\right)$$

$$\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\left(-\frac{2\sqrt{3}}{3}\right)$$

$$\frac{2\sqrt{3}}{3} + \left(\frac{-6}{9}\right) \rightarrow \frac{2\sqrt{3}}{3} + \frac{-2}{3}$$

$$\frac{2\sqrt{3}-2}{3}$$

$$\sec^2\left(\pi/6\right) + \tan\left(\frac{7\pi}{6}\right)\csc\left(-\frac{2\pi}{3}\right) = \boxed{\frac{2\sqrt{3}-2}{3}}$$

2½ out of 3

award full marks
– ½ mark for procedural error (not squaring the value of secant)

Question 37 R5

The graph of $f(x) = 3x + 7$ is reflected over the y-axis.

Determine the equation of the new function.

Solution

$y = -3x + 7$

or

$y = f(-x)$

1 mark

Exemplar 1

$y =$ $3f(-x) + 7$

½ out of 1

award full marks
–½ mark for procedural error

Exemplar 2

$y =$ $-(3x+7)$

0 out of 1

Question 38 R14

Determine the equations of all of the asymptotes of the function:

$$y = \frac{2x+1}{x-3}$$

Solution

Horizontal asymptote at $y = 2$	1 mark for horizontal asymptote
Vertical asymptote at $x = 3$	1 mark for vertical asymptote

2 marks

Exemplar 1

horizontal asymptote : $y \neq 2$

vertical asymptote : $x \neq 3$

2 out of 2

award full marks
E7 (notation error)

Exemplar 2

HA = 2
VA = 3

1 out of 2

award full marks
– 1 mark for concept error (not stating variable in equation)

Exemplar 3

$x = 2$
$y = 3$

1 out of 2

award full marks
– 1 mark for concept error (interchanging asymptotes)

Question 39 — R11

One of the zeros of $p(x) = x^3 + 6x^2 - 32$ is $x = 2$. Determine all of the other zeros of $p(x)$.

Solution

$$\begin{array}{c|cccc} 2 & 1 & 6 & 0 & -32 \\ & \downarrow & 2 & 16 & 32 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

1 mark for synthetic division (or any equivalent strategy)

$0 = x^2 + 8x + 16$

½ mark for the other factors

$0 = (x + 4)(x + 4)$

$x = -4$

½ mark for the other zeros

2 marks

Exemplar 1

$$\begin{array}{r|rrrr} 2 & 1 & 6 & 0 & -32 \\ & & 2 & 16 & 32 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

$P(2) = (2)^3 + 6(2)^2 - 32 = 0$

$8 + 24 - 32 = 0$

$0 = 0$

$x^2 + 8x + 16$

$(x+4)(x+4)$

$x = -4$

2 out of 2

award full marks
E2 (changing an expression to an equation in line 4)

Exemplar 2

$$\begin{array}{r|rrrr} 2 & 1 & +6 & +0 & -32 \\ & & 4 & 20 & 40 \\ \hline & 2 & 10 & 20 & +8 \end{array}$$

½ out of 2

+ 1 mark for synthetic division
– ½ mark for procedural error

Exemplar 3

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 0 & -32 \\ & & -2 & -16 & -32 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

$(x-2)(x^2+8x+16) = P(x)$

$(x-2)(x-4)(x-4) = P(x)$

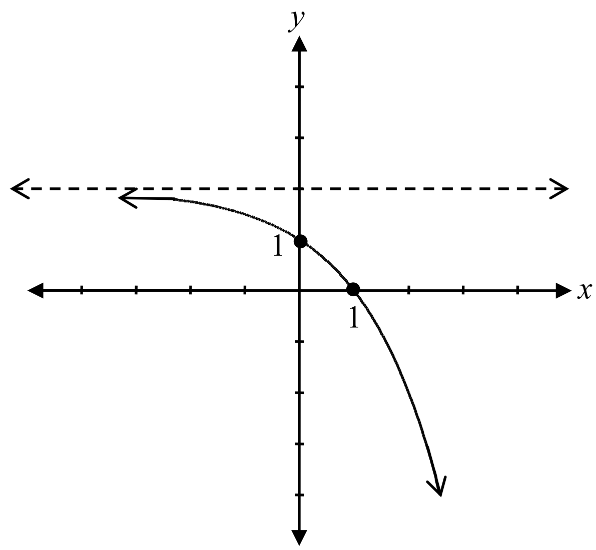
$x = 4$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 2
E7 (notation error in line 2, did not show the equation equal to zero before solving)

Sketch the graph of $y = -2^x + 2$.

