

BLM 6-4 Section 6.3 Extra Practice

1. a) $\frac{1}{\sin x}$ b) $\frac{1}{\sin^2 x}$ c) $\cos x$

2. a) $\tan x$ b) $\frac{\sin x - 2}{5}$ c) $\frac{\cos x + 2}{7}$

d) $\sin x - 1$

3. a) Example:

$$\begin{aligned} \text{Left side} &= \csc^2 x (1 - \cos^2 x) \\ &= \frac{1}{\sin^2 x} (\sin^2 x) \\ &= 1 \\ &= \text{Right side} \end{aligned}$$

b) Example:

$$\begin{aligned} \text{Left side} &= (\tan x - 1)^2 \\ &= \tan^2 x - 2 \tan x + 1 \\ &= \frac{\sin^2 x - 2 \sin x \cos x + \cos^2 x}{\cos^2 x} \\ \text{Right side} &= \frac{1}{\cos^2 x} - \frac{2 \sin x}{\cos x} \\ &= \frac{1 - 2 \sin x \cos x}{\cos^2 x} \\ &= \frac{\sin^2 x - 2 \sin x \cos x + \cos^2 x}{\cos^2 x} \\ &= \text{Left side} \end{aligned}$$

c) Example:

$$\begin{aligned} \text{Right side} &= \cos x \\ \text{Left side} &= \frac{\sin^2 x + \cos^2 x}{\sec x} \\ &= 1 \div \frac{1}{\cos x} \\ &= \cos x \\ &= \text{Right side} \end{aligned}$$

4. a) Example:

$$\text{Right side} = \tan x$$

$$\begin{aligned} \text{Left side} &= \frac{1 + \tan x}{1 + \cot x} \\ &= 1 + \frac{\sin x}{\cos x} \div \left(1 + \frac{\cos x}{\sin x} \right) \\ &= \frac{\cos x + \sin x}{\cos x} \times \frac{\sin x}{\sin x + \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= \text{Right side} \end{aligned}$$

b) Example:

$$\text{Right side} = \cot x$$

$$\begin{aligned} \text{Left side} &= \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} \\ &= \frac{1}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} \\ &= \frac{\cos^2 x}{\sin x \cos x} \\ &= \cot x \\ &= \text{Right side} \end{aligned}$$

c) Example:

$$\text{Right side} = \csc x$$

$$= \frac{1}{\sin x}$$

$$\begin{aligned} \text{Left side} &= \frac{\cot x + \tan x}{\sec x} \\ &= \cos x \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) \\ &= \frac{\cos^2 x + \sin^2 x}{\sin x} \\ &= \frac{1}{\sin x} \\ &= \text{Right side} \end{aligned}$$

5. a) Example:

$$\begin{aligned} \text{Left side} &= \frac{\csc x + \cot x}{\tan x + \sin x} \\ &= \frac{1 + \cos x}{\sin x} \div \frac{\sin x + \sin x \cos x}{\cos x} \\ &= \frac{1 + \cos x}{\sin x} \times \frac{\cos x}{\sin x(1 + \cos x)} \\ &= \frac{\cos x}{\sin^2 x} \end{aligned}$$

$$\text{Right side} = \cot x \csc x$$

$$= \frac{\cos x}{\sin^2 x}$$

$$= \text{Left side}$$

b) Example:

$$\text{Right side} = \tan x$$

$$\text{Left side} = \frac{\sin x + \tan x}{\cos x + 1}$$

$$= \frac{\sin x \cos x + \sin x}{\cos x} \times \frac{1}{\cos x + 1}$$

$$= \frac{\sin x(\cos x + 1)}{\cos x} \times \frac{1}{\cos x + 1}$$

$$= \tan x$$

$$= \text{Right side}$$

c) Example:

$$\text{Right side} = \cot x$$

$$\text{Left side} = \frac{\cos x + 1}{\sin x + \tan x}$$

$$= (\cos x + 1) \times \frac{\cos x}{\sin x(\cos x + 1)}$$

$$= \cot x$$

$$= \text{Right side}$$

6. a) Example:

