Lesson 1.1 Square Roots of Perfect Squares Practice (pages 11–13)

Check

3. Each square root represents the side length of the shaded square.

a) The side length of the shaded square is: $\frac{1}{2}$

So, $\sqrt{0.25} = \frac{1}{2}$, or 0.5

b) The side length of the shaded square is: $\frac{3}{4}$

So, $\sqrt{\frac{9}{16}} = \frac{3}{4}$, or 0.75

c) The side length of the shaded square is: $\frac{4}{5}$

So, $\sqrt{\frac{16}{25}} = \frac{4}{5}$, or 0.8

4. A perfect square is a number that can be written as a product of two equal factors.
The whole numbers we consider are: 1, 2, 3, …
Their squares are: $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, …

a) The whole numbers from 1 to 100 that are perfect squares are:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100

b) The square roots of the numbers in part a are:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10

5. a)
$$\begin{aligned}\sqrt{0.36} &= \sqrt{\frac{36}{100}} \\ &= \frac{6}{10} \\ &= 0.6\end{aligned}$$

b)
$$\begin{aligned}\sqrt{0.49} &= \sqrt{\frac{49}{100}} \\ &= \frac{7}{10} \\ &= 0.7\end{aligned}$$

c)
$$\begin{aligned}\sqrt{0.81} &= \sqrt{\frac{81}{100}} \\ &= \frac{9}{10} \\ &= 0.9\end{aligned}$$

d)
$$\begin{aligned}\sqrt{0.16} &= \sqrt{\frac{16}{100}} \\ &= \frac{4}{10} \\ &= 0.4\end{aligned}$$

e) $\sqrt{\dfrac{1}{36}} = \dfrac{1}{6}$

f) $\sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$

g) $\sqrt{\dfrac{64}{100}} = \dfrac{8}{10}$
$= \dfrac{4}{5}$

h) $\sqrt{\dfrac{36}{16}} = \dfrac{6}{4}$
$= \dfrac{3}{2}$

6. **a)** The whole numbers from 101 to 400 that are perfect squares are:
121, 144, 169, 196, 225, 256, 289, 324, 361, 400

b) The square roots of the numbers in part a are:
11, 12, 13, 14, 15, 16, 17, 18, 19, 20

7. Write each number as a product of two equal factors.

a) $\sqrt{\dfrac{169}{9}} = \sqrt{\dfrac{13}{4} \times \dfrac{13}{4}}$
$= \dfrac{13}{4}$

b) $\sqrt{\dfrac{400}{196}} = \sqrt{\dfrac{20}{14} \times \dfrac{20}{14}}$
$= \dfrac{20}{14}$
$= \dfrac{10}{7}$

c) $\sqrt{\dfrac{256}{361}} = \sqrt{\dfrac{16}{19} \times \dfrac{16}{19}}$
$= \dfrac{16}{19}$

d) $\sqrt{\dfrac{225}{289}} = \sqrt{\dfrac{15}{17} \times \dfrac{15}{17}}$
$= \dfrac{15}{17}$

e) $\sqrt{144} = \sqrt{12 \times 12}$
$= 12$

f) $\sqrt{0.0225} = \sqrt{\frac{225}{10\,000}}$

$= \sqrt{\frac{15}{100} \times \frac{15}{100}}$

$= \frac{15}{100}$, or 0.15

g) $\sqrt{0.0121} = \sqrt{\frac{121}{10\,000}}$

$= \sqrt{\frac{11}{100} \times \frac{11}{100}}$

$= \frac{11}{100}$, or 0.11

h) $\sqrt{3.24} = \sqrt{\frac{324}{100}}$

$= \sqrt{\frac{18}{10} \times \frac{18}{10}}$

$= \frac{18}{10}$, or 1.8

i) $\sqrt{0.0324} = \sqrt{\frac{324}{10\,000}}$

$= \sqrt{\frac{18}{100} \times \frac{18}{100}}$

$= \frac{18}{100}$, or 0.18

j) $\sqrt{0.0169} = \sqrt{\frac{169}{10\,000}}$

$= \sqrt{\frac{13}{100} \times \frac{13}{100}}$

$= \frac{13}{100}$, or 0.13

Apply

8. a) 0.12

Write 0.12 as a fraction.

$0.12 = \frac{12}{100}$ Simplify the fraction. Divide the numerator and denominator by 4.

$= \frac{3}{25}$

This fraction is in simplest form.

The denominator can be written as 5×5, but the numerator cannot be written as a product of equal factors.

So, $\frac{3}{25}$, or 0.12, is not a perfect square.

b) 0.81
Write 0.81 as a fraction.
$0.81 = \frac{81}{100}$
$\frac{81}{100}$ can be written as $\frac{9}{10} \times \frac{9}{10}$.
So, $\frac{81}{100}$, or 0.81, is a perfect square.

c) 0.25
Write 0.25 as a fraction.
$0.25 = \frac{25}{100}$
$\frac{25}{100}$ can be written as $\frac{5}{10} \times \frac{5}{10}$.
So, $\frac{25}{100}$, or 0.25, is a perfect square.

d) 1.69
Write 1.69 as a fraction.
$1.69 = \frac{169}{100}$
$\frac{169}{100}$ can be written as $\frac{13}{10} \times \frac{13}{10}$.
So, $\frac{169}{100}$, or 1.69, is a perfect square.

e) $\frac{9}{12}$
Simplify the fraction first. Divide the numerator and denominator by 3.
$\frac{9}{12} = \frac{3}{4}$
The fraction is in simplest form.
Look for a fraction that, when multiplied by itself, gives $\frac{3}{4}$.
The denominator can be written as $4 = 2 \times 2$, but the numerator cannot be written as a product of equal factors. So, $\frac{9}{12}$ is not a perfect square.

f) $\frac{36}{81}$
Simplify the fraction first. Divide the numerator and denominator by 9.
$\frac{36}{81} = \frac{4}{9}$
Since $4 = 2 \times 2$ and $9 = 3 \times 3$, we can write:
$\frac{4}{9} = \frac{2}{3} \times \frac{2}{3}$
Since $\frac{4}{9}$ can be written as a product of two equal fractions, it is a perfect square.
So, $\frac{36}{81}$ is a perfect square.

g) $\frac{81}{49}$

This fraction is in simplest form. So, look for a fraction that when multiplied by itself gives $\frac{81}{49}$.

The numerator can be written as 81 = 9 × 9, and the denominator can be written as 49 = 7 × 7, so we can write:

$\frac{81}{49} = \frac{9}{7} \times \frac{9}{7}$

Since $\frac{81}{49}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{81}{49}$ is a perfect square.

h) $\frac{75}{27}$

Simplify the fraction first. Divide the numerator and denominator by 3.

$\frac{75}{27} = \frac{25}{9}$

Since 25 = 5 × 5, and 9 = 3 × 3, we can write:

$\frac{25}{9} = \frac{5}{3} \times \frac{5}{3}$

Since $\frac{25}{9}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{75}{27}$ is also a perfect square.

i) 0.081

Write 0.081 as a fraction.

$0.081 = \frac{81}{1000}$

The numerator can be written as 81 = 9 × 9, but the denominator cannot be written as a product of equal factors. So, 0.081 is not a perfect square.

j) $\frac{25}{10}$

Simplify the fraction first. Divide the numerator and denominator by 5.

$\frac{25}{10} = \frac{5}{2}$

This fraction is in simplest form.

Neither 5 nor 2 can be written as a product of equal factors, so $\frac{5}{2}$ is not a perfect square, and $\frac{25}{10}$ is not a perfect square.

k) 2.5

Write 2.5 as a fraction.

$$2.5 = \frac{25}{10}$$
$$= \frac{5}{2}$$

Neither 5 nor 2 can be written as a product of equal factors, so 2.5 is not a perfect square.

l) $\frac{8}{50}$

Simplify the fraction first. Divide the numerator and denominator by 2.

$\frac{8}{50} = \frac{4}{25}$

Since 4 = 2 × 2 and 25 = 5 × 5, we can write:

$\frac{4}{25} = \frac{2}{5} \times \frac{2}{5}$

Since $\frac{4}{25}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{8}{50}$ is also a perfect square.

9. a) The number whose square root is 0.3 can be represented as the area of a square with side length 0.3 units:

$0.3^2 = 0.3 \times 0.3 = 0.09$

So, 0.3 is a square root of 0.09.

b) The number whose square root is 0.12 can be represented as the area of a square with side length 0.12 units:

$0.12^2 = 0.12 \times 0.12 = 0.0144$

So, 0.12 is a square root of 0.0144.

c) The number whose square root is 1.9 can be represented as the area of a square with side length 1.9 units.

$1.9^2 = 1.9 \times 1.9 = 3.61$

So, 1.9 is a square root of 3.61.

d) The number whose square root is 3.1 can be represented as the area of a square with side length 3.1 units.