Solution



1 mark for shape of an exponential function
1 mark for vertical reflection
1 mark for asymptotic behaviour approaching $y = 2$

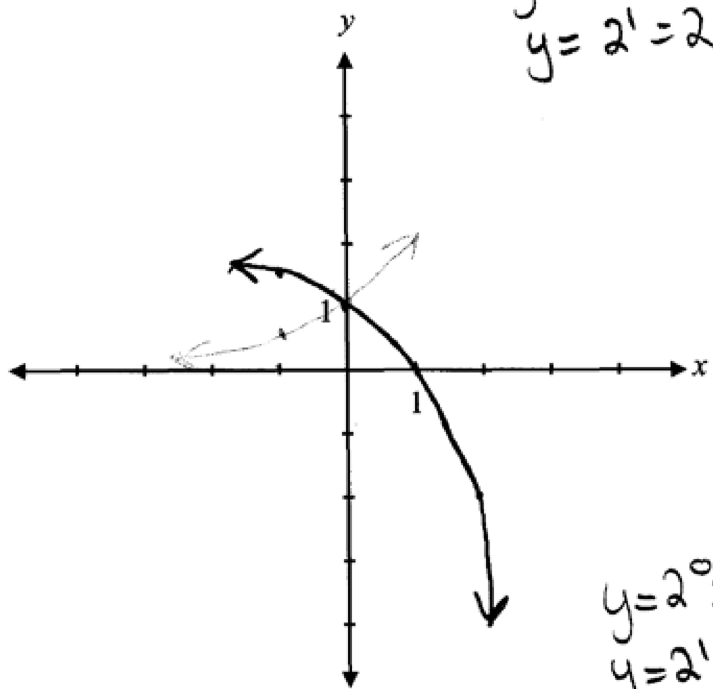
3 marks

Exemplar 1

$y = 2^{-1} = \frac{1}{2}$ $\quad (-1, \frac{1}{2}) \rightarrow (-1, 1.5)$

$y = 2^{0} = 1$ $\quad (0, 1) \rightarrow (0, 1)$

$y = 2^{1} = 2$ $\quad (1, 2) \rightarrow (1, 0)$



$(x, y) \rightarrow (x, -y+2)$

$y = 2^{0} = 1$ $\quad (0, 1) \rightarrow (0, 1)$

$y = 2^{1} = 2$ $\quad (1, 2) \rightarrow (1, 0)$

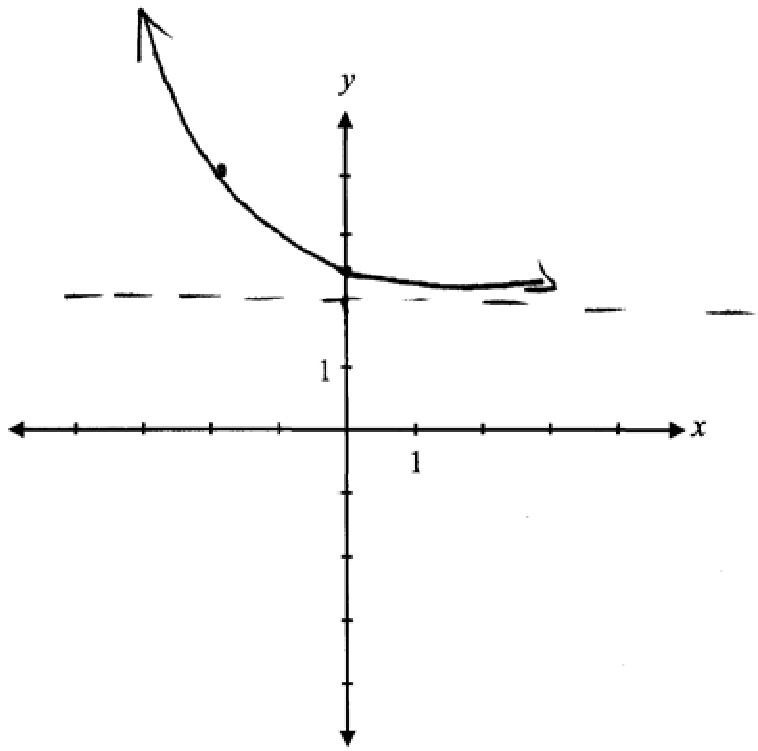
$y = 2^{2} = 4$ $\quad (2, 4) \rightarrow (2, -2)$

