NORTHLANDS PARKWAY
COLLEGIATE

Grade 12 Pre-Calculus

Practice Booklet

Introduction

Welcome to Grade 12 Pre-Calculus at NPC! This practice booklet has been created to help you learn the topics we cover in this course. It is essential that students take time to practice our topics outside of class as we generally have to use much of our class time to get through the material. It would be important to attempt as much of this practice as possible. The more variety of questions you see and experience, the more prepared you will be for assessments, including the provincial exam at the end of the course that makes up 30% of your final mark. If you feel you need more examples or practice, the table of contents includes where you should look in the course textbook (available upon request). Practice in addition to this booklet and what is available in the textbook is available upon request.

The questions that have been compiled for your practice in this booklet are almost entirely former provincial exam questions. Each question will include from which exam it came from and its question number on that exam. This will allow you to go online to Mr. Sumner's website (mrmsumner.weebly.com) and find the answers and mark breakdowns in the marking guides for each exam. The questions in this booklet have also been organized according to the outcome they were designed to test. As material is covered in class, you will be kept up to date as to which outcomes have been covered. Simply have a look at the table of contents to quickly find where you need to look for the relevant practice questions. Also, take note of the number of questions for each outcome. Four previous exams have been used to compile these questions and it will quickly become clear based on how much practice there is as to which outcomes may have a greater emphasis on the exam.

If you ever have any questions or encounter any difficulties, please speak to your teacher as soon as possible. Good luck!

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Solve: Give a solution for a problem or determine the value(s) of a variable.

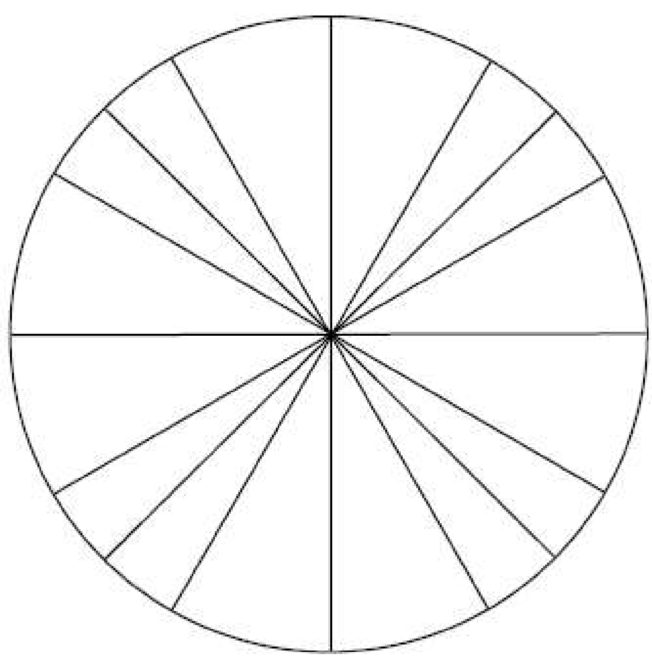
Verify: Establish the truth of a statement by substitution or comparison.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

State: Give an answer without an explanation or justification.

Describe: Use words to provide the process or to report details of the response.

Unit Circle (can be used if needed)



Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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R1 Demonstrate an understanding of operations on, and compositions of, functions.

1. Given $f(x) = x^2 + x - 4$ and $g(x) = \sqrt{x+5}$, Taz was asked to find $f(g(x))$.

   Taz's solution:

   $f(g(x)) = \left(\sqrt{x+5}\right)^2 + x - 4$

   $= x + 5 + x - 4$

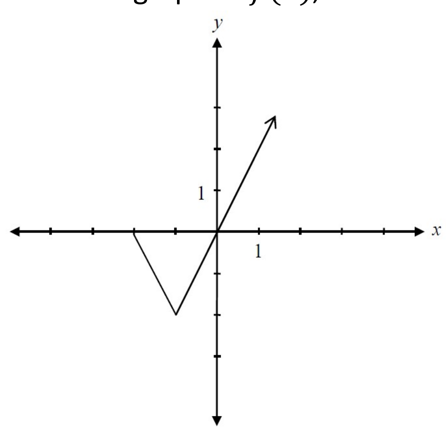
   $= 2x + 1, \ x \geq -5$

   Describe the error in Taz's solution. (2016-Jan-27)

2. Given the point (-12, -18) on the graph of $f(x)$, determine the new points after the following transformations of $f(x)$. (2016-Jan-41)

   a. $\frac{1}{f(x)}$ b. $f(-x) + 10$

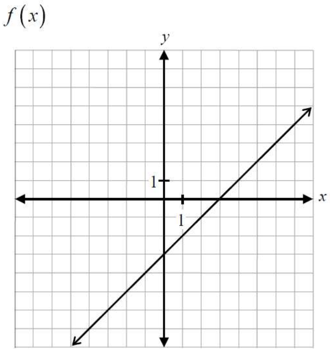
3. Given the graph of $f(x)$,



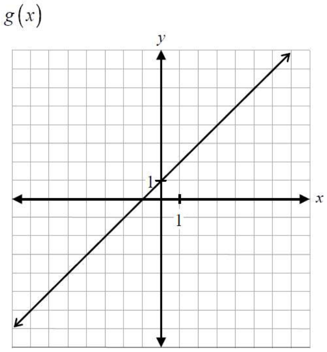
   sketch the graph of $y = |f(2x)| + 1$. (2016-Jan-43)

4. Given the following graphs of $f(x) = x - 3$ and $g(x) = x + 1$,

$f(x)$

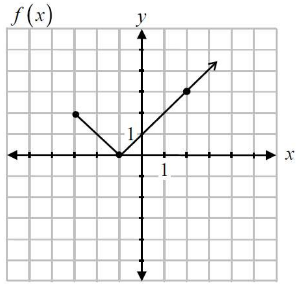


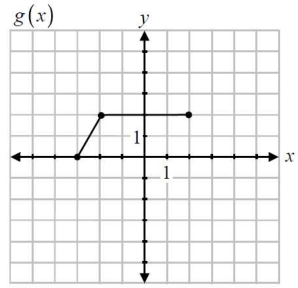
$g(x)$



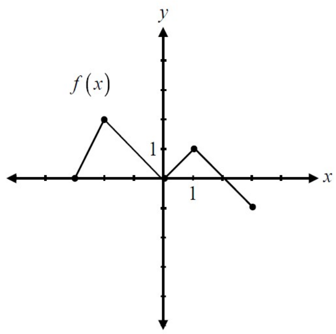
   sketch the graph of $h(x) = (f \cdot g)(x)$. (2016-Jan-45)

5. Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f+g)(x)$. (2016-Jun-7)





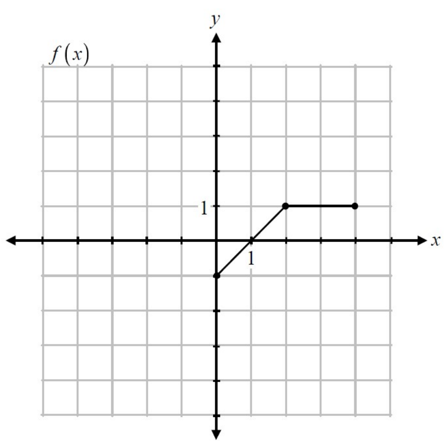
6. Given the graph of $y = f(x)$, sketch the graph of $y = 2|f(x+1)|$. (2016-Jun-10)



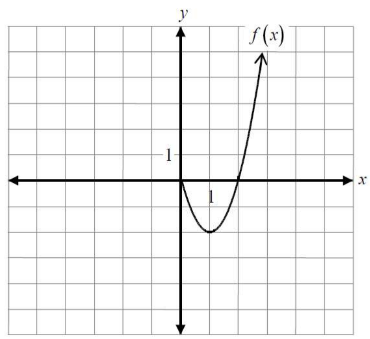
7. Given $f(x) = 2x - 1$ and $g(x) = x^2 + 1$:

   a. Determine $f(x) \cdot g(x)$.
   b. Determine $g(g(x))$. (2016-Jun-13)

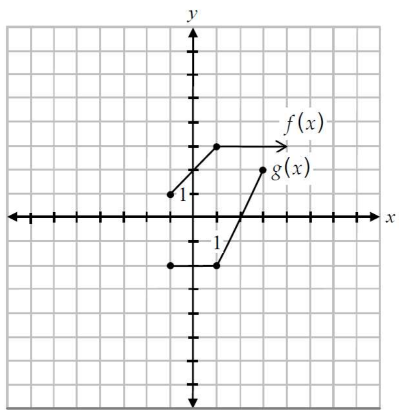
8. Given the function $f(x)$, sketch the graph of the reciprocal, $\frac{1}{f(x)}$. (2016-Jun-36)