$3.1^2 = 3.1 \times 3.1 = 9.61$

So, 3.1 is a square root of 9.61.

e) $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3}$

$= \frac{4}{9}$

So, $\frac{2}{3}$ is a square root of $\frac{4}{9}$.

f) $\left(\frac{5}{6}\right)^2 = \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{36}$

So, $\frac{5}{6}$ is a square root of $\frac{25}{36}$.

g) $\left(\frac{1}{7}\right)^2 = \frac{1}{7} \times \frac{1}{7}$

$= \frac{1}{49}$

So, $\frac{1}{7}$ is a square root of $\frac{1}{49}$.

h) $\left(\frac{2}{5}\right)^2 = \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

So, $\frac{2}{5}$ is a square root of $\frac{4}{25}$.

10. a) $\sqrt{12.25} = \sqrt{\frac{1225}{100}}$

$= \sqrt{\frac{35}{10} \times \frac{35}{10}}$

$= \frac{35}{10}$, or 3.5

b) $\sqrt{30.25} = \sqrt{\frac{3025}{100}}$

$= \sqrt{\frac{55}{10} \times \frac{55}{10}}$

$= \frac{55}{10}$, or 5.5

c) $\sqrt{20.25} = \sqrt{\frac{2025}{100}}$

$= \sqrt{\frac{45}{10} \times \frac{45}{10}}$

$= \frac{45}{10}$, or 4.5

d) $\sqrt{56.25} = \sqrt{\frac{5625}{100}}$

$= \sqrt{\frac{75}{10} \times \frac{75}{10}}$

$= \frac{75}{10}$, or 7.5

11. a) i) $36.0 = \frac{36}{1}$

Since 36 = 6 × 6 and 1 = 1 × 1, we can write:

$\frac{36}{1} = \frac{6}{1} \times \frac{6}{1}$

Since $\frac{36}{1}$ can be written as a product of two equal fractions, it is a perfect square.

So, 36.0 is also a perfect square.

ii) $3.6 = \frac{36}{10}$ Simplify the fraction. Divide the numerator and denominator by 2.

$= \frac{18}{5}$

Since neither 18 nor 5 can be written as a product of equal factors, 3.6 is not a perfect square.

iii) $0.36 = \frac{36}{100}$ Simplify the fraction. Divide the numerator and denominator by 4.

$= \frac{9}{25}$

Since 9 = 3 × 3 and 25 = 5 × 5, we can write:

$\frac{9}{25} = \frac{3}{5} \times \frac{3}{5}$

Since $\frac{9}{25}$ can be written as a product of two equal fractions, it is a perfect square.

So, 0.36 is also a perfect square.

iv) $0.036 = \frac{36}{1000}$ Simplify the fraction. Divide the numerator and denominator by 4.

$= \frac{9}{250}$

The numerator can be written as 9 = 3 × 3, but the denominator cannot be written as a product of equal factors.

So, 0.036 is not a perfect square.

v) $0.0036 = \frac{36}{10\,000}$ Simplify the fraction. Divide the numerator and denominator by 4.

$= \frac{9}{2500}$

Since 9 = 3 × 3 and 2500 = 50 × 50, we can write:

$\frac{9}{2500} = \frac{3}{50} \times \frac{3}{50}$

Since $\frac{9}{2500}$ can be written as a product of two equal fractions, it is a perfect square.

So, 0.0036 is also a perfect square.

vi) $0.000\,36 = \frac{36}{100\,000}$ Simplify the fraction. Divide the numerator and denominator by 4.

$= \frac{9}{25\,000}$

The numerator can be written as 9 = 3 × 3, but the denominator cannot be written as a product of equal factors.

So, 0.000 36 is not a perfect square.

b) i) $\sqrt{36.0} = 6$

ii) $\sqrt{3.6} = 1.897\ 366\ 596\ldots$
$\doteq 1.9$

iii) $\sqrt{0.36} = 0.6$

iv) $\sqrt{0.036} = 0.189\ 736\ 659\ 6\ldots$
$\doteq 0.19$

v) $\sqrt{0.0036} = 0.06$

vi) $\sqrt{0.000\ 36} = 0.018\ 973\ 666\ldots$
$\doteq 0.19$

c) Answers will vary, but can include: When the denominators are 1, 100, and 10 000, the decimal is a perfect square; alternate decimals in part b are perfect squares. The decimals that are perfect squares differ by a factor of 100; the square roots of these decimals differ by a factor of 10.

d) You can use the square roots of whole numbers to determine the square roots of decimals when, if you convert the decimal to a fraction, both the numerator and denominator are perfect squares.

12. a) i) Since $\sqrt{9}=3$, then $\sqrt{900}=30$, and $\sqrt{90\ 000}=300$

ii) Since $\sqrt{9}=3$, then $\sqrt{900}=30$

iii) $\sqrt{0.09} = \sqrt{\frac{9}{100}}$
$= \frac{3}{10}$
$= 0.3$

iv) $\sqrt{0.0009} = \sqrt{\frac{9}{10\ 000}}$
$= \frac{3}{100}$
$= 0.03$

b) i) $\sqrt{0.0025} = \sqrt{\frac{25}{10\ 000}}$
$= \frac{5}{100}$
$= 0.05$

ii) $\sqrt{0.25} = \sqrt{\frac{25}{100}}$
$= \frac{5}{10}$
$= 0.5$

iii) Since $\sqrt{25} = 5$, then $\sqrt{2500} = 50$

iv) Since $\sqrt{25} = 5$, then $\sqrt{2500} = 50$ and $\sqrt{250\,000} = 500$

c) Answers will vary. For example:
I chose the whole number 16. I know that $\sqrt{16}$ = 4. Then:
$\sqrt{0.16}$ = 0.4
$\sqrt{0.0016}$ = 0.04
$\sqrt{0.000\,016}$ = 0.004
I know the square roots are correct because the equivalent fractions have denominators 100, 10 000, and 1 000 000. Based on my answers from parts a and b, I know fractions with these denominators can be written as a product of equal factors. The numerator 16 is a perfect square, and the denominators are also perfect squares.
I can use a calculator to check:
0.4 × 0.4 = 0.16
0.04 × 0.04 = 0.0016
0.04 × 0.04 = 0.000 016

13. a) i) $\sqrt{12.25} = \sqrt{\frac{1225}{100}}$
$= \sqrt{\frac{35}{10} \times \frac{35}{10}}$
$= \frac{35}{10}$
$= 3.5$
The square root corresponds to the letter C.

ii) $\sqrt{\frac{121}{25}} = \sqrt{\frac{11}{5} \times \frac{11}{5}}$
$= \frac{11}{5}$, or 2.2
The square root corresponds to the letter A.

iii) $\sqrt{16.81} = \sqrt{\frac{1681}{100}}$
$= \sqrt{\frac{41}{10} \times \frac{41}{10}}$
$= \frac{41}{10}$
$= 4.1$
The square root corresponds to the letter E.

iv) $\sqrt{\frac{81}{100}} = \sqrt{\frac{9}{10} \times \frac{9}{10}}$
$= \frac{9}{10}$, or 0.9
The square root corresponds to the letter B.

v) $\sqrt{0.09} = \sqrt{\frac{9}{100}}$

$= \sqrt{\frac{3}{10} \times \frac{3}{10}}$

$= \frac{3}{10}$, or 0.3

The square root corresponds to the letter F.

vi) $\sqrt{\frac{841}{25}} = \sqrt{\frac{29}{5} \times \frac{29}{5}}$

$= \frac{29}{5}$, or 5.8

The square root corresponds to the letter D.

b) Answers will vary. For example:

Let $G = \sqrt{5.76}$

$\sqrt{5.76} = \sqrt{\frac{576}{100}}$

$= \sqrt{\frac{24}{10} \times \frac{24}{10}}$

$= \frac{24}{10}$, or 2.4

Let $H = \sqrt{1.69}$

$\sqrt{1.69} = \sqrt{\frac{169}{100}}$

$= \sqrt{\frac{13}{10} \times \frac{13}{10}}$

$= \frac{13}{10}$, or 1.3

Let $J = \sqrt{0.3025}$

$\sqrt{0.3025} = \sqrt{\frac{3025}{10\,000}}$

$= \sqrt{\frac{55}{100} \times \frac{55}{100}}$

$= \frac{55}{100}$, or 0.55

The number line will appear as follows:

14. a) The side length of a square is the square root of its area.

$\sqrt{5.76} = \sqrt{\frac{576}{100}}$

$= \sqrt{\frac{24}{10} \times \frac{24}{10}}$

$= \frac{24}{10}$, or 2.4

The side length of the square is 2.4 cm.

b) The perimeter of a square is 4 times its side length.
$4 \times 2.4 = 9.6$
The perimeter of the square is 9.6 cm.