---

3 out of 3

award full marks
E10 (asymptote omitted but still implied)

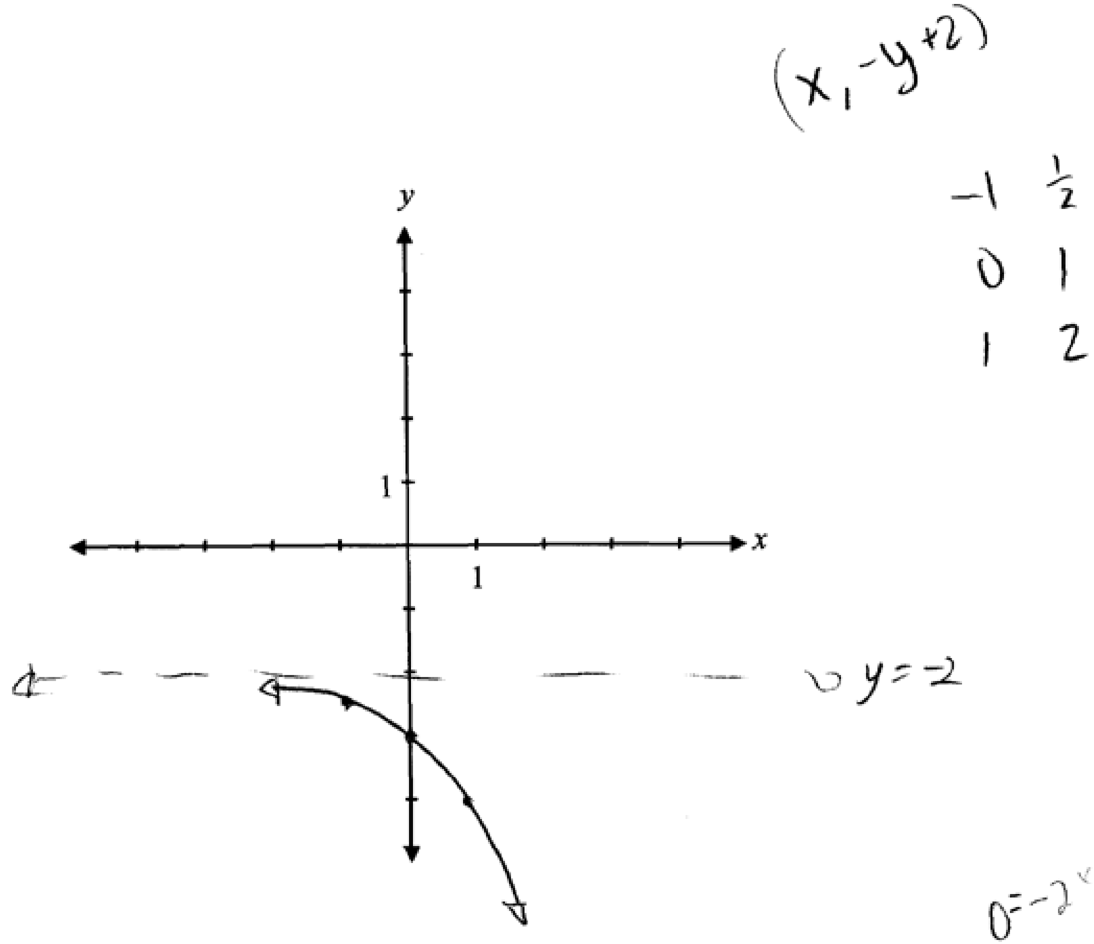
Exemplar 2



2 out of 3

+ 1 mark for shape of an exponential function
+ 1 mark for asymptotic behaviour approaching $y = 2$