9. Given the graph of $f(x)$, sketch the graph of $y = \left|\frac{1}{2}f(x-1)\right|$. (2017-Jan-16)



10. Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = (f \cdot g)(x)$. (2017-Jan-18)



11. Given $f(x) = \frac{1}{x-2}$ and $g(x) = x + 5$,

    a. Determine the equation for $f(g(x))$.
    b. Sketch the graph of $f(g(x))$. (2017-Jan-31)

12. Given the function $f(x) = \frac{2}{x} - 1$, justify why $f(f(2))$ is undefined. (2017-Jan-41)

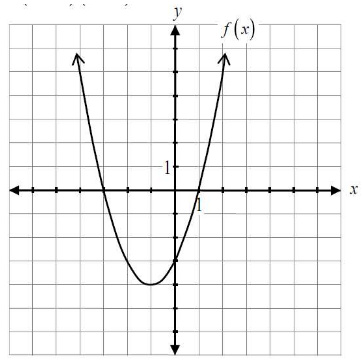
13. Given $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + 1$,

    a. Determine $g(f(x))$.
    b. Explain why the domain of $g(f(x))$ is restricted. (2017-Jun-41)

14. Given $f(x) = x^2 + 5x + 6$, $g(x) = x + 3$, and $h(x) = f(x) - g(x)$,

    a. Determine $h(x)$.
    b. Sketch the graph of $y = h(x)$. (2017-Jun-49)

15. Given the graph of $f(x) = (x + 3)(x - 1)$,

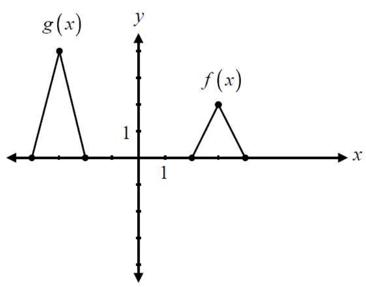


a. Sketch the graph of $g(x) = \frac{1}{f(x)}$.
b. Describe how to sketch the graph of $h(x) = |f(x)|$. (2017-Jun-29)

R2 Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.
R3 Demonstrate an understanding of the effects of horizontal and vertical compressions and stretches on the graphs of functions and their related equations.

R5 Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including through the x-axis, y-axis, and line y=x.

1. Determine an equation for $g(x)$ as a transformation of $f(x)$. (2016-Jan-7)



2. Describe the transformations of $y = f(x)$ when asked to sketch the graph of $y = -f(x-4)$. (2016-Jan-9)

3. If $P(3, 5)$ is a point on the graph of $y = f(x)$, identify the corresponding point on the graph of $y = f(x-1) + 7$. (2017-Jun-18)

   a. (2, 12)
   b. (4, -2)
   c. (2, -2)
   d. (4, 12)

4. If the range of $y = f(x)$ is $-3 \leq y \leq 6$, determine the range of $y = 2f(3x)$. (2017-Jun-16)

5. The graph of $y = f(x)$ contains the point (a, b). The graph of $g(x)$ is a transformation of the graph of $f(x)$ and contains the point (3a, b). Identify the function that represents $g(x)$. (2016-Jan-15)

   a. $g(x) = f(3x)$
   b. $g(x) = 3f(x)$
   c. $g(x) = f\left(\frac{x}{3}\right)$
   d. $g(x) = \frac{1}{3}f(x)$

6. The point (-2, 4) is on the graph of $f(x)$. State the coordinates of the corresponding point when $f(x)$ is reflected over the y-axis. (2016-Jun-6)

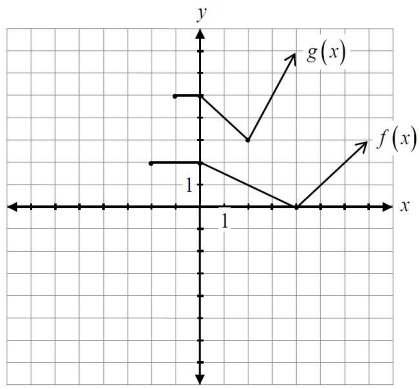
7. The graph of $f(x) = 3x + 7$ is reflected over the y-axis. Determine the equation of the new function. (2017-Jan-37)

8. Identify how the graph of $y = 3^x$ is transformed to the graph of $y = 3^{-x}$. (2017-Jan-19)

   a. Reflected over the x-axis
   b. Reflected over the y-axis
   c. Reflected over both the x-axis and y-axis
   d. Reflected over the line $y = x$

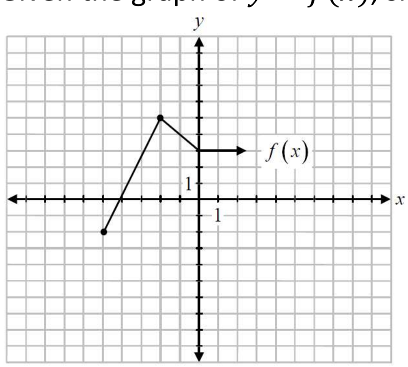
R4 Apply translations, compressions, and stretches to the graphs and equations of functions.

1. State the equation of $g(x)$ in terms of $f(x)$. (2017-Jan-11)



2. Describe the transformations used to obtain the graph of the function $y = 5f(x+1)$ from the graph of $y = f(x)$. (2017-Jan-14)

3. Given the graph of $y = f(x)$, sketch the graph of $y = f(-x+4)$. (2017-Jun-6)



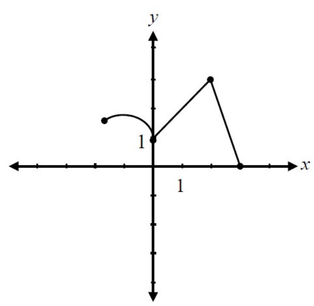
R6 Demonstrate an understanding of inverses of relations.

1. Given $f(x) = \frac{2}{x-1}$, determine the equation of the inverse, $f^{-1}(x)$. (2016-Jan-35)

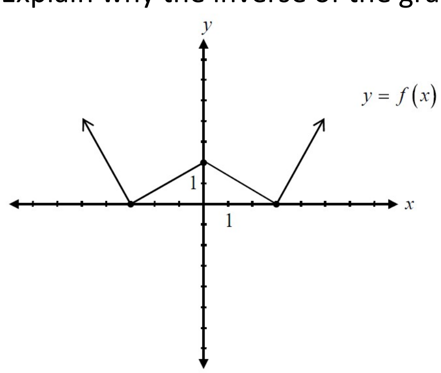
2. Given $f(x) = 3x + 2$, identify $f^{-1}(x)$. (2016-Jun-21)

   a. $f^{-1}(x) = -3x - 2$
   b. $f^{-1}(x) = 2x + 3$
   c. $f^{-1}(x) = \frac{x}{3} - 2$
   d. $f^{-1}(x) = \frac{x-2}{3}$

3. Describe how to determine the range of the inverse of the following graph. (2016-Jun-38)



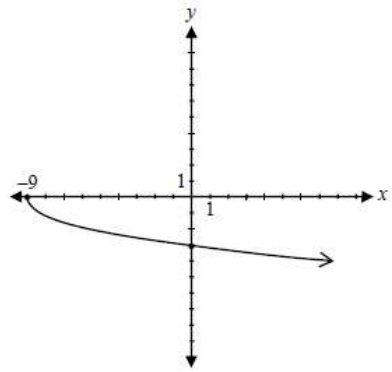
4. Explain why the inverse of the graph of $y = f(x)$ is not a function. (2017-Jan-12)



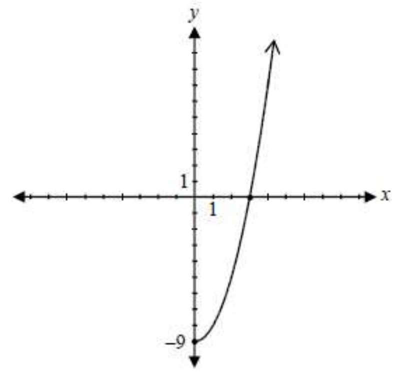
5. Determine algebraically, if $f(x) = \frac{1}{x+5}$ and $g(x) = \frac{1}{x-5}$ are inverses of each other. (2017-Jun-12)

6. Identify the graph of $f^{-1}(x)$ if $f(x) = x^2 - 9, x \geq 0$. (2017-Jan-24)

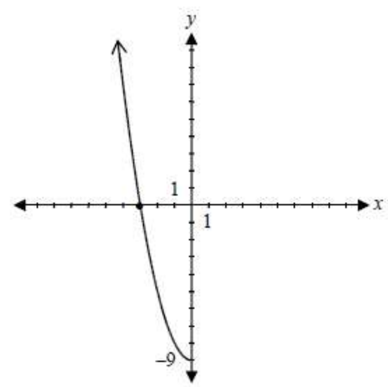
a)



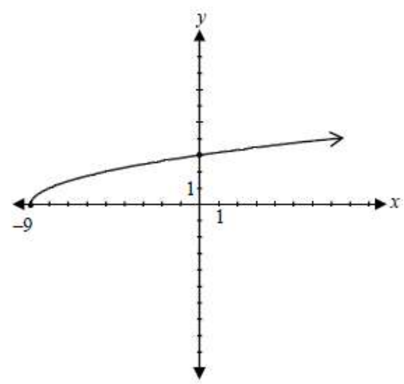
b)



c)



d)



R7 Demonstrate an understanding of logarithms.