15. The side length of a square is the square root of its area.
a) Least area: 6.25 km^2
Least side length, in kilometres: $\sqrt{6.25} = 2.5$
The least possible side length is 2.5 km.

b) Greatest area: 10.24 km^2
Greatest side length, in kilometres: $\sqrt{10.24} = 3.2$
The greatest possible side length is 3.2 km.

c) The area of the square, in square kilometres, is: $(2.8)^2 = 7.84$

16. No, the student is not correct.
Check by calculating $0.02^2 = 0.0004$, which is not equal to 0.04.
The correct square root is:

$$\begin{aligned}\sqrt{0.04} &= \sqrt{\frac{4}{100}} \\ &= \sqrt{\frac{2}{10} \times \frac{2}{10}} \\ &= \frac{2}{10}, \text{ or } 0.2\end{aligned}$$

17. Answers may vary. For example:
a) The name *Pythagorean triple* is appropriate because the three numbers (the triplet) can be the side lengths of a right triangle and they satisfy the Pythagorean Theorem.

b) The following numbers are the Pythagorean triples with squares between 1 and 400:

Triple	Equation
3, 4, 5	$3^2 + 4^2 = 5^2$
6, 8, 10	$6^2 + 8^2 = 10^2$
9, 12, 15	$9^2 + 12^2 = 15^2$
12, 16, 20	$12^2 + 16^2 = 20^2$
5, 12, 13	$5^2 + 12^2 = 13^2$
8, 15, 17	$8^2 + 15^2 = 17^2$

Take It Further

18. To determine the perfect squares between 0.64 and 0.81, take the square root of 0.64 and 0.81.

$$\begin{aligned}\sqrt{0.64} &= \sqrt{\frac{64}{100}} \\ &= \sqrt{\frac{8}{10}\times\frac{8}{10}} \\ &= \frac{8}{10}\text{, or } 0.8\end{aligned}$$

$$\begin{aligned}\sqrt{0.81} &= \sqrt{\frac{81}{100}} \\ &= \sqrt{\frac{9}{10}\times\frac{9}{10}} \\ &= \frac{9}{10}\text{, or } 0.9\end{aligned}$$

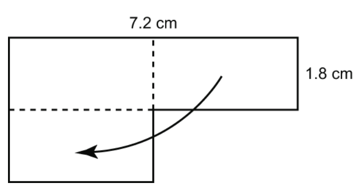
The squares of all numbers between 0.8 and 0.9 are between 0.64 and 0.81.
For example: $0.85^2 = 0.7225$, $0.875^2 = 0.765\,625$, and $0.89^2 = 0.7921$.
So, 0.7225, 0.765 625, and 0.7921 are all perfect squares between 0.64 and 0.81.

19. a) The area of a rectangle is its length multiplied by its width:
$7.2 \text{ cm} \times 1.8 \text{ cm} = 12.96 \text{ cm}^2$
So, the area of the square is also 12.96 cm^2.
Then, the side length of the square is:

$$\begin{aligned}\sqrt{12.96} &= \sqrt{\frac{1296}{100}} \\ &= \sqrt{\frac{36}{10}\times\frac{36}{10}} \\ &= \frac{36}{10}\text{, or } 3.6\end{aligned}$$

The square has side length 3.6 cm.

b) Only one cut is needed. The student can cut the 7.2-cm by 1.8-cm rectangle in half lengthwise and place the two pieces together to make a square with side length 3.6 cm.



Lesson 1.2 **Square Roots of Non-Perfect Squares** **Practice (pages 18–20)**

Check

4. **a)** The two perfect squares with a square root closest to $\sqrt{3.5}$ are 1 and 4; $\sqrt{1}=1$ and $\sqrt{4}=2$

b) The two perfect squares with a square root closest to $\sqrt{13.5}$ are 9 and 16; $\sqrt{9}=3$ and $\sqrt{16}=4$.

c) The two perfect squares with a square root closest to $\sqrt{53.5}$ are 49 and 64; $\sqrt{49}=7$ and $\sqrt{64}=8$.

d) The two perfect squares with a square root closest to $\sqrt{73.5}$ are 64 and 81; $\sqrt{64}=8$ and $\sqrt{81}=9$.

e) The two perfect squares with a square root closest to $\sqrt{93.5}$ are 81 and 100; $\sqrt{81}=9$ and $\sqrt{100}=10$.

f) The two perfect squares with a square root closest to $\sqrt{113.5}$ are 100 and 121; $\sqrt{100}=10$ and $\sqrt{121}=11$.

5. **a)** $\sqrt{\frac{5}{10}} = \sqrt{0.5}$

The two perfect squares with a square root closest to $\sqrt{0.5}$ are 0.49 and 0.64; $\sqrt{0.49}=0.7$ and $\sqrt{0.64}=0.8$.

b) $\sqrt{\frac{55}{10}} = \sqrt{5.5}$

The two perfect squares with a square root closest to $\sqrt{5.5}$ are 4 and 9; $\sqrt{4}=2$ and $\sqrt{9}=3$.

c) $\sqrt{\frac{95}{10}} = \sqrt{9.5}$

The two perfect squares with a square root closest to $\sqrt{9.5}$ are 9 and 16; $\sqrt{9}=3$ and $\sqrt{16}=4$.

d) $\sqrt{\frac{595}{10}} = \sqrt{59.5}$

The two perfect squares with a square root closest to $\sqrt{59.5}$ are 49 and 64; $\sqrt{49}=7$ and $\sqrt{64}=8$.

e) $\sqrt{\frac{795}{10}} = \sqrt{79.5}$

The two perfect squares with a square root closest to $\sqrt{79.5}$ are 64 and 81; $\sqrt{64}=8$ and $\sqrt{81}=9$.

f) $\sqrt{\frac{1095}{10}} = \sqrt{109.5}$

The two perfect squares with a square root closest to $\sqrt{109.5}$ are 100 and 121; $\sqrt{100}=10$ and $\sqrt{121}=11$.

Apply

6. Estimates will vary; for example:

a) $\sqrt{\frac{8}{10}} = \sqrt{\frac{80}{100}}$ and

$\sqrt{\frac{80}{100}} \doteq \sqrt{\frac{81}{100}}$

$= \sqrt{\frac{9}{10} \times \frac{9}{10}}$

$= \frac{9}{10}$

So, $\sqrt{\frac{8}{10}} \doteq \frac{9}{10}$.

b) $\sqrt{\frac{17}{5}} = \sqrt{\frac{85}{25}}$

In the fraction $\frac{85}{25}$, 85 is close to the perfect square 81.

So, $\sqrt{\frac{85}{25}}$ is close to $\sqrt{\frac{81}{25}}$.

$\sqrt{\frac{85}{25}} \doteq \sqrt{\frac{81}{25}}$

$= \sqrt{\frac{9}{5} \times \frac{9}{5}}$

$= \frac{9}{5}$

So, $\sqrt{\frac{17}{5}} \doteq \frac{9}{5}$.

c) In the fraction $\frac{7}{13}$, 7 is close to the perfect square 9, and 13 is close to the perfect square 16.

So, $\sqrt{\frac{7}{13}} \doteq \sqrt{\frac{9}{16}}$

$\sqrt{\frac{9}{16}} = \frac{3}{4}$

So, $\sqrt{\frac{7}{13}} \doteq \frac{3}{4}$.

Or, another way:

$\sqrt{\frac{7}{13}} \doteq \sqrt{\frac{49}{100}}$ and $\sqrt{\frac{49}{100}} = \frac{7}{10}$.

So, $\sqrt{\frac{7}{13}} \doteq \frac{7}{10}$.

d) In the fraction $\frac{29}{6}$, 29 is close to the perfect square 25, and 6 is close to the perfect square 4.

So, $\sqrt{\frac{29}{6}} \doteq \sqrt{\frac{25}{4}}$

$\sqrt{\frac{25}{4}} = \frac{5}{2}$

So, $\sqrt{\frac{29}{6}} \doteq \frac{5}{2}$.

Or, another way:

$\sqrt{\frac{29}{6}} \doteq \sqrt{\frac{169}{36}}$ and $\sqrt{\frac{169}{36}} = \frac{13}{6}$.

So, $\sqrt{\frac{29}{6}} \doteq \frac{13}{6}$.

7. Approximations will vary. For example:

a) Think of the closest perfect squares on either side of 4.5: 4 and 9.

$\sqrt{4} = 2$ and $\sqrt{9} = 3$

$\sqrt{4.5}$ is between $\sqrt{4}$ and $\sqrt{9}$, and closer to $\sqrt{4}$.

So, $\sqrt{4.5}$ is between 2 and 3, and closer to 2.

Choose 2.1 as an estimate.

To check, evaluate: $2.1^2 = 4.41$

This is very close to 4.5, so $\sqrt{4.5} \doteq 2.1$.

b) Think of the closest perfect squares on either side of 14.5: 9 and 16.