Exemplar 3



2 out of 3

+ 1 mark for shape of an exponential function
+ 1 mark for vertical reflection

Question 41 R1

Given the function $f(x) = \frac{2}{x} - 1$, justify why $f(f(2))$ is undefined.

Solution

$$\begin{aligned} f(2) &= \frac{2}{2} - 1 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

$f(f(2)) = \frac{2}{0} - 1$, which is undefined because the denominator cannot be zero.

1 mark for justification

1 mark

Exemplar 1

$f(2) = \frac{2}{2} - 1$

$f(2) = 1 - 1$

$f(2) = 0$

$\boxed{\text{∴ Not it's not defined because if you plug 2 into x then it becomes 1 } (\frac{2}{2} = 1) \text{ and then you do 1 - 1 which is equal to 0, so not defined.}}$

0 out of 1

Exemplar 2

$\frac{2}{2} - 1$

0

$\frac{2}{0} - 1$

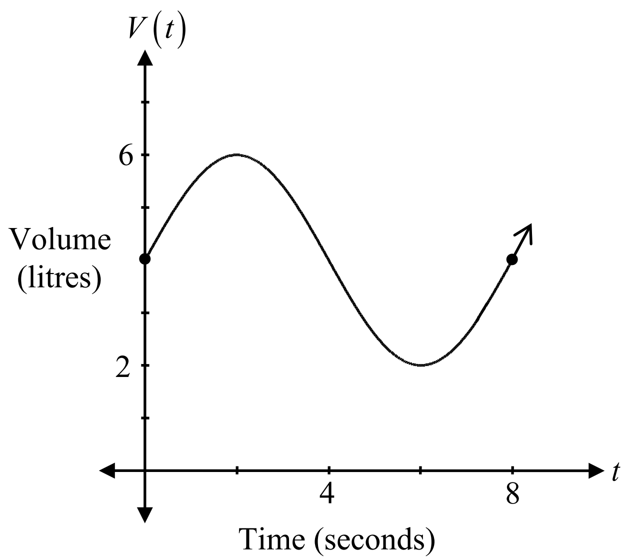
Undefined. 2 is divided by zero which is impossible

1 out of 1

award full marks
E7 (notation error)

Question 42 T4

The following graph represents the volume of air in an adult's lungs. If $V(t)$ is the volume of air in litres and t is the time in seconds, determine an equation that represents this sinusoidal function.



Solution

$$V(t) = 2\sin\left(\frac{\pi}{4}t\right) + 4$$

or

$$V(t) = 2\cos\left[\frac{\pi}{4}(t-2)\right] + 4$$

1 mark for amplitude
½ mark for period
½ mark for consistent value of *b*
1 mark for vertical translation

3 marks

Exemplar 1

$V(t) = 2\sin\left[\frac{\pi}{3}(\theta)\right] + 4$

2 out of 3

+ 1 mark for amplitude
+ 1 mark for vertical translation
E3 (variable introduced without being defined)

Exemplar 2

$V(t) = 2\sin x + 4$

2 out of 3

+ 1 mark for amplitude
+ 1 mark for vertical translation
E3 (variable introduced without being defined)

Exemplar 3

$V(t) = 2\cos\left(\frac{\pi}{4}x\right) + 4$

2 out of 3

award full marks
– 1 mark for concept error (using the cosine function without a horizontal translation)
E3 (variable introduced without being defined)

Question 43 R9

Explain why the domain of $y = \log_2(x-1)$ is $x > 1$.

Solution

The argument of a logarithmic function must be positive.