1. Evaluate: $\log_4 2$. (2016-Jan-23)

2. Estimate the value of $\log_2 5$. Justify your answer. (2016-Jan-25)

3. Solve $7^{\log_7 2} = x$. (2016-Jun-18)

   a. $x = 1$
   b. $x = 2$
   c. $x = 7$
   d. $x = 49$

4. Identify the logarithmic form of $5^x = 6$. (2017-Jan-20)

   a. $\log_5 x = 6$
   b. $\log_5 6 = x$
   c. $\log_6 x = 5$
   d. $\log_6 5 = x$

5. Justify why 4.7 is a better estimate than 4.3 for the value of $\log_2 26$. (2017-Jan-34)

6. Identify the equation $\log_a b = c$ in exponential form.

   a. $b^c = a$
   b. $a^c = b$
   c. $a^b = c$
   d. $c^a = b$

R8 Demonstrate an understanding of the product, quotient, and power laws of logarithms.

1. Solve: $4\log_3 2 - \frac{1}{3}\log_3 8 = \log_3 a$. (2016-Jan-36)

2. Using the laws of logarithms, fully expand this expression: $\log_a\left(\frac{x^3}{y\sqrt{z}}\right)$. (2016-Jan-38)

3. Using the laws of logarithms, fully expand this expression: $\log_2\left(\frac{w^3x}{y-1}\right)$. (2016-Jun-8)

4. Expand using the laws of logarithms: $\log\left(\frac{a}{b^4}\right)$. (2017-Jan-10)

5. If the $\log 6 = p$, $\log 5 = r$ and $\log 2 = q$, express $\log 60$ in terms of p, q and r. (2017-Jan-28)

6. Evaluate: $\log_2 80 - \log_2 10$. (2017-Jun-46)

R9 Graph and analyze exponential and logarithmic functions.

1. Sketch the graph of the function $f(x) = 3\log_2(x+1)$. (2016-Jan-28)

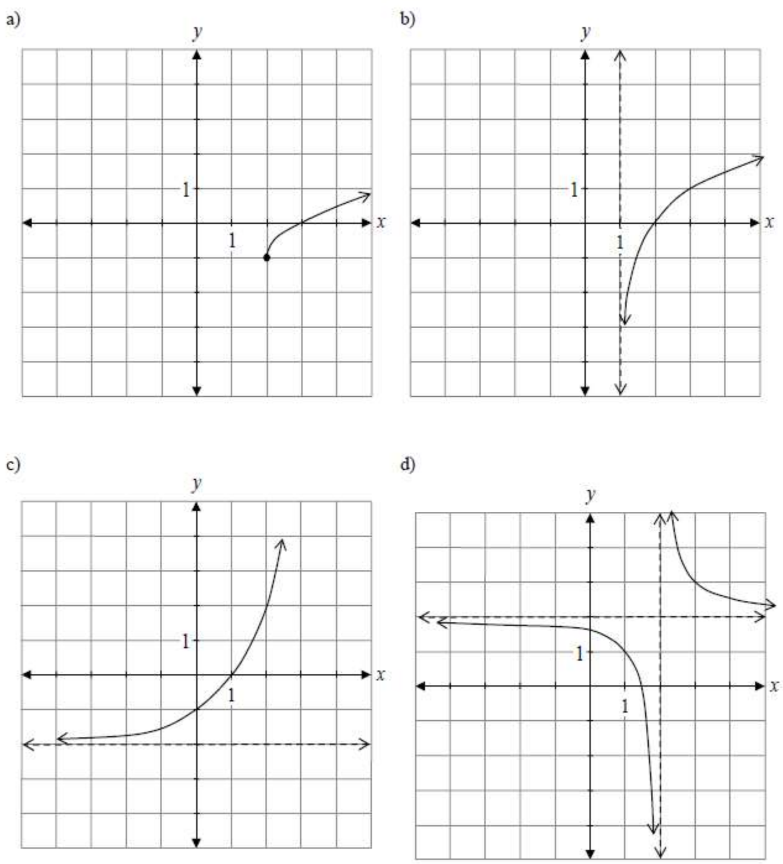
2. Given $f(x) = 2^x + 1$, state the equation of the horizontal asymptote. (2016-Jan-44)

3. Sketch the graph of $f(x) = 3^x + 2$. (2016-Jun-31)

4. Sketch the graph of $y = -2^x + 2$. (2017-Jan-40)

5. Explain why the domain of $y = \log_2(x-1)$ is $x > 1$. (2017-Jan-43)

6. Identify which of the following graphs represents a logarithmic function. (2017-Jun-22)



7. Describe how the value of m in the equation $y = \log_3(x-m), m \in R$, affects the asymptote on the graph of $y = \log_3 x$. (2017-Jun-30)

8. Determine the x-intercept of the graph of $f(x) = e^x - 1$. (2017-Jun-44)

R10 Solve problems that involve exponential and logarithmic equations.

1. Solve: $6(5)^{3x+2} = 9^{2-x}$. (2016-Jan-4)

2. Sheeva's bank is lending her \$50 000 at an annual interest rate of 6%, compounded monthly, to purchase a car. Given the last payment will be a partial payment, determine how many full monthly payments of \$800 Sheeva will have to make. The formula below may be used.

   $$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

   where $PV =$ the present value of the amount borrowed

   $R =$ the amount of each periodic payment

   $$i = \frac{\text{annual interest rate (as a decimal)}}{\text{the number of compounding periods per year}}$$

   $n =$ the number of equal periodic payments

   Express your answer as a whole number. (2016-Jun-4)

3. Solve the following equation: $\log_3(x+3) + \log_3(x-5) = 2$. (2016-Jun-23)

4. Solve $9^{2x+1} = 27^x$. (2016-Jun-42)

5. Peter invests \$560 per month at an annual interest rate of 4.2%, compounded monthly. Determine how many monthly investments he will need to make to obtain at least \$500 000. Express your answer as a whole number. Use the formula:

   $$\text{FV} = \frac{\text{R}\left[(1+i)^n - 1\right]}{i}$$

   where FV = the future value

   R = the investment amount each period

   $$i = \frac{\text{the annual interest rate}}{\text{the number of compounding periods per year}}$$

   $n =$ the number of investments

   (2017-Jan-3)

6. Solve the following equation algebraically: $\log(x^2+5) - \log(x^2+1) = \log 3$ (2017-Jan-13)

7. A water filtration system which removes impurities from a sample of water can be modelled by

   $P = 0.25(0.55)^n$, where:
   P = the percentage of impurities remaining, in decimal form
   n = the number of filters

   Determine, algebraically, how many filters are required so that less than 1% of the impurities remain in the water sample. Express your answer as a whole number. (2017-Jun-3)

8. Solve algebraically: $25^x = \left(\frac{1}{5}\right)^{-3x+1}$. (2017-Jun-31)

9. Solve algebraically: $2\log_a 3 + \log_a 4 = 2, where\ a > 0$. (2017-Jun-42)

R11 Demonstrate an understanding of factoring polynomials of degree greater than 2, but no more than 5 with integral coefficients.

1. A student must determine the factors of $5x^4 - 2x^3 + 4x - 1$. He used 5, -2, 4, and -1 as the coefficients of the polynomial when using synthetic division. Explain the student's error. (2016-Jan-8)

2. Given that $(x + 3)$ is one of the factors, express $2x^3 + 7x^2 + 2x - 3$ as a product of factors. (2016-Jan-13)

3. Given the polynomial function $P(x) = x^4 - 5x^2 - 2x + 6$, if $P(1) = 0$, identify which statement is true. (2016-Jun-15)

   a. The y-intercept is 1.
   b. $P(x)$ has a factor of $(x + 1)$.
   c. The graph has a zero at 1.
   d. The graph has a zero at -1.