$\sqrt{9} = 3$ and $\sqrt{16} = 4$

$\sqrt{14.5}$ is between $\sqrt{9}$ and $\sqrt{16}$, and closer to $\sqrt{16}$.

So, $\sqrt{14.5}$ is between 3 and 4, and closer to 4.

Choose 3.8 as an estimate.

To check, evaluate: $3.8^2 = 14.44$

This is very close to 14.5, so $\sqrt{14.5} \doteq 3.8$.

c) Think of the closest perfect squares on either side of 84.5: 81 and 100.

$\sqrt{81} = 9$ and $\sqrt{100} = 10$

$\sqrt{84.5}$ is between $\sqrt{81}$ and $\sqrt{100}$, and closer to $\sqrt{81}$.

So, $\sqrt{84.5}$ is between 9 and 10, and closer to 9.

Choose 9.2 as an estimate.

To check, evaluate: $9.2^2 = 84.64$

This is very close to 84.5, so $\sqrt{84.5} \doteq 9.2$.

d) Think of the closest perfect squares on either side of 145.5: 144 and 169

$\sqrt{144} = 12$ and $\sqrt{169} = 13$

$\sqrt{145.5}$ is between $\sqrt{144}$ and $\sqrt{169}$, and closer to $\sqrt{144}$.

So, $\sqrt{145.5}$ is between 12 and 13, and closer to 12.

Choose 12.1 as an estimate.

To check, evaluate: $12.1^2 = 146.41$

This is very close to 145.5, so $\sqrt{145.5} \doteq 12.1$.

e) Think of the closest perfect squares on either side of 284.5: 256 and 289
$\sqrt{256} = 16$ and $\sqrt{289} = 17$
$\sqrt{284.5}$ is between $\sqrt{256}$ and $\sqrt{289}$, and closer to $\sqrt{289}$.
So, $\sqrt{284.5}$ is between 16 and 17, and closer to 17.
Choose 16.9 as an estimate.
To check, evaluate: $16.9^2 = 285.61$
This is very close to 284.5, so, $\sqrt{284.5} \doteq 16.9$.

f) Think of the closest perfect squares on either side of 304.5: 289 and 324
$\sqrt{289} = 17$ and $\sqrt{324} = 18$
$\sqrt{304.5}$ is between $\sqrt{289}$ and $\sqrt{324}$, and closer to $\sqrt{289}$.
So, $\sqrt{304.5}$ is between 17 and 18, and closer to 17.
Choose 17.4 as an estimate.
To check, evaluate: $17.4^2 = 302.76$
This is very close to 304.5, so $\sqrt{304.5} \doteq 17.4$.

8. a) 29.5 is between 25 and 36, and closer to 25,
So, $\sqrt{29.5}$ is between $\sqrt{25}$ and $\sqrt{36}$, and closer to $\sqrt{25}$.
$\sqrt{25} = 5$ and $\sqrt{36} = 6$
So, $\sqrt{29.5}$ is between 5 and 6, and closer to 5.
Choose an estimate of 5.4.
To check, evaluate: $5.4^2 = 29.16$, which is close to 29.5.
So, $\sqrt{29.5} \doteq 5.4$.

b) $\frac{5}{2} = 2.5$ is between 1 and 4.
$\sqrt{\frac{5}{2}} = \sqrt{2.5}$, which is between $\sqrt{1}$ and $\sqrt{4}$.
$\sqrt{1} = 1$ and $\sqrt{4} = 2$
So, $\sqrt{\frac{5}{2}}$ is between 1 and 2.
Choose an estimate of 1.5.
To check, evaluate: $1.5^2 = 2.25$; this is close to 2.5, but less than 2.5.
Choose an estimate of 1.6.
To check, evaluate: $1.6^2 = 2.56$
2.5 is closer to 2.56 than to 2.25, so the better estimate is 1.6.
So, $\sqrt{\frac{5}{2}} \doteq 1.6$.

9. a) Since $2.2^2 = 4.84$, or 4.8 to the nearest tenth, the estimate is incorrect: too high
Choose 2.1 as an estimate; $2.1^2 = 4.41$, which is a closer estimate.
So, $\sqrt{4.4} \doteq 2.1$.

b) Since $0.3^2 = 0.09$, or 0.1 to the nearest tenth, the estimate is incorrect: too low
Choose 0.8 as an estimate; $0.8^2 = 0.64$, which is a closer estimate.
So, $\sqrt{0.6} \doteq 0.8$.

c) The estimate is correct. I used a calculator.
$\sqrt{6.6} \doteq 2.569$, or 2.6 to the nearest tenth.

d) Since $0.2^2 = 0.04$, the estimate is incorrect: too low
Choose 0.6 as an estimate; $0.6^2 = 0.36$
Choose 0.7 as an estimate; $0.7^2 = 0.49$
0.4 is closer to 0.36, so $\sqrt{0.4} \doteq 0.6$ is a closer estimate.

10. Answers may vary.
a) $3^2 = 9$ and $4^2 = 16$, so any number between 9 and 16 has a square root between 3 and 4.
Two decimals between 9 and 16 are 10.24 and 12.25.
$\sqrt{10.24} = 3.2$ and $\sqrt{12.25} = 3.5$

b) $7^2 = 49$ and $8^2 = 64$, so any number between 49 and 64 has a square root between 7 and 8.
Two decimals between 49 and 64 are 50.41 and 59.29.
$\sqrt{50.41} = 7.1$ and $\sqrt{59.29} = 7.7$

c) $12^2 = 144$ and $13^2 = 169$, so any number between 144 and 169 has a square root between 12 and 13.
Two decimals between 144 and 169 are 158.36 and 166.41.
$\sqrt{158.36} \doteq 12.6$ and $\sqrt{166.41} = 12.9$

d) $1.5^2 = 2.25$ and $2.5^2 = 6.25$, so any number between 2.25 and 6.25 has a square root between 1.5 and 2.5. Two decimals between 2.25 and 6.25 are 3.0 and 3.5.
$\sqrt{3.0} \doteq 1.7$ and $\sqrt{3.5} \doteq 1.9$

e) $4.5^2 = 20.25$ and $5.5^2 = 30.25$, so any number between 20.25 and 30.25 has a square root between 4.5 and 5.5. Two decimals between 20.25 and 30.25 are 22.09 and 29.16.
$\sqrt{22.09} = 4.7$ and $\sqrt{29.16} = 5.4$

11. I used benchmarks to estimate the value of each square root, because they provided a small range of numbers to work with.
a) 4.5 is between the perfect squares 4 and 9, and closer to 4.
So, $\sqrt{4.5}$ is between 2 and 3, and closer to 2.
Estimate $\sqrt{4.5}$ as 2.1.
To check, evaluate: $2.1^2 = 4.41$
This is very close to 4.5, so $\sqrt{4.5} \doteq 2.1$.

b) $\frac{17}{2} = 8.5$
8.5 is between the perfect squares 4 and 9, and closer to 9.
So, $\sqrt{\frac{17}{2}}$ is between 2 and 3, and closer to 3.
Estimate $\sqrt{\frac{17}{2}}$ as 2.9.
To check, evaluate: $2.9^2 = 8.41$, which is very close to 8.5.
So, $\sqrt{\frac{17}{2}} \doteq 2.9$

c) 0.15 is between the perfect squares 0.09 and 0.16, and closer to 0.16.
So, $\sqrt{0.15}$ is between 0.3 and 0.4, and closer to 0.4.
Estimate $\sqrt{0.15}$ as 0.39.
To check, evaluate: $0.39^2 = 0.1521$, which is very close to 0.15.
So, $\sqrt{0.15} \doteq 0.39$

d) In the fraction $\frac{10}{41}$, 10 is close to the perfect square 9, and 41 is close to the perfect square 36.

So, $\sqrt{\frac{10}{41}} \doteq \sqrt{\frac{9}{36}}$

$\sqrt{\frac{9}{36}} = \frac{3}{6}$, or $\frac{1}{2}$

So, $\sqrt{\frac{10}{41}} \doteq \frac{1}{2}$, or 0.5

e) 0.7 is between the perfect squares 0.64 and 0.81, and closer to 0.64.

So, $\sqrt{0.7}$ is between 0.8 and 0.9, and closer to 0.8.

Estimate $\sqrt{0.7}$ as 0.84.

To check, evaluate: $0.84^2 = 0.7056$, which is very close to 0.7.

So, $\sqrt{0.7} \doteq 0.84$

f) In the fraction $\frac{8}{45}$, 8 is close to the perfect square 9, and 45 is close to the perfect square 49.

So, $\sqrt{\frac{8}{45}} \doteq \sqrt{\frac{9}{49}}$

$\sqrt{\frac{9}{49}} = \frac{3}{7}$

So, $\sqrt{\frac{8}{45}} \doteq \frac{3}{7}$, or about 0.4

g) 0.05 is between the perfect squares 0.04 and 0.09, and closer to 0.04.