1 mark

Exemplar 1

$2^y = (x-1)$

If it were to be 1 or less than one you would get $(x-1)$ to be negative or zero. Which would be impossible to hake because $2^y = (x-1)$ and you couldnt get a negetive when you have an unknown exponent (y) over 2.

1 out of 1

Exemplar 2

because you cannot have a negative log or = to zero

½ out of 1

award full marks
– ½ mark for terminology error in explanation (negative logarithm)

Exemplar 3

Because it would normally
be $x>0$, but since there
is an $(x-1)$ it is shifted
to the right once.

1 out of 1

Exemplar 4

because there is a translation
of 1 unit to the right and the
base graph starts at 0.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

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Question 44 T2

The point $(-2, 7)$ is on the terminal arm of an angle in standard position.

Determine the coordinates of the corresponding point, $P(\theta)$, on the unit circle.

Solution

$$x^2 + y^2 = r^2$$

$$(-2)^2 + (7)^2 = r^2$$

½ mark for substitution of $x = \pm 2$ and $y = 7$

$$4 + 49 = r^2$$

$$53 = r^2$$

½ mark for solving for r

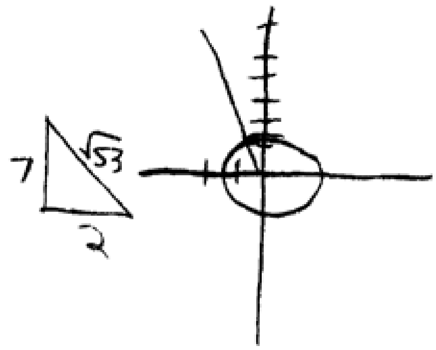
$$\sqrt{53} = r$$

$$P(\theta) = \left(\frac{-2}{\sqrt{53}}, \frac{7}{\sqrt{53}} \right)$$

1 mark for $P(\theta)$ (½ mark for each coordinate)

2 marks

Exemplar 1

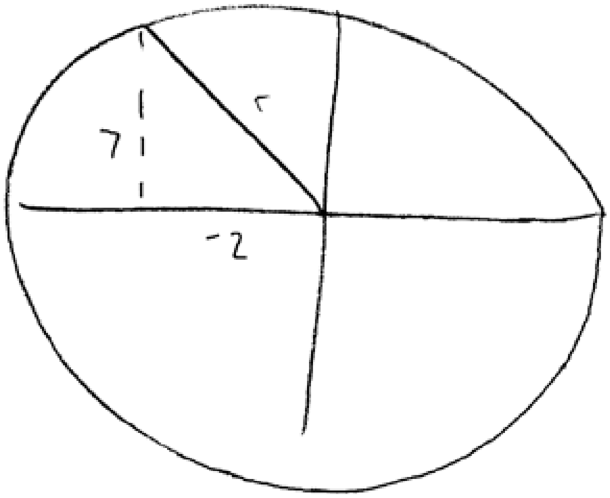


$$\boxed{\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$$

1 out of 2

+ ½ mark for substitution of $x = 2$ and $y = 7$
+ ½ mark for solving for r

Exemplar 2



$-2^2 + 7^2 = r^2$

$4 + 49 = r2$

$53 = r^2$

$\sqrt{53} = r$

$\sin\theta = \frac{7}{\sqrt{53}}$

$\cos\theta = \frac{-2}{\sqrt{53}}$

$P\theta$

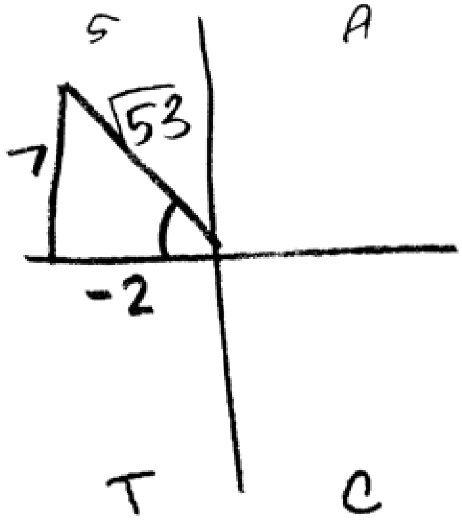
$x = \frac{-2}{\sqrt{53}}$

$y = \frac{7}{\sqrt{53}}$

2 out of 2

award full marks
E4 (missing brackets but still implied in line 1)