$$\text{Right side} = \cos^2 x - \sin^2 y$$

$$\text{Left side} = \cos(x + y)\cos(x - y)$$

$$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$$

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x(1 - \sin^2 y) - \sin^2 y(1 - \cos^2 x)$$

$$= \cos^2 x - \sin^2 y \cos^2 x - \sin^2 y + \sin^2 y \cos^2 x$$

$$= \cos^2 x - \sin^2 y$$

$$= \text{Right side}$$

b) Example:

$$\text{Right side} = \cot x$$

$$= \frac{\cos x}{\sin x}$$

$$\text{Left side} = \frac{1 + \cos 2x}{\sin 2x}$$

$$= \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \text{Right side}$$

c) Example:

$$\text{Right side} = (\sin x + \cos x)^2$$

$$= \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$= 1 + 2\sin x \cos x$$

$$\text{Left side} = 1 + \sin 2x$$

$$= 1 + 2\sin x \cos x$$

$$= \text{Right side}$$

d) Example:

$$\text{Right side} = \frac{2}{1 + \cos 2x}$$

$$= \frac{2}{1 + 2\cos^2 x - 1}$$

$$= \frac{1}{\cos^2 x}$$

$$\text{Left side} = \sec^2 x$$

$$= \frac{1}{\cos^2 x}$$

$$= \text{Right side}$$

7. a) Verify for $x = 30^\circ$:

$$\text{Left side} = \sec^4 30^\circ - \sec^2 30^\circ$$

$$= \frac{16}{9} - \frac{4}{3}$$

$$= \frac{4}{9}$$

$$\text{Right side} = \tan^4 30^\circ + \tan^2 30^\circ$$

$$= \frac{1}{9} + \frac{1}{3}$$

$$= \frac{4}{9}$$

$$= \text{Left side}$$

Example:

$$\text{Left side} = \sec^4 x - \sec^2 x$$

$$= \frac{1}{\cos^2 x} \left(\frac{1 - \cos^2 x}{\cos^2 x} \right)$$

$$= \frac{\sin^2 x}{\cos^4 x}$$

$$\text{Right side} = \tan^4 x + \tan^2 x$$

$$= \tan^2 x(\tan^2 x + 1)$$

$$= \frac{\sin^2 x}{\cos^2 x} \left(\frac{1}{\cos^2 x} \right)$$

$$= \frac{\sin^2 x}{\cos^4 x}$$

$$= \text{Left side}$$

b) Verify for $x = 30^\circ$:

$$\text{Left side} = \cos 30^\circ + \cos 30^\circ \tan^2 30^\circ$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\text{Right side} = \sec 30^\circ$$

$$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= \text{Left side}$$

Example:

$$\text{Left side} = \cos x + \cos x \tan^2 x$$

$$= \frac{\cos^3 x + \cos x \sin^2 x}{\cos^2 x}$$

$$= \frac{\cos x(\cos^2 x + \sin^2 x)}{\cos^2 x}$$

$$= \frac{1}{\cos x}$$

$$\text{Right side} = \sec x$$

$$= \frac{1}{\cos x}$$

$$= \text{Left side}$$

8. a) Example:

$$\text{Left side} = \tan \theta$$

$$\text{Right side} = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

$$= \text{Left side}$$

b) $\theta \neq \frac{n\pi}{2}; n \in \mathbb{Z}$

9. Example:

$$\text{Left side} = 1 + \sin 2x$$

$$= 1 + 2\sin x \cos x$$

$$\text{Right side} = (\sin x + \cos x)^2$$

$$= \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$= 1 + 2\sin x \cos x$$

$$= \text{Left side}$$

10. Example:

$$\text{Left side} = \cos 3x + 1$$

$$= \cos(2x + x) + 1$$

$$= \cos 2x \cos x - \sin 2x \sin x + 1$$

$$= \cos x(2\cos^2 x - 1) - 2\sin x(\sin x \cos x) + 1$$

$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x + 1$$

$$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) + 1$$

$$= 4\cos^3 x - 3\cos x + 1$$

$$\text{Right side} = 4\cos^3 x - 3\cos x + 1$$

$$= \text{Left side}$$