So, $\sqrt{0.05}$ is between 0.2 and 0.3, and closer to 0.2.

Estimate $\sqrt{0.05}$ as 0.22.

To check, evaluate: $0.22^2 = 0.0484$, which is very close to 0.05.

So, $\sqrt{0.05} \doteq 0.22$

h) In the fraction $\frac{90}{19}$, 90 is close to the perfect square 81, and 19 is close to the perfect square 16.

So, $\sqrt{\frac{90}{19}} \doteq \sqrt{\frac{81}{16}}$

$\sqrt{\frac{81}{16}} = \frac{9}{4}$

So, $\sqrt{\frac{90}{19}} \doteq \frac{9}{4}$

12. Estimates may vary. I used benchmarks.

a) In the fraction $\frac{3}{8}$, 3 is close to the perfect square 4, and 8 is close to the perfect square 9.

So, $\sqrt{\frac{3}{8}} \doteq \sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{3}{8}} \doteq \frac{2}{3}$, or about 0.7 to the nearest tenth

b) In the fraction $\frac{5}{12}$, 5 is close to the perfect square 4, and 12 is close to the perfect square 9.

So, $\sqrt{\frac{5}{12}} \doteq \sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{5}{12}} \doteq \frac{2}{3}$, or about 0.7 to the nearest tenth

c) In the fraction $\frac{13}{4}$, 13 is close to the perfect square 16, and 4 is a perfect square.

So, $\sqrt{\frac{13}{4}} \doteq \sqrt{\frac{16}{4}}$

$\sqrt{\frac{16}{4}} = 2$

So, $\sqrt{\frac{13}{4}} \doteq 2.0$

d) I wrote the fraction as a decimal, then used benchmarks.

$\frac{25}{3} \doteq 8.3$

The closest perfect squares on either side of 8.3 are 4 and 9.

$\sqrt{4} = 2$ and $\sqrt{9} = 3$

8.3 is closer to 9, so choose 2.9 as a possible estimate for a square root.

To check, evaluate: $2.9^2 = 8.41$, which is close to 8.3.

So, $\sqrt{\frac{25}{3}} \doteq 2.9$

13. Write the Pythagorean Theorem in each right triangle to determine the unknown length.

a) $h^2 = 1.2^2 + 0.5^2$

$= 1.44 + 0.25$

$= 1.69$

$h = \sqrt{1.69}$

$= 1.3$

The unknown length h is 1.3 cm.

b) $h^2 = 1.5^2 + 2.2^2$

$= 2.25 + 4.84$

$= 7.09$

$h = \sqrt{7.09}$

$\doteq 2.663$

The unknown length h is about 2.7 cm.

c) $s^2 + 2.8^2 = 5.6^2$

$s^2 = 5.6^2 - 2.8^2$

$= 31.36 - 7.84$

$= 23.52$

$s = \sqrt{23.52}$

$\doteq 4.85$

The unknown length s is about 4.9 cm.

d) $s^2 + 2.4^2 = 2.5^2$
$s^2 = 2.5^2 - 2.4^2$
$= 6.25 - 5.76$
$= 0.49$
$s = \sqrt{0.49}$
$= 0.7$
The unknown length s is 0.7 cm.

14. For decimals:
$0.5^2 = 0.25$ and $0.6^2 = 0.36$
Any number between 0.25 and 0.36 has a square root between 0.5 and 0.6.
For example:
$0.51^2 = 0.2601$; $0.52^2 = 0.2704$; … $0.599^2 = 0.358\ 801$

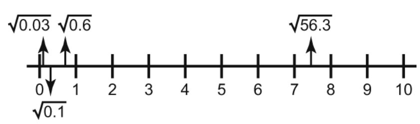
For fractions:
$0.5 = \frac{5}{10}$ and $0.6 = \frac{6}{10}$
Write these fractions with denominator 100: $\frac{50}{100}$ and $\frac{60}{100}$
$\left(\frac{50}{100}\right)^2 = \frac{2500}{10\ 000}$ and $\left(\frac{60}{100}\right)^2 = \frac{3600}{10\ 000}$
Any number between $\frac{2500}{10\ 000}$ and $\frac{3600}{10\ 000}$ will have a square root between $\frac{50}{100}$ and $\frac{60}{100}$, or 0.5 and 0.6.
For example:
$\left(\frac{53}{100}\right)^2 = \frac{2809}{10\ 000}$; $\left(\frac{56}{100}\right)^2 = \frac{3136}{10\ 000}$; and $\left(\frac{59}{100}\right)^2 = \frac{3481}{10\ 000}$

15. a) $\sqrt{0.1} \doteq 0.32$

b) $\sqrt{56.3} \doteq 7.5$

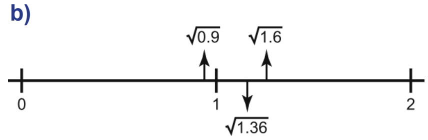
c) $\sqrt{0.6} \doteq 0.78$

d) $\sqrt{0.03} \doteq 0.17$



16. a) $\sqrt{0.9} \doteq 0.95$; this is placed too far to the left on the number line.
$\sqrt{0.25} = 0.5$; this is correctly placed.
$\sqrt{0.5} \doteq 0.71$; this is correctly placed.
$\sqrt{1.6} \doteq 1.26$; this is placed too far to the left.
$\sqrt{1.44} = 1.2$; this is correctly placed.
$\sqrt{1.36} \doteq 1.17$; this is placed too far to the right.
$\sqrt{3.6} \doteq 1.90$; this is correctly placed.

b)



17. a) $\sqrt{52.9}$

$\sqrt{52.9} = 7.2732...$
$\doteq 7.3$

b) $\sqrt{5.29}$

$\sqrt{5.29} = 2.3$

c) $\sqrt{2.25}$

$\sqrt{2.25} = 1.5$

d) $\sqrt{22.5}$

$\sqrt{22.5} = 4.7434...$
$\doteq 4.7$

The square roots in parts a and d are approximate, because the decimals do not terminate or repeat.

18. a) The numbers greater than 1 have square roots that are less than the given number.
For example: $\sqrt{2} \doteq 1.4142$, $\sqrt{4} = 2$, $\sqrt{10} \doteq 3.1623$, $\sqrt{100} = 10$

b) The number must be 0 or 1 for its square root to be equal to the number.
$\sqrt{0} = 0$, $\sqrt{1} = 1$

c) The numbers less than 1 have square roots that are greater than the given number.
For example: $\sqrt{0.1} \doteq 0.3162$, $\sqrt{0.01} = 0.1$, $\sqrt{0.09} = 0.3$, $\sqrt{0.025} \doteq 0.1581$

19. Answers will vary. For example:

a) The number with a square root of 1 is 1. So, any number between 0 and 1 has a square root between 0 and 1. For example: 0.64
$\sqrt{0.64}$ is between 0 and 1.

b) The number with a square root of 1.5 is 2.25. The number with a square root of 2 is 4.
So, any number between 2.25 and 4 has a square root between 1.5 and 2. For example: 3
$\sqrt{3} \doteq 1.7321$ is between 1.5 and 2.

c) Write $\frac{1}{2}$ and $\frac{3}{4}$ as decimals:

$\frac{1}{2} = 0.5$ and $\frac{3}{4} = 0.75$

The number with a square root of 0.5 is 0.25. The number with a square root of 0.75 is 0.5625. So, any number between 0.25 and 0.5625 has a square root between $\frac{1}{2}$ and $\frac{3}{4}$. For example: 0.4, or $\frac{4}{10}$

$\sqrt{0.4} = \sqrt{\frac{4}{10}}$
$\doteq 0.63$

So, $\sqrt{0.4}$ is between $\frac{1}{2}$ and $\frac{3}{4}$.

d) Write $3\frac{3}{4}$ as a decimal:

$3\frac{3}{4} = 3.75$

The number with a square root of 3.75 is 14.0625.
The number with a square root of 4 is 16.
So, any number between 14.0625 and 16 has a square root between $3\frac{3}{4}$ and 5.

For example: 15

$\sqrt{15} \doteq 3.873$ is between $3\frac{3}{4}$ and 4.

20. a) $AB^2 = 0.5^2 + 1.75^2$
$= 0.25 + 3.0625$
$= 3.3125$
$AB = \sqrt{3.3125}$
$\doteq 1.82$
AB is about 1.82 km.

b) $AB^2 = 1.25^2 + 2^2$
$= 1.5625 + 4$
$= 5.5625$
$AB = \sqrt{5.5625}$
$\doteq 2.3585$
AB is about 2.36 km.