Exemplar 3



$$7^2 + 2^2 = c^2$$

$$49 + 4 = c^2$$

$$\sqrt{53} = c^2$$

$$\sqrt{53} = c$$

$$\sin\theta = \frac{7}{\sqrt{53}}$$

$$\cos\theta = \frac{^-2}{\sqrt{53}}$$

$$\tan\theta = \frac{7}{-2}$$

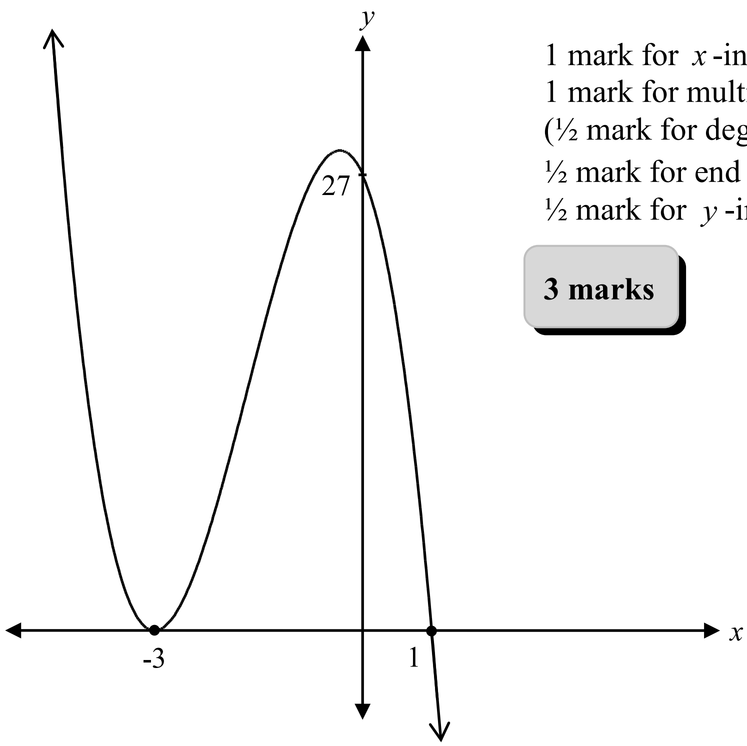
2 out of 2

award full marks
E1 (final answer not stated)

Sketch a graph of $P(x)$ that satisfies all of the following conditions:

- $P(x)$ is a polynomial function of degree 3.
- $P(x)$ has a zero at -3 with a multiplicity of 2.
- $P(x)$ has a zero at 1.
- $P(x)$ has a leading coefficient of -3.