4. When $P(x) = 3x^4 - kx^3 + 5x - 14$ is divided by $(x + 2)$, the remainder is -8. Determine the value of k. (2016-Jun-28)

5. Using the remainder theorem, identify which value of x results in a remainder of zero given $p(x) = x^3 + 7x^2 + 14x + 8$. (2017-Jan-25)

   a. 1
   b. 0
   c. -1
   d. -3

6. One of the zeros of $p(x) = x^3 + 6x^2 - 32$ is $x = 2$. Determine all of the other zeros of $p(x)$. (2017-Jan-39)

7. Express $p(x) = x^3 - 2x^2 - 4x + 8$ as a product of factors. (2017-Jun-28)

R12 Graph and analyze polynomials functions.

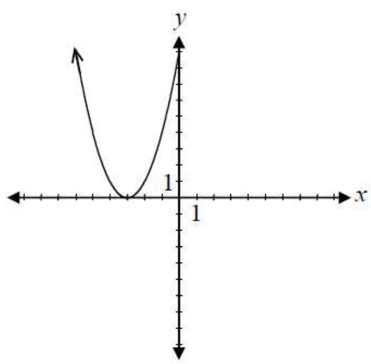
1. The roots of the polynomial equation $3(x-2)(x+1)^2 = 0$ are $x = 2$ and $x = -1$. Explain what these roots represent on the graph of $p(x) = 3(x-2)(x+1)^2$. (2016-Jan-6)

2. Identify the maximum number of x-intercepts for a polynomial function of degree 3. (2016-Jan-14)

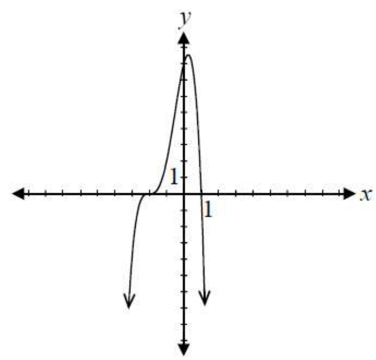
   a. 1
   b. 2
   c. 3
   d. 4

3. Identify which of the following graphs of polynomial functions has a zero with a multiplicity of 3. (2016-Jan-19)

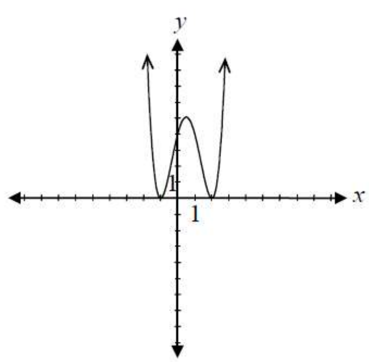
a)



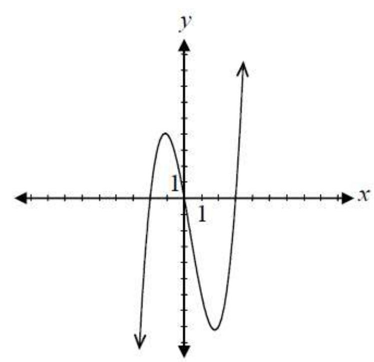
b)



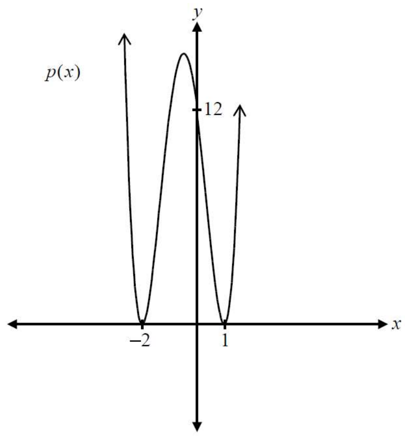
c)



d)

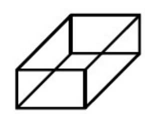


4. Determine the equation of the polynomial function, $p(x)$, represented by the graph. (2016-Jan-22)



5. Sketch the graph of the polynomial function with the following characteristics:
   y-intercept of -9
   zeros at -1 and 3
   the zero at -1 has a multiplicity of 1 and the zero at 3 has a multiplicity of 2
   (2016-Jun-34)

6. The volume of a planter, in the shape of a rectangular prism, can be modelled by the polynomial function $V(x) = x^3 + 3x^2 - 34x + 48$. Determine all the factors of the function, $V(x)$, which represent possible dimensions of this planter. (2016-Jun-37)



7. Explain why $f(x) = (x+2)^3(x-1)^{\frac{1}{2}}$ is not a polynomial function. (2017-Jan-17)

8. Sketch a graph of $P(x)$ that satisfies all of the following conditions:

   $P(x)$ is a polynomial function of degree 3
   $P(x)$ has a zero at -3 with a multiplicity of 2
   $P(x)$ has a zero at 1
   $P(x)$ has a leading coefficient of -3

   (2017-Jan-45)

9. Match the following equations with their graphs. (2017-Jan-27)

Place the appropriate letter
in this column.

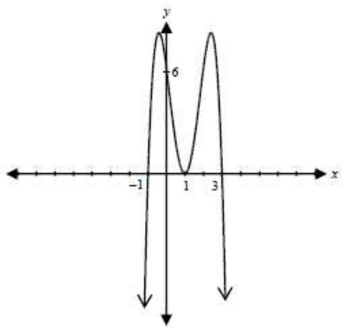
$f(x) = (x-1)^3(x+1)(x-3)$ ____________

$g(x) = (x+1)^2(x-1)(x+3)$ ____________

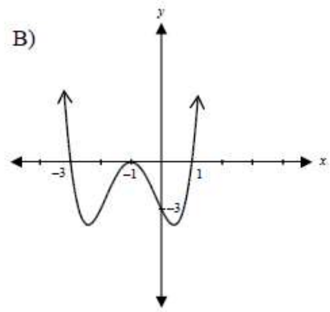
$h(x) = -2(x-1)^2(x+1)(x-3)$ ____________

$k(x) = 2(x+1)^2(x-1)(x+3)$ ____________

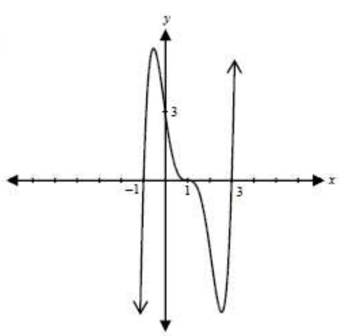
A)



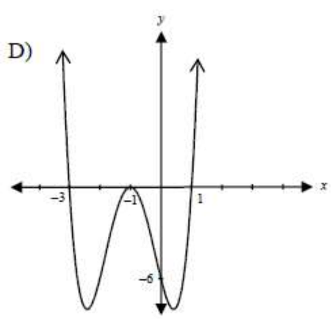
B)



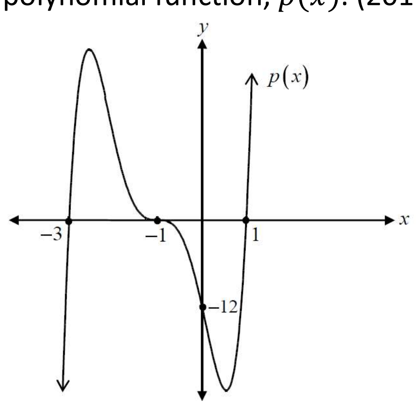
C)



D)



10. Determine, algebraically, the value of the leading coefficient of the graph of the polynomial function, $p(x)$. (2017-Jun-10)



11. If the volume of a box is represented by $V(x) = (x + 4)(x + 2)(x - 1)$, identify a possible value of x. (2017-Jun-23)

    a. -4
    b. -1
    c. 1
    d. 4

12. Describe a difference between the graphs of $y = f(x)$ and $y = g(x)$. (2017-Jun-33)

$f(x) = -2(x + 1)^2(x + 3)$
$g(x) = 2(x + 1)^2(x + 3)$

13. Describe the relationship between the zeros of the function $f(x) = (2x - 1)(x + 3)^2$, the roots of the equation $(2x - 1)(x + 3)^2 = 0$, and the x-intercepts of the graph of $y = f(x)$. (2017-Jun-35)

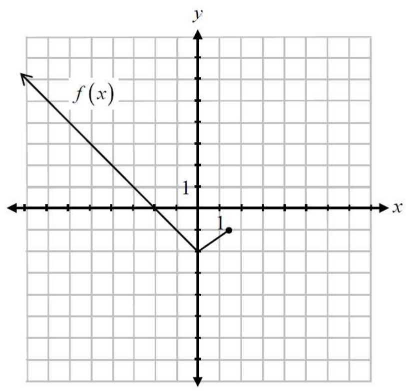
R13 Graph and analyze radical functions.