21. a) i) $\sqrt{0.005} = 0.0707107\ldots$
$\doteq 0.0707$

ii) $\sqrt{0.5} = 0.7071068\ldots$
$\doteq 0.7071$

iii) $\sqrt{50} = 7.0710678\ldots$
$\doteq 7.0711$

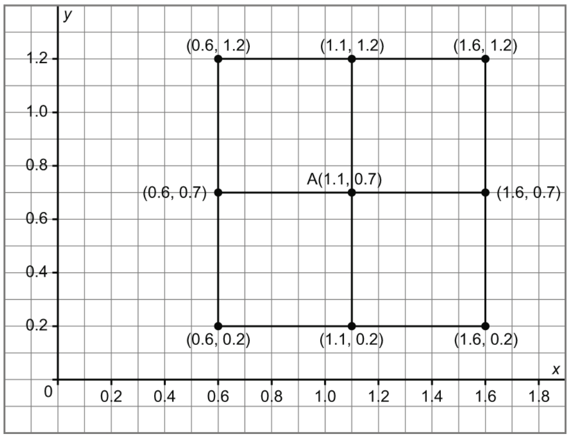
iv) $\sqrt{5000} = 70.7106781\ldots$
$\doteq 70.7107$

v) $\sqrt{500\,000} = 707.1067812\ldots$
$\doteq 707.1068$

b) Each square root is ten times greater than the previous one.
The previous two square roots in the pattern less than $\sqrt{0.005}$ will be: $\sqrt{0.000\,05} \doteq 0.007\,071$ and $\sqrt{0.000\,0005} \doteq 0.000\,707\,1$. The next two square roots in the pattern greater than $\sqrt{500\,000}$ will be: $\sqrt{50\,000\,000} \doteq 7\,071.0678$ and $\sqrt{5\,000\,000\,000} \doteq 70\,710.678\,1$.

22. $\sqrt{0.6} \doteq 0.775$ and $\sqrt{0.61} \doteq 0.781$
So, all numbers between 0.775 and 0.781 have squares between 0.6 and 0.61.
For example: $0.776^2 \doteq 0.6022$, $0.779^2 \doteq 0.6068$, and $0.78^2 = 0.6084$, which are all between 0.6 and 0.61.

23. The square has area 0.25 square units.
Take the square root of its area to determine the side length.
$\sqrt{0.25} = 0.5$
So, the side length of the square is 0.5 units.
The scale is 1 grid square represents 0.1 units. So, 0.5 units represent 5 grid squares.
Since one vertex of the square is (1.1, 0.7), the other vertices of one possible square are:
(1.1, 0.2), (0.6, 0.2), and (0.6, 0.7).



It is possible to get other vertices by considering different positions of the square:
(1.1, 0.2), (1.6, 0.2), and (1.6, 0.7) or
(1.6, 0.7), (1.6, 1.2), and (1.1, 1.2) or
(1.1, 1.2), (0.6, 1.2), and (0.6, 0.7).

24. a) The area of the original photograph, in square centimetres, is $(5.5)^2$, or 30.25 cm^2.
The area of the larger photograph is: 30.25 $cm^2 \times 2$ = 60.5 cm^2
To determine the side length of the square, take the square root of the area of the square: $\sqrt{60.5} \doteq 7.8$
So, the side length of the larger photograph is approximately 7.8 cm.

b) For example, a square with side length 5.5 cm has an area of $(5.5)^2$, or 30.25 cm^2.
Double the side length of the square: 5.5 cm $\times$ 2 = 11 cm
A square with side length 11 cm has an area of $(11)^2$, or 121 cm^2, which is 30.25 $cm^2 \times 4$.
So, doubling the side length of a square does not double the area; it increases the area by a factor of 4.

Mid-Unit Review (page 21)

Lesson 1.1

1. a) $\sqrt{\frac{25}{36}}$

The square is divided into 36 equal parts, so the area of one small square is $\frac{1}{36}$ square units.

25 squares are shaded, so the shaded square has an area of $\frac{25}{36}$ square units.

The side length of the shaded square is equal to the square root of its area. Each side length is 5 one-sixths, so the square root of $\sqrt{\frac{25}{36}}$ is $\frac{5}{6}$.

b) $\sqrt{0.36}$

The square is divided into 100 equal parts, so the area of one small square is $\frac{1}{100}$ square units.

36 squares are shaded so the shaded square has an area of $\frac{36}{100}$ square units.

The side length of the shaded square is equal to the square root of its area. Each side of the shaded square is six-tenths. The square root of $\sqrt{0.36}$ is $\frac{6}{10}$, or 0.6.

2. a) $1.4^2 = 1.96$

b) $\left(\frac{3}{8}\right)^2 = \frac{9}{64}$

c) $\left(\frac{7}{4}\right)^2 = \frac{49}{16}$

d) $0.5^2 = 0.25$

3. a) $\sqrt{0.04}$

$$\begin{aligned}\sqrt{0.04} &= \sqrt{\frac{4}{100}}\\ &= \sqrt{\frac{2}{10}\times\frac{2}{10}}\\ &= \frac{2}{10}\text{, or } 0.2\end{aligned}$$

b) $\sqrt{\frac{1}{16}}$

$$\begin{aligned}\sqrt{\frac{1}{16}} &= \sqrt{\frac{1}{4}\times\frac{1}{4}}\\ &= \frac{1}{4}\end{aligned}$$

c) $\sqrt{1.96}$

$$\begin{aligned}\sqrt{1.96} &= \sqrt{\frac{196}{100}}\\ &= \sqrt{\frac{14}{10}\times\frac{14}{10}}\\ &= \frac{14}{10}, \text{ or } 1.4\end{aligned}$$

d) $\sqrt{\frac{4}{81}}$

$$\begin{aligned}\sqrt{\frac{4}{81}} &= \sqrt{\frac{2}{9}\times\frac{2}{9}}\\ &= \frac{2}{9}\end{aligned}$$

e) $\sqrt{1.69}$

$$\begin{aligned}\sqrt{1.69} &= \sqrt{\frac{169}{100}}\\ &= \sqrt{\frac{13}{10}\times\frac{13}{10}}\\ &= \frac{13}{10}, \text{ or } 1.3\end{aligned}$$

f) $\sqrt{\frac{121}{49}}$

$$\begin{aligned}\sqrt{\frac{121}{49}} &= \sqrt{\frac{11}{7}\times\frac{11}{7}}\\ &= \frac{11}{7}\end{aligned}$$

g) $\sqrt{0.09}$

$$\begin{aligned}\sqrt{0.09} &= \sqrt{\frac{9}{100}}\\ &= \sqrt{\frac{3}{10}\times\frac{3}{10}}\\ &= \frac{3}{10}, \text{ or } 0.3\end{aligned}$$

h) $\sqrt{\frac{289}{100}}$

$$\begin{aligned}\sqrt{\frac{289}{100}} &= \sqrt{\frac{17}{10}\times\frac{17}{10}}\\ &= \frac{17}{10}\end{aligned}$$

4. a) $\sqrt{3.24}$

$$\sqrt{3.24} = \sqrt{\frac{324}{100}}$$
$$= \sqrt{\frac{18}{10} \times \frac{18}{10}}$$
$$= \frac{18}{10}, \text{ or } 1.8$$

b) $\sqrt{90.25}$

$$\sqrt{90.25} = \sqrt{\frac{9025}{100}}$$
$$= \sqrt{\frac{95}{10} \times \frac{95}{10}}$$
$$= \frac{95}{10}, \text{ or } 9.5$$

c) $\sqrt{2.56}$

$$\sqrt{2.56} = \sqrt{\frac{256}{100}}$$
$$= \sqrt{\frac{16}{10} \times \frac{16}{10}}$$
$$= \frac{16}{10}, \text{ or } 1.6$$

5. a) To determine the side length of the square, take the square root of its area.

$\sqrt{148.84} = 12.2$

The side length of the square is 12.2 cm.

b) The perimeter of a square is 4 times its side length.

$4 \times 12.2 = 48.8$

The perimeter of the square is 48.8 cm.

6. Use a calculator to determine: $0.04^2 = 0.0016$, which is different than 0.16.
So, the student is incorrect.

$$\sqrt{0.16} = \sqrt{\frac{16}{100}}$$
$$= \sqrt{\frac{4}{10} \times \frac{4}{10}}$$
$$= \frac{4}{10}, \text{ or } 0.4$$

7. a) $\frac{9}{64}$

Write $\frac{9}{64}$ as a product of two equal fractions.

$\frac{9}{64} = \frac{3}{8} \times \frac{3}{8}$

So, $\frac{9}{64}$ is a perfect square.

b) 3.6

$3.6 = \frac{36}{10}$, or $\frac{18}{5}$ in simplest form.

$\frac{18}{5}$ cannot be written as a product of two equal fractions.

So, 3.6 is not a perfect square.

c) $\frac{6}{9}$

The denominator can be written as $9 = 3 \times 3$, but the numerator cannot be written as a product of equal factors.