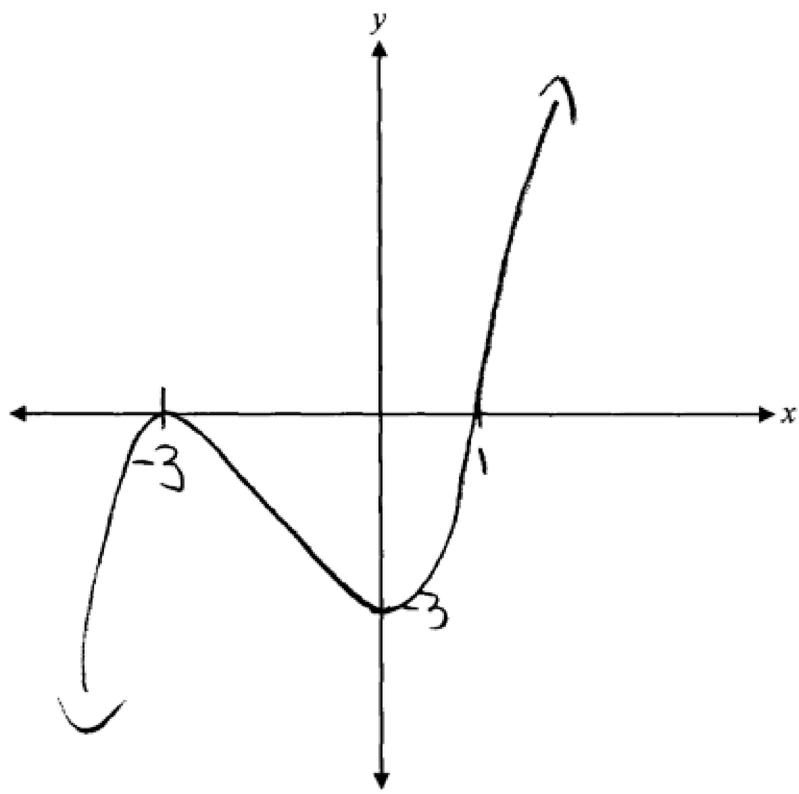
Solution



1 mark for x-intercepts
1 mark for multiplicity
(½ mark for degree 2 at $x = -3$, ½ mark for degree 1 at $x = 1$)
½ mark for end behaviour
½ mark for y-intercept