1. Sketch the graph of $f(x) = 3\sqrt{x-2} + 1$. (2016-Jan-39)

2. Determine

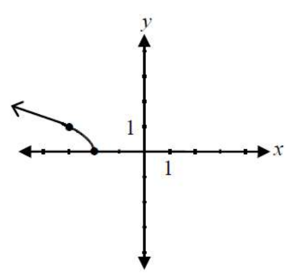
    a. The domain of the graph of the function $f(x) = \sqrt{x^2 - 4}$
    b. Explain why the domain of $f(x) = \sqrt{x^2 - 4}$ is restricted. (2016-Jan-40)

3. Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. (2016-Jun-14)

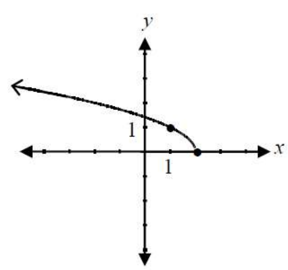


4. Identify the graph that corresponds to the function $f(x) = -\sqrt{(x-2)}$. (2016-Jun-17)

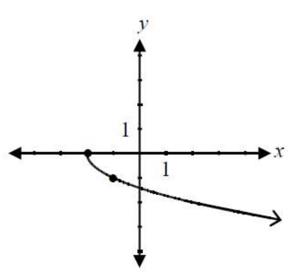
a)



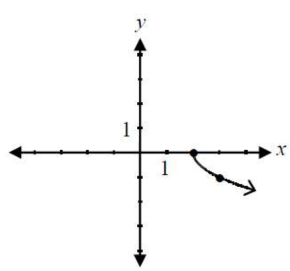
b)



c)



d)

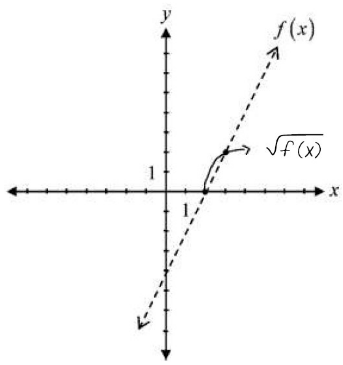


5. Identify the function that has a domain of $x \leq -2$ and a range of $y \geq 3$. (2016-Jun-20)

    a. $y = \sqrt{x+2} + 3$
    b. $y = \sqrt{-(x+2)} + 3$
    c. $y = -\sqrt{x-2} - 3$
    d. $y = -\sqrt{-(x-2)} - 3$

6. Sketch the graph of the function $y = \sqrt{2x} + 1$. (2016-Jun-39)

7. Suah was given the graph of $f(x)$ and asked to graph $y = \sqrt{f(x)}$. Her solution is given on the graph below.

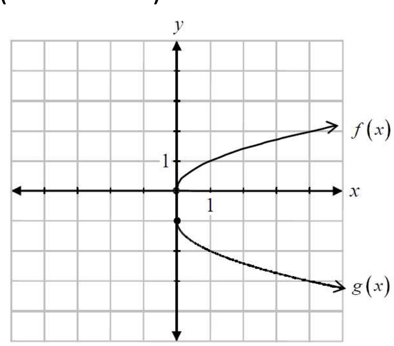


Describe the error Suah made when sketching the graph of $y = \sqrt{f(x)}$. (2016-Jun-41)

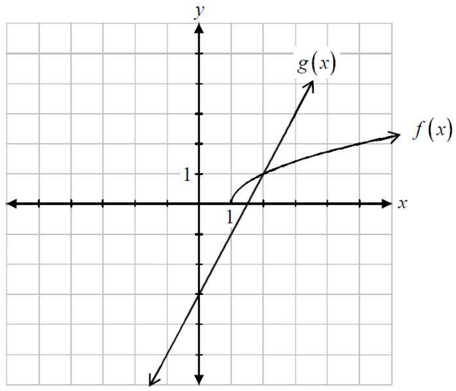
8. Sketch the graph of $y = \sqrt{-2x} + 1$. (2017-Jan-29)

9. Determine the domain and range of $f(x) = \sqrt{x-5} - 1$. (2017-Jan-33)

10. Describe the transformations applied to the graph of $f(x)$ to obtain the graph of $g(x)$. (2017-Jun-9)



11. Using the graphs of $y = f(x)$ and $y = g(x)$, solve $f(x) = g(x)$. (2017-Jun-13)



12. Match the following radical functions with their graphs. (2017-Jun-27)

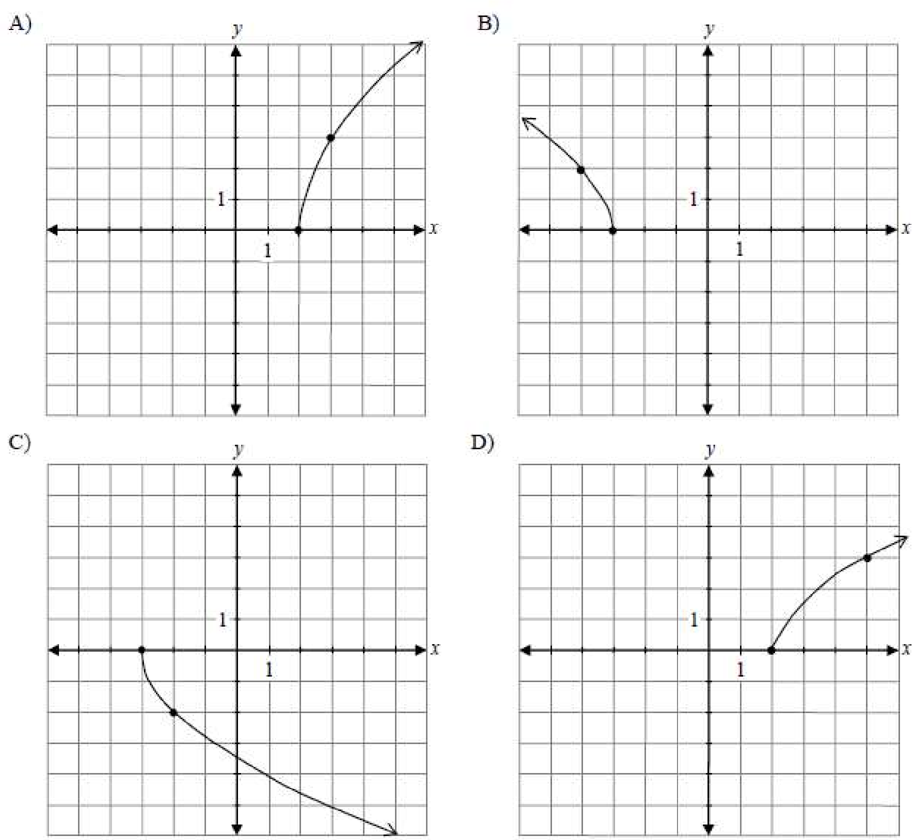
Place the appropriate letter in this column.

$f(x) = 2\sqrt{-(x+3)}$ _______

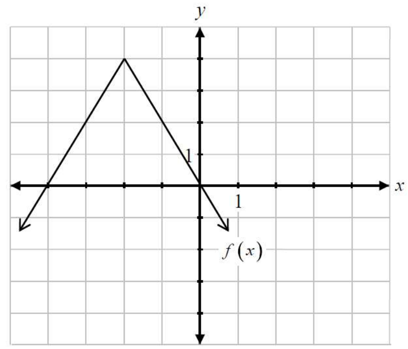
$g(x) = -2\sqrt{(x+3)}$ _______

$h(x) = 3\sqrt{(x-2)}$ _______

$k(x) = \sqrt{3(x-2)}$ _______



13. Given the graph of $y = f(x)$, sketch the graph of $\sqrt{f(x)}$. (2017-Jun-34)



R14 Graph and analyze rational functions.

1. Identify which of the following statements is true for the rational $f(x) = \frac{4(x-1)(x-2)}{(x-1)(x+3)}$.
   (2016-Jan-21)

   a. The equation of the horizontal asymptote is $y = 4$.
   b. The equation of the vertical asymptote is $x = 1$.
   c. The y-intercept is 0.
   d. There is a point of discontinuity (hole) when x= 2.