So, $\frac{6}{9}$ is not a perfect square.

d) 5.76

$5.76 = \frac{576}{100}$, or $\frac{144}{25}$ in simplest form.

Write $\frac{144}{25}$ as a product of two equal fractions: $\frac{144}{25} = \frac{12}{5} \times \frac{12}{5}$

So, 5.76 is a perfect square.

Lesson 1.2

8. Estimates will vary; for example:

a) 5.6 is between the perfect squares 4 and 9, and closer to 4.

So, $\sqrt{5.6}$ is between 2 and 3, and closer to 2.

Estimate $\sqrt{5.6}$ as 2.4.

To check, evaluate: $2.4^2 = 5.76$

This is very close to 5.6, so $\sqrt{5.6} \doteq 2.4$

b) Write $\frac{9}{10}$ as a decimal.

$\frac{9}{10} = 0.9$, which is between the perfect squares 0.81 and 1.

So, $\sqrt{\frac{9}{10}}$ is between 0.9 and 1.

Estimate $\sqrt{\frac{9}{10}}$ as 0.95.

To check, evaluate: $0.95^2 = 0.9025$

This is very close to 0.9, so $\sqrt{\frac{9}{10}} \doteq 0.95$

c) 42.8 is between the perfect squares 36 and 49.

So, $\sqrt{42.8}$ is between 6 and 7.

Estimate $\sqrt{42.8}$ as 6.5.

To check, evaluate: $6.5^2 = 42.45$

This is very close to 42.8, so $\sqrt{42.8} \doteq 6.5$

d) Write $\frac{356}{10}$ as a decimal:

$\frac{356}{10} = 35.6$

35.6 is between the perfect squares 25 and 36, and closer to 36.
So $\sqrt{35.6}$ is between 5 and 6, and closer to 6.
Choose a possible value for the square root; for example, 5.9
To check, evaluate: $5.9^2 = 34.81$
This is too low, so choose 5.95.
Evaluate: $5.95^2 = 35.4$
This is very close to 35.6, so $\sqrt{\frac{356}{10}} \doteq 5.95$

e) 0.056 is close to the perfect squares 0.04 and 0.09, and closer to 0.04.
So, $\sqrt{0.056}$ is between 0.2 and 0.3, and closer to 0.2.
Estimate $\sqrt{0.056}$ as 0.24.
To check, evaluate: $0.24^2 = 0.0576$
This is very close to 0.056, so $\sqrt{0.056} \doteq 0.24$

f) $\frac{9}{100}$ can be written as the product of two equal fractions: $\frac{3}{10} \times \frac{3}{10}$

So, $\sqrt{\frac{9}{100}} = \frac{3}{10}$, or 0.3

9. Write the Pythagorean Theorem in each triangle to determine the unknown length.

a) $h^2 = 2.5^2 + 1.6^2$
$= 6.25 + 2.56$
$= 8.81$
$h = \sqrt{8.81}$
$\doteq 2.97$
The unknown length h is about 3.0 cm.

b) $s^2 + 0.9^2 = 4.1^2$
$s^2 = 4.1^2 - 0.9^2$
$= 16.81 - 0.81$
$= 16$
$s = \sqrt{16}$
$= 4$
The unknown length s is 4 cm.

10. a) Correct: $\sqrt{0.09} = 0.3$, since $0.3^2 = 0.09$

b) Incorrect; $0.4^2 = 0.16$
$\sqrt{1.7} \doteq 1.3$

c) Correct; $\sqrt{8.5} \doteq 2.9$, since $2.9^2 = 8.41$, which is close to 8.5

d) Correct; $\sqrt{27.5} \doteq 5.2$, since $5.2^2 = 27.04$, which is close to 27.5

11. Answers will vary.

a) $4^2 = 16$ and $8^2 = 64$
So, any decimals between 16 and 64 will have square roots between 4 and 8.
For example: 20.25, 33.64
$\sqrt{20.25} = 4.5$ and $\sqrt{33.64} = 5.8$

b) $0.7^2 = 0.49$ and $0.9^2 = 0.81$
So, any decimals between 0.49 and 0.81 will have square roots between 0.7 and 0.9.
For example: 0.5625, 0.64
$\sqrt{0.5625} = 0.75$ and $\sqrt{0.64} = 0.8$

c) $1.25^2 = 1.5625$ and $1.35^2 = 1.8225$
So, any decimals between 1.5625 and 1.8225 will have square roots between 1.25 and 1.35.
For example, 1.69, 1.7
$\sqrt{1.69} = 1.3$ and $\sqrt{1.7} \doteq 1.304$

d) $0.25^2 = 0.0625$ and $0.35^2 = 0.1225$
So, any decimals between 0.0625 and 0.1225 will have square roots between 0.25 and 0.35.
For example, 0.09, 0.1024
$\sqrt{0.09} = 0.3$ and $\sqrt{0.1024} = 0.32$

e) $4.5^2 = 20.25$ and $5.5^2 = 30.25$
So, any decimals between 20.25 and 30.25 will have square roots between 4.5 and 5.5.
For example, 22.09, 28.09
$\sqrt{22.09} = 4.7$ and $\sqrt{28.09} = 5.3$

f) $0.05^2 = 0.0025$ and $0.1^2 = 0.01$
So, any decimals between 0.0025 and 0.01 will have square roots between 0.05 and 0.1.
For example, 0.0036, 0.0049
$\sqrt{0.0036} = 0.06$ and $\sqrt{0.0049} = 0.07$

Lesson 1.3 **Surface Areas of Objects Made from Right Rectangular Prisms** **Practice (pages 30–32)**

Check

4. **a)** The composite object has 3 cubes.
Each cube has 6 faces, so the total number of faces is: 3 × 6 = 18.
The cubes overlap in 2 places, so there are 2 × 2, or 4 faces that are not part of the surface area.
The surface area of the object, in square units, is: 18 – 4 = 14

b) The composite object has 4 cubes.
The total number of faces is 4 × 6 = 24.
The cubes overlap in 3 places, so there are 3 × 2, or 6 faces that are not part of the surface area.
The surface area of the object, in square units, is: 24 – 6 = 18

c) The composite object has 5 cubes.
The total number of faces is 5 × 6 = 30.
The cubes overlap in 4 places, so there are 2 × 4, or 8 faces that are not part of the surface area.
The surface area of the object, in square units, is: 30 – 8 = 22

d) The composite object has 5 cubes.
The total number of faces is 5 × 6 = 30.
The cubes overlap in 5 places, so there are 2 × 5, or 10 faces that are not part of the surface area.
The surface area of the object, in square units, is: 30 – 10 = 20

e) The composite object has 5 cubes.
The total number of faces is 5 × 6 = 30.
The cubes overlap in 4 places, so there are 2 × 4, or 8 faces that are not part of the surface area.
The surface area of the object, in square units, is: 30 – 8 = 22

f) The composite object has 6 cubes.
The total number of faces is 6 × 6 = 36.
The cubes overlap in 5 places, so there are 2 × 5, or 10 faces that are not part of the surface area.
The surface area of the object, in square units, is: 36 – 10 = 26

Apply

5. **a)** **i)** The composite object has 4 cubes.
The total number of faces is 4 × 6 = 24.
The cubes overlap in 3 places, so there are 2 × 3, or 6 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: 24 – 6 = 18

ii) The composite object has 4 cubes.
The total number of faces is 4 × 6 = 24.
The cubes overlap in 3 places, so there are 2 × 3, or 6 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: 24 – 6 = 18

iii) The composite object has 4 cubes.
The total number of faces is 4 × 6 = 24.
The cubes overlap in 3 places, so there are 2 × 3, or 6 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: 24 – 6 = 18

b) The surface areas are the same because the composite objects are made from the same number of cubes and the cubes overlap in the same number of places.

6. a) i) The composite object has 5 cubes. The total number of faces is $5 \times 6 = 30$.
The cubes overlap in 5 places, so there are 2×5, or 10 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: $30 - 10 = 20$

ii) The composite object has 5 cubes. The total number of faces is $5 \times 6 = 30$.
The cubes overlap in 5 places, so there are 2×5, or 10 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: $30 - 10 = 20$

iii) The composite object has 5 cubes. The total number of faces is $5 \times 6 = 30$.
The cubes overlap in 4 places, so there are 2×4, or 8 faces that are not part of the surface area.
The surface area of the object, in square centimetres, is: $30 - 8 = 22$

b) The composite objects do not overlap in the same number of places, so their surface areas are not the same.

7. Explanations may vary.
For example: There are views for which some of the faces are hidden. So, I cannot use 6 views to determine the surface area of this object.