3 marks

Exemplar 1



2 out of 3

+ 1 mark for x-intercepts
+ 1 mark for multiplicity

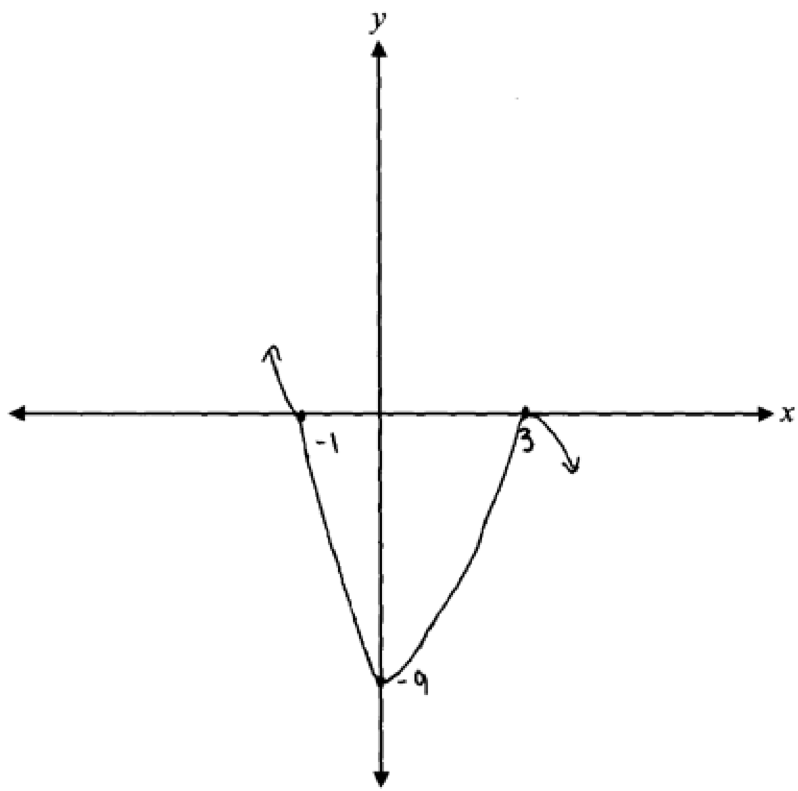
Exemplar 2

$x = -3$

$(3)^2$

$x = -1$

$-3(3)^2(1)^1$



1½ out of 3

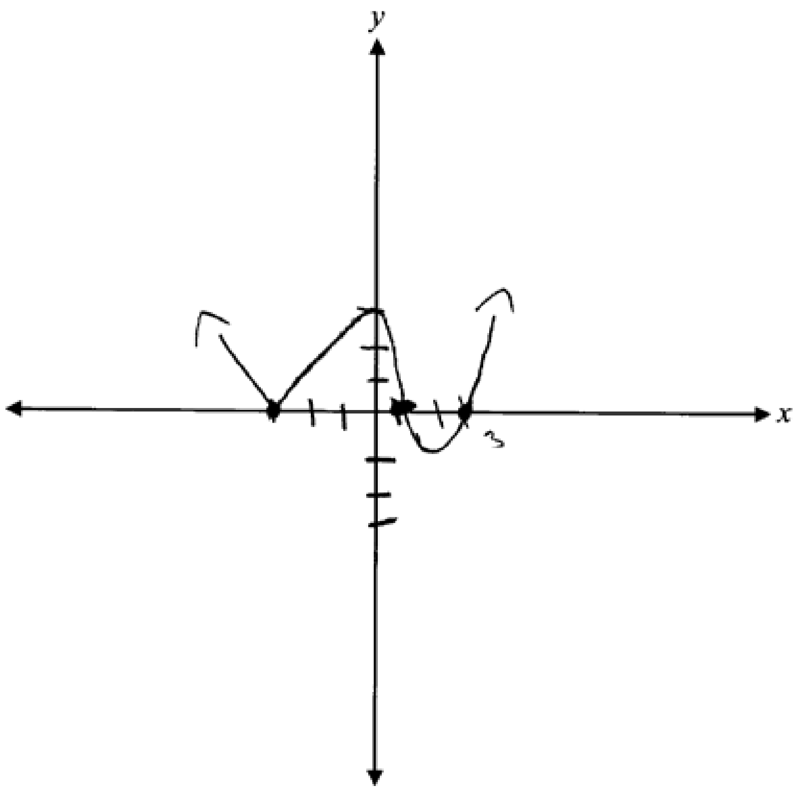
+ 1 mark for multiplicity (consistent with incorrect x-intercepts)
+ ½ mark for end behaviour

Exemplar 3

$\downarrow \quad \uparrow$

$x^3 + 8^2$

$(x-3)(x+3)^2(x-1)$



1 out of 3

+ 1 mark for multiplicity

Question 46 P1

Determine how many 4-digit numbers greater than 4000 can be made using the digits 2, 3, 4, 5, and 6 if repetitions are not allowed.

Solution

$$\underset{4,5,6}{\underline{3}} \bullet \underbrace{\underline{4} \bullet \underline{3} \bullet \underline{2}}_{\text{remaining options}} = 72$$

1 mark for restriction of first digit
1 mark for fundamental counting principle

2 marks

Exemplar 1

$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} = 140$

½ out of 2

+ 1 mark for fundamental counting principle
– ½ mark for arithmetic error

Exemplar 2

$\underset{4\text{-}6}{\underline{3}} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 180$

1½ out of 2

award full marks
– ½ mark for procedural error

Exemplar 3

$\underset{5}{\underline{1}} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} = 24$

$\underset{6}{\underline{1}} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} = 24$

$\therefore 48$

1 out of 2

+ 1 mark for fundamental counting principle

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Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

Question(s) affected: ______________________________

Action taken or rationale for assigning marks: ______________________________

Follow-up: ______________________________

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Decision: ______________________________

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Marker's Signature: ______________________________

Principal's Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
11	R4	2
12	R6	1
14	R4	2
16	R1, R2, R3	3
18	R1	2
24	R6	1
31a)	R1	1
37	R5	1
41	R1	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
2a)	T1	1
2b)	T1	1
7	T1	1
8	T1	1
21	T4	1
26	T3	1
36	T3	3
42	T4	3
44	T2	2
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
1	P3	1
4	P2	2
6	P4	2
15	P3	3
22	P4	1
30	P2	1
46	P1	2
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
17	R12	1
25	R11	1
27	R12	2
39	R11	2
45	R12	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
5	T5	3
9a)	T6	1
9b)	T6	3
19	T6	1
23	T5	1
32a)	T6	3
32b)	T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	3
10	R8	2
13	R10	2
20	R7	1
28	R8	2
34	R7	1
40	R9	3
43	R9	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
29	R13	4
31b)	R14	3
33	R13	2
35	R14	2
38	R14	2