2. Write an equation of a rational function that would not have any vertical asymptotes.
   (2016-Jan-29)

3. Sketch the graph of the following function: $y = \frac{(x+3)(x-3)}{x(x-3)}$
   (2016-Jan-31)

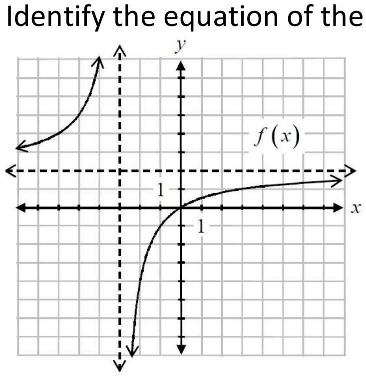
4. Sketch the graph of the function $y = \frac{2x+2}{x^2-1}$. (2016-Jun-25)

5. Describe the difference between the graph of $f(x) = \frac{7(x+2)}{x+2}$ and the graph of
   $g(x) = \frac{7(x-2)}{x+2}$ at $x = -2$. (2016-Jun-30)

6. Sketch the graph for the function: $f(x) = \frac{x(x-2)(x-4)}{(x-2)}$
   (2017-Jan-35)

7. Determine the equations for all of the asymptotes of the function: $y = \frac{2x+1}{x-3}$
   (2017-Jan-38)

8. Identify the equation of the function, $f(x)$, for the following graph.



a) $f(x) = \frac{2x}{x+3}$

b) $f(x) = \frac{2}{x+3}$

c) $f(x) = \frac{2x^2}{x(x+3)}$

d) $f(x) = \frac{3x^2}{x(x+2)}$

(2017-Jun-26)

9. Describe how to determine the equation of the horizontal asymptote of a rational function when the degree of the polynomial in the numerator and the degree of the polynomial in the denominator are equal. (2017-Jun-38)

10. Sketch the graph of the function $f(x) = \frac{-1}{(x-1)^2}$ and determine the range. (2017-Jun-40)

T1 Demonstrate an understanding of angles in standard position, expressed in degrees and radians.

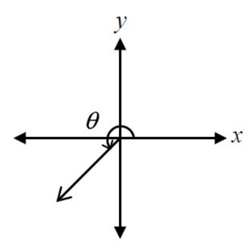
1. A pizza with a diameter of 15 inches is cut into equal slices, each with a central angle of $36°$. Determine the length of the crust on the outer edge of one slice of pizza. (2016-Jan-1)

2. The angle 2.95 radians, in standard position, terminates in quadrant: (2016-Jan-16)

   a. I
   b. II
   c. III
   d. IV

3. A wheel has a diameter of 20 cm and rotates through a central angle of $252°$. Determine how far the wheel rolled. (2016-Jun-1)

4. Identify a possible value for the angle θ sketch in standard position. (2016-Jun-22)

   a. 2
   b. 3
   c. 4
   d. 5

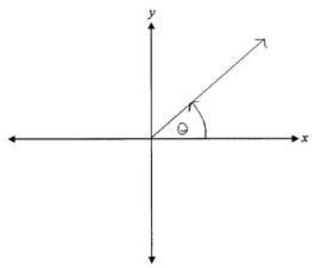
5. State a coterminal angle for $\theta = \frac{9\pi}{4}$. (2016-Jun-24)

6. Given $\theta = 40°$, (2017-Jan-2)

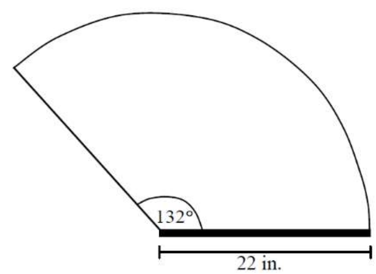
   a. Convert θ to radians.
   b. Determine the coterminal angles of θ where $\theta \in R$.

7. Determine the radius of a circle which has an arc length of 5 cm with a central angle of 3 radians. (2017-Jan-7)

8. Tyler incorrectly sketched the angle $\theta = -\frac{7\pi}{4}$ in standard position. Describe his error. (2017-Jan-8)



9. A section of a car windshield is cleaned by a wiper as shown in the diagram below. The arm of the wiper is 22 inches, and it rotates through a central angle of $132°$. Determine the length of the arc that is created by the tip of the wiper. (2017-Jun-1)



10. An angle in standard position measures $\frac{3\pi}{4}$. Determine in which quadrant the terminal arm of this angle is located after a rotation of 3 radians. Justify your answer. (2017-Jun-14)

11. Identify a coterminal angle for $\theta = -\frac{\pi}{3}$. (2017-Jun-24)

   a. $\frac{\pi}{3}$
   b. $\frac{4\pi}{3}$
   c. $\frac{7\pi}{3}$
   d. $\frac{11\pi}{3}$

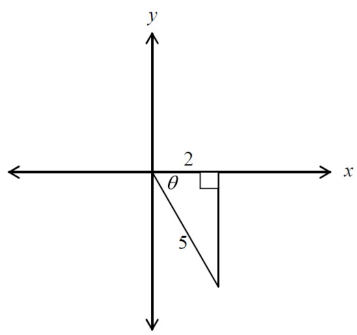
T2 Develop and apply the equation of the unit circle.

1. If θ terminates in quadrant III and $\cos\theta = -\frac{6}{7}$, determine the exact value of $\tan\theta$. (2016-Jan-26)

2. Given $\cot\theta = -\frac{1}{3}$, where θ is in quadrant II, determine the exact value of $\sin\theta$. (2016-Jun-35)

3. The point (-2, 7) is on the terminal arm of an angle in standard position. Determine the coordinates of the corresponding point, $P(\theta)$, on the unit circle. (2017-Jan-44)

4. Given the following triangle, determine $\csc\theta$. (2017-Jan-7)



T3 Solve problems, using the six trigonometric ratios for angles, expressed in radians and degrees.

1. Evaluate: $\left(\cos\frac{11\pi}{3}\right)\left(\csc\frac{11\pi}{6}\right)$
   (2016-Jan-24)

2. Explain why there is no solution for the equation $\csc\theta = -\frac{1}{2}$. (2016-Jan-42)

3. Evaluate $\cos\left(\cos\left(\frac{3\pi}{2}\right)\right)$. (2017-Jan-26)

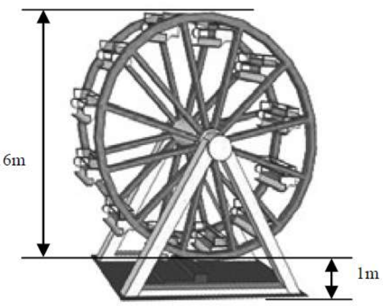
   a. 1
   b. $\frac{1}{2}$
   c. 0
   d. -1

4. Evaluate: $\sec^2\left(\frac{\pi}{6}\right)+\tan\left(\frac{7\pi}{6}\right)\csc\left(-\frac{2\pi}{3}\right)$
   (2017-Jan-36)

5. Evaluate: $\frac{\cot\left(-\frac{5\pi}{6}\right)}{\sin\left(\frac{17\pi}{3}\right)}$
   (2017-Jun-39)

T4 Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems.

1. Jose and Dana get on a Ferris wheel, which is 1 metre off the ground. The diameter of the Ferris wheel is 16 metres. Their ride lasts for 4 minutes, in which time the Ferris wheel makes one revolution. Determine the values of A, B, C, and D, if the sinusoidal function that models the situation is $h(t) = A\cos[B(t-C)] + D$, where h is the height at which Jose and Dana are located on the Ferris wheel, from the ground, in metres, and t is the time, in minutes. (2016-Jan-33)



2. Sketch the graph of at least one period of the function $y = 3\cos(\pi x) - 1$. (2016-Jan-37)

3. Sketch the graph of $y = -\sin\left(\frac{\pi}{2}(x-1)\right) + 3$ over the domain [0, 6]. (2016-Jun-27)

4. Given the following characteristics of a sinusoidal function:
   An amplitude of 2
   A vertical translation down 3 units
   A period of $\frac{\pi}{4}$

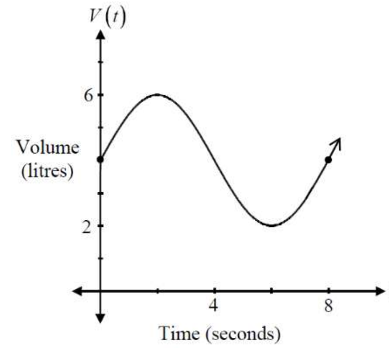
   a. Determine an equation of this sinusoidal function in the form $y = a\sin b(x-c) + d$.
   b. Determine the range of this function.