8. Subtract the area of the overlapping faces when calculating the total surface area of the composite object.

a) Surface area of the smaller rectangular prism:
Area of top, bottom, front, and back faces: $4(2 \times 1) = 8$
Area of left and right faces: $2(1 \times 1) = 2$

Surface area of the larger rectangular prism:
Area of top and bottom faces: $2(5 \times 3)= 30$
Area of front and back faces: $2(5 \times 2) = 20$
Area of left and right faces: $2(3 \times 2) = 12$

In square centimetres, total surface area – overlap = $8 + 2 + 30 + 20 + 12 - 2(2 \times 1) = 68$

b) Surface area of top rectangular prism:
Area of top and bottom faces: $2(2 \times 2) = 8$
Area of front, back, left, and right faces: $4(2 \times 1) = 8$

Surface area of middle rectangular prism:
Area of top and bottom faces: $2(4 \times 3) = 24$
Area of front and back faces: $2(4 \times 2) = 16$
Area of left and right faces: $2(3 \times 2) = 12$

Surface area of bottom rectangular prism:
Area of top and bottom faces: $2(6 \times 4) = 48$
Area of front and back faces: $2(6 \times 3) = 36$
Area of left and right faces: $2(3 \times 4) = 24$

In square centimetres, total surface area – overlap =
$8 + 8 + 24 + 16 + 12 + 48 + 36 + 24 - 2(4 \times 3) - 2(2 \times 2) = 144$

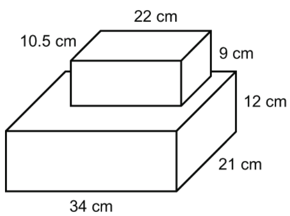
c) Surface area of the left rectangular prism:
Area of top and bottom faces: 2(2.5 × 4.5) = 22.5
Area of front and back faces: 2(5.5 × 2.5) = 27.5
Area of left and right faces: 2(5.5 × 4.5) = 49.5

Surface area of the middle rectangular prism:
Area of top and bottom faces: 2(3.5 × 3.5) = 24.5
Area of front, back, left, and right faces: 4(1.5 × 3.5) = 21

Surface area of the right rectangular prism:
Area of top and bottom faces: 2(2.5 × 5.5) = 27.5
Area of front and back faces: 2(2.5 × 6.5) = 32.5
Area of left and right faces: 2(5.5 × 6.5) = 71.5

In square centimetres, total surface area – overlap =
22.5 + 27.5 + 49.5 + 24.5 + 21 + 27.5 + 32.5 + 71.5 – 4(3.5 × 1.5) = 255.5

9. Answers will vary depending on the boxes used. For example:



a) The area of overlap is equal to the area of the bottom of the tissue box. We calculate the overlap by multiplying this area by 2.
Area overlap = 2 × 22 × 10.5
= 462

b) To calculate the surface area of the composite object, we determine the total surface area of both boxes and subtract the area of overlap.

Surface area of the tissue box:
Area of top and bottom faces: 2(22 × 10.5) = 462
Area of front and back faces: 2(22 × 9) = 396
Area of side faces: 2(9 × 10.5) = 189
Surface area = 462 + 396 + 189
= 1047

Surface area of the shoe box:
Area of top and bottom faces: 2(34 × 21) = 1428
Area of front and back faces: 2(34 × 12) = 816
Area of side faces: 2(21 × 12) = 504
Surface area = 1428 + 816 + 504
= 2748

In square centimetres,
total surface area = surface area tissue box + surface area shoe box – overlap
= 1047 + 2748 – 462
= 3333

10. a) Do not include the base in the calculation.

Surface area of garage:
Area of roof: 7.8 × 5.0 = 39
Area of left and right walls: 2(5.0 × 3.8) = 38
Area of front and back walls: 2(3.8 × 7.8) = 59.28

Surface area of shed:
Area of roof: 3.9 × 2.5 = 9.75
Area of left and right walls: 2(2.5 × 3.8) = 19
Area of front and back walls: 2(3.9 × 3.8) = 29.64

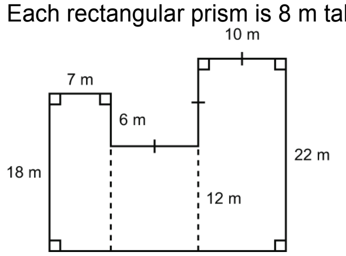
In square metres, total surface area – overlap =
39 + 38 + 59.28 + 9.75 + 19 + 29.64 – 2(3.9 × 3.8) = 165.03
The surface area of the building is 165.03 m^2.

b) Surface area of the building, in square metres, without doors, window, and roof:
165.03 – (2 × 3) – (2 × 1) – (1 × 1) – (7.8 × 5.0) – (2.5 × 3.9) = 107.28

Cost of siding:
$15 × 107.28 = $1609.2

It will cost $1609.20 to cover this building with siding.

11. Divide the building into three rectangular prisms.
Each rectangular prism is 8 m tall.



Since this is a building, we do not include the area of the floor.

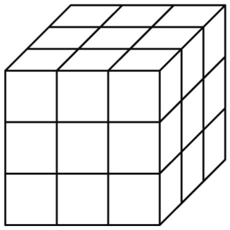
Surface area of left prism:
Area of roof: 18 × 7 = 126
Area of front and back walls: 2(8 × 7) = 112
Area of left and right walls: 2(18 × 8) = 288

Surface area of middle prism:
Area of roof: 10 × 12 = 120
Area of front and back walls: 2(10 × 8) = 160
Area of left and right walls: 2(12 × 8) = 192

Surface area of right prism:
Area of roof: 10 × 22 = 220
Area of front and back walls: 2(10 × 8) = 160
Area of left and right walls: 2(22 × 8) = 352

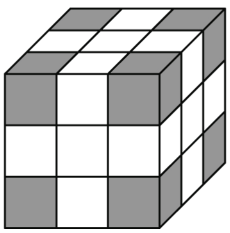
In square metres, total surface area of building – overlap =
126 + 112 + 288 + 120 + 160 + 192 + 220 + 160 + 352 – 4(12 × 8) = 1346

12. I built a 3 by 3 by 3 cube.

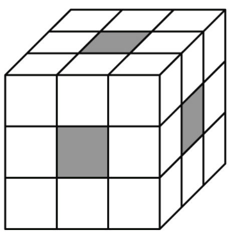


a) The surface area, in square units, of the large cube is: 6 × 9 = 54

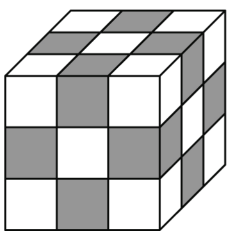
b) 9 ways: I can remove each of the 8 cubes at the corners, and the small interior cube at the centre of the big cube.



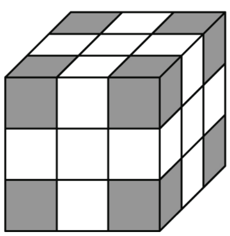
c) i) $1 \times 6 = 6$; 6 cubes



ii) 12 cubes



iii) 8 cubes

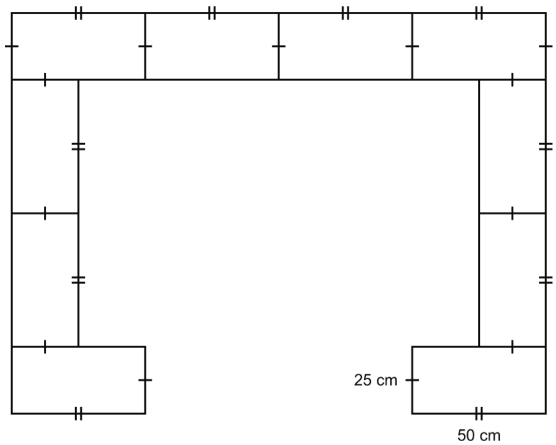


iv) 1 cube (at centre)

v) 0 cubes

I could check by building a cube with linking cubes.

13. a) Answers will vary depending on students' castles.
For example:



b) I used 3 layers of 10 blocks of ice.
The castle has a height of 300 cm.

Outside perimeter = $10 \times 50 + 4 \times 25$
= $500 + 100$
= 600

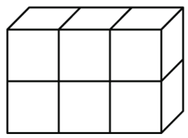
So, outside surface area = outside perimeter × height of castle
= 600×300
= $180\ 000$
The outside surface area of the castle is 180 000 cm^2.

Inside perimeter = $2 \times 25 + 7 \times 50$
= $50 + 350$
= 400

So, inside surface area = inside perimeter × height of castle + area of top + area of entrance
= $400 \times 300 + 10 \times 25 \times 50 + 2 \times 25 \times 300$
= $120\ 000 + 12\ 500 + 15\ 000$
= $147\ 500$
The inside surface area of the castle is 147 000 cm^2.

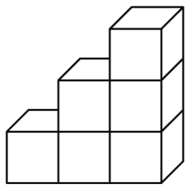
Take It Further

14. a) Answers will vary. For example:
I built a composite object using 2 rows of 3 cubes.