   (2016-Jun-40)

5. Given $f(\theta) = 3\cos 2\theta - 1$ and $g(\theta) = \sin\theta + 1$, identify which statement is true. (2017-Jan-21)

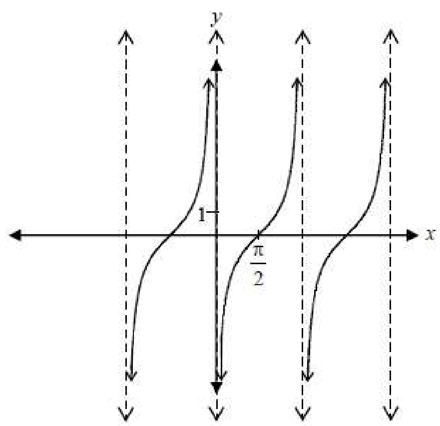
   a. Both functions have the same period.
   b. Both functions have the same amplitude.
   c. Both functions have the same minimum value.
   d. Both functions have the same maximum value.

6. The following graph represents the volume of air in an adult's lungs. If $V(t)$ is the volume of air in litres and t is the time in seconds, determine an equation that represents this sinusoidal function. (2017-Jan-42)

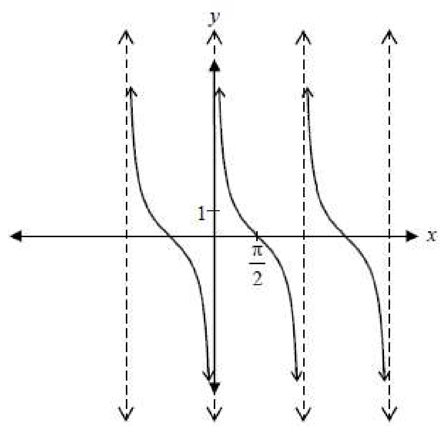


7. Identify the graph of $y = \tan x$. (2017-Jun-21)

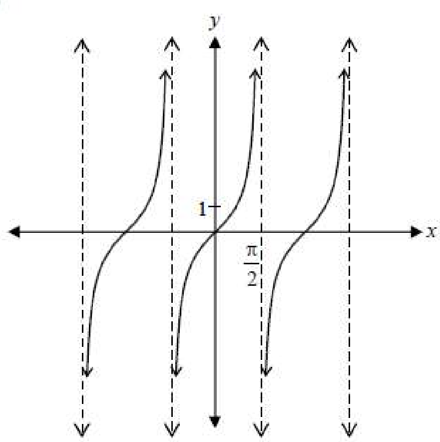
a)



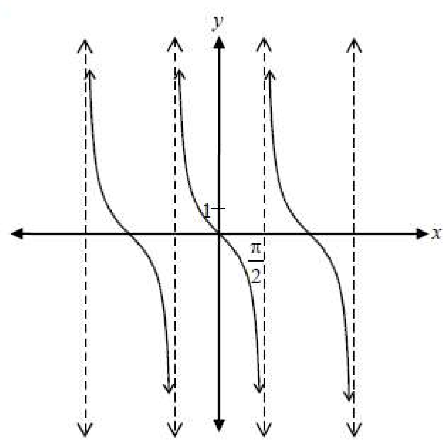
b)



c)



d)



8. Sketch a graph of at least one period of the function $f(x) = \cos\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right] - 3$. (2017-Jun-36)

9. State the amplitude of $f(x) = -2\sin(x - \pi) - 1$. (2017-Jun-47)

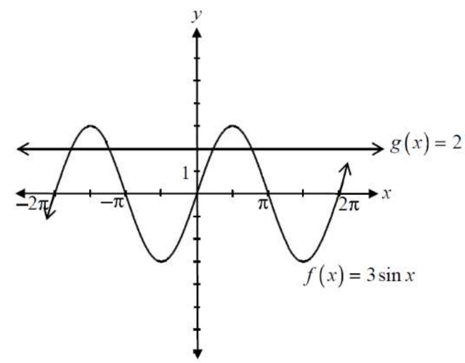
T5 Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

1. Solve the following equation over the interval $[0, 2\pi]$. (2016-Jan-3)

   $$\sin^2\theta + 6\sin\theta - 2 = 0$$

2. Solve $(2\sin\theta - 1)(\sin\theta + 1) = 0$ where $\theta \in R$. (2016-Jan-5)

3. Describe how to use the graphs of $f(x) = 3\sin x$ and $g(x) = 2$ to solve the equation $3\sin x = 2$. (2016-Jan-11)



4. Solve the following equation over the interval $[0, 2\pi]$. (2016-Jun-2)

   $$3\sin^2\theta - 10\sin\theta - 8 = 0$$

5. Identify the equation that has a general solution of $\left.\begin{array}{l}\theta = \dfrac{\pi}{6} + 2\pi k \\ \theta = \dfrac{5\pi}{6} + 2\pi k\end{array}\right\}$ where $k \in \mathbb{Z}$. (2016-Jun-19)

   a. $\sin\theta = \frac{1}{2}$
   b. $\cos\theta = \frac{1}{2}$
   c. $\sin\theta = \frac{\sqrt{3}}{2}$
   d. $\cos\theta = \frac{\sqrt{3}}{2}$

6. Describe the error that was made when solving the following equation:

$$\sin^2\theta + \sin\theta - 2 = 1$$

$$\sin^2\theta + \sin\theta = 3$$

$$\sin\theta(\sin\theta + 1) = 3$$

$\sin\theta = 3$ $\quad$ $\sin\theta + 1 = 3$

$\sin\theta = 2$

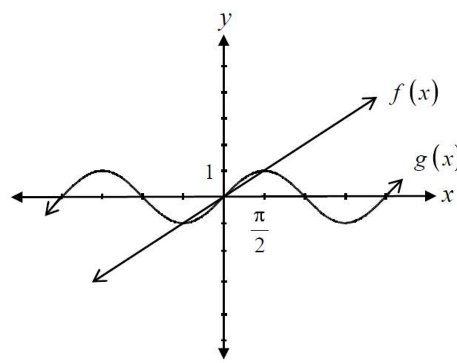
$\therefore$ No solution $\quad$ $\therefore$ No solution

(2016-Jun-33)

7. Solve the following equation algebraically over the interval $0 \le \theta \le 2\pi$. (2017-Jan-5)

$$2\cos^2\theta + 9\cos\theta - 5 = 0$$

8. Given the graphs of $f(x)$ and $g(x)$, identify the choice with all the solutions of the equation $f(x) = g(x)$. (2017-Jan-23)



   a. $x = -2\pi, -\pi, 0, \pi, 2\pi$
   b. $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$
   c. $x = \frac{\pi}{2}$
   d. $x = -1, 0, 1$

9. Solve the following equation algebraically over the interval $[0, 2\pi]$. (2017-Jun-5)

$$6\sin^2\theta + \sin\theta - 1 = 0$$

10. Maurice incorrectly solved the equation, $\sin\theta + 1 = 0$, over the interval $[0, 360°]$.

$$\sin\theta + 1 = 0$$

$$\sin\theta = -1$$

$$\sin\theta = 270°$$

Describe his error. (2017-Jun-17)

11. Solve $\sec\theta + 2 = 0$ over the interval $[0, 2\pi]$. (2017-Jun-43)

T6 Prove trigonometric identities using reciprocal identities, quotient identities, Pythagorean identities, sum or difference identities, double-angle identities.

1. Prove the identity below for all permissible values of θ. (2016-Jan-10)

   $$\sin\theta + \frac{\cos\theta}{\tan\theta} = \frac{1}{\cos\theta\tan\theta}$$

2. Evaluate: $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$
   (2016-Jan-17)

   a. $\frac{1}{2}$
   b. $\frac{\sqrt{2}}{2}$
   c. 1
   d. $\sqrt{2}$

3. A non-permissible value of x for the function $f(x) = \frac{1}{\cos x +}$ is:
   (2016-Jan-20)

   a. -1
   b. 0
   c. π
   d. $\frac{3\pi}{2}$

4. Determine the exact value of $\tan 75°$. (2016-Jan-30)

5. Solve the following equation algebraically for θ, where $0 \le \theta \le 2\pi$:
   $2\cos 2\theta = 1$
   (2016-Jun-9)

6. Prove the identity for all permissible values of θ:

   $$\cos\theta + \tan\theta\sin\theta = \frac{\tan\theta\sin\theta}{1-\cos^2\theta}$$

   (2016-Jun-11)

7. Given that $\cos\alpha = \frac{7}{12}$ where α is in quadrant IV, and $\sin\beta = \frac{3}{5}$ where β is in quadrant I, determine the exact value of:

   a. $\sin(\alpha - \beta)$
   b. $\csc(\alpha - \beta)$

   (2016-Jun-29)

8. Given the identity $\sec\theta + \cos\theta = \dfrac{2 - \sin^2\theta}{\cos\theta}$

   a. Determine the non-permissible values of θ, over the interval $0 \leq \theta \leq 2\pi$.
   b. Prove the identity for all permissible values of θ.

   (2017-Jan-9)

9. Identify the trigonometric function that is equivalent to $\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$. (2017-Jan-19)

   a. $\sin\frac{2\pi}{7}$
   b. $\sin\frac{7\pi}{12}$
   c. $\cos\frac{2\pi}{7}$
   d. $\cos\frac{7\pi}{12}$

10. Given that $\sin\alpha = \frac{3}{7}$, where α is in Quadrant II, and $\cos\beta = \frac{4}{5}$, where β is in Quadrant IV, determine the exact value of:

   a. $\sin(\alpha - \beta)$
   b. $\cos 2\alpha$

   (2017-Jan-32)

11. Prove the following identity for all permissible values of θ. (2017-Jun-15)

$$\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot\theta$$

12. Solve $\cos 2\theta = 0$, where $\theta \in R$. (2017-Jun-32)

13. Verify that $\theta = \frac{4\pi}{3}$ is a solution of the equation $4\cos^2\theta - 1 = 0$. (2017-Jun-37)

14. Determine the exact value of $\cos 15°$. (2017-Jun-48)

Extra Identity Practice:

1. Rewrite each expression in terms of sine and cosine only. Then simplify.

a) $\dfrac{\sec x}{\tan x}$

b) $\dfrac{\cot^2 x}{1 - \sin^2 x}$

c) $\dfrac{\csc x - \sin x}{\cot x}$

2. Factor and simplify each rational trigonometric expression.

a) $\dfrac{\tan x - \tan x \sin^2 x}{\cos^2 x}$

b) $\dfrac{\sin^2 x + \sin x - 6}{5\sin x + 15}$

c) $\dfrac{\cos^2 x - 4}{7\cos x - 14}$

d) $\dfrac{\sin^2 x \tan x - \tan x}{\sin x \tan x + \tan x}$

3. Use the Pythagorean identities to prove each identity for all permissible values of x.

a) $\csc^2 x(1 - \cos^2 x) = 1$

b) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$

c) $\dfrac{\sin^2 x + \cos^2 x}{\sec x} = \cos x$

4. Prove each identity. Use a common denominator to express two terms as one term, when necessary.

a) $\dfrac{1 + \tan x}{1 + \cot x} = \tan x$

b) $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$

c) $\dfrac{\cot x + \tan x}{\sec x} = \csc x$

5. Prove each identity, using factoring.

a) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

b) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

c) $\dfrac{\cos x + 1}{\sin x + \tan x} = \cot x$

6. Prove the following algebraically.

a) $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

b) $\dfrac{1 + \cos 2x}{\sin 2x} = \cot x$

c) $1 + \sin 2x = (\sin x + \cos x)^2$

d) $\sec^2 x = \dfrac{2}{1 + \cos 2x}$

7. Verify each equation is true for $x = 30°$. Then prove each equation is an identity.

a) $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

b) $\cos x + \cos x \tan^2 x = \sec x$

8. a) Prove that $\tan\theta = \dfrac{1 - \cos 2\theta}{\sin 2\theta}$.

b) State any non-permissible values.

9. Prove the following identity.
$1 + \sin 2x = (\sin x + \cos x)^2$

10. Prove the following identity.
$\cos 3x + 1 = 4\cos^3 x - 3\cos x + 1$

P1 Apply the fundamental counting principle to solve problems.

1. A hockey arena has 5 doors. Determine the number of ways that you can enter through one door and exit through a different door. (2016-Jan-12)

2. An employee asked 10 people in an ice cream shop to wait in line. Determine the number of different arrangements possible if two of the people, Jamie and John, refused to stand next to each other in the line. (2016-Jun-5)

3. Raoul has 8 shirts, 5 pairs of pants, and 3 hats. He adds the options together and determines he has 16 different outfits to wear.
   Raoul made an error in calculating the number of different outfits. Describe how to determine the correct number of outfits. (2016-Jun-12)

4. Determine how many 4-digit numbers greater than 4000 can be made using the digits 2, 3, 4, 5, and 6 if repetitions are not allowed. (2017-Jan-46)

5. Frank, Liam, Chan, and Thao are going to a movie. Determine the number of ways they can sit in a row of four chairs, if Frank and Chan must sit beside each other.
   (2017-Jun-11)

P2 Determine the number of permutations of *n* elements taken *r* at a time to solve problems.

1. Solve algebraically:

   $${}_{n}P_{3} = 4!(n-1)$$

   (2016-Jan-34)

2. Ishmael has 4 dogs, 5 cats, and 3 horses. If he arranges all of them in a row, determine how many ways they can be arranged if each type of animal must be grouped together. (2017-Jan-4)

3. Justify why the letters of the word FRANCE have a greater number of possible arrangements than the letters of the word CANADA. (2017-Jan-30)

4. Solve algebraically.

   $${}_{n}P_{2} = 9n$$

   (2017-Jun-8)

P3 Determine the number of combinations of *n* different elements taken *r* at a time to solve problems.

1. There are 9 girls and 7 boys in a math class from which a committee of 5 is to be chosen.

   a. How many different committees of 5 can be formed if one of the boys, William, must be on the committee?
   b. How many different committees of 5 can be formed if there must be 2 girls and 3 boys on the committee?

   (2016-Jan-2)

2. There are 6 different books that are being distributed evenly amongst three people. Identify which expression represents the number of possible combinations. (2016-Jun-16)

   a) ${}_6C_2 \cdot {}_6C_2 \cdot {}_6C_2$

   b) ${}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2$

   c) ${}_2C_2 \cdot {}_2C_2 \cdot {}_2C_2$

   d) $3 \cdot {}_6C_2$

3. Solve algebraically:

   $${}_nC_3 = n - 2$$

   (2016-Jun-32)

4. There are 24 different movies Kiandra can download to her computer. Determine the number of ways she can select 15 movies. (2017-Jan-1)

5. Solve algebraically:

   $${}_nC_2 = 3n + 4$$

   (2017-Jan-15)

6. There are 20 boys and 11 girls who can be selected to be on a team. Determine the number of ways that 7 boys and 5 girls can be selected for this team. (2017-Jun-2)

7. Identify the value of *n* in the equation ${}_{n}C_{3} = {}_{n}C_{6}$.   (2017-Jun-25)

   a. 3
   b. 6
   c. 9
   d. 18

P4 Expand powers of a binomial in a variety of ways, including using the binomial theorem.

1. Identify which of the following represents the 5th term in the expansion of $(4x^2 - 2y^3)^{15}$. (2016-Jan-18)

   a) $_{15}C_5 (4x^2)^{10} (-2y^3)^5$

   b) $_{15}C_5 (4x^2)^{11} (-2y^3)^4$

   c) $_{15}C_4 (4x^2)^{10} (-2y^3)^5$

   d) $_{15}C_4 (4x^2)^{11} (-2y^3)^4$

2. In the binomial expansion of $\left(\frac{1}{x^3} - 2x^2\right)^9$, determine which term contains x^3. (2016-Jan-32)

3. Determine and simplify the fourth term in the expansion of $(2x^4 - 3y^2)^8$. (2016-Jun-3)

4. Justify why the binomial expansion of $(x + x^3)^7$ does not have a term containing x^{10}. (2016-Jun-26)

5. Determine which term contains $\frac{1}{x^6}$ in the binomial expansion of $\left(\frac{2}{x^3} + 3x^2\right)^7$. (2017-Jan-6)

6. Identify the fourth term in the expansion of $(x + y)^5$. (2017-Jan-22)

   a. $10x^4y$
   b. $10x^3y^2$
   c. $10x^2y^3$
   d. $10xy^4$

7. In the binomial expansion of $\left(x^2 - \frac{2}{y}\right)^8$, determine the middle term in simplified form. (2017-Jun-4)

8. Given the fifth row of Pascal's triangle, determine the values of the next row. (2017-Jun-45)

1 4 6 4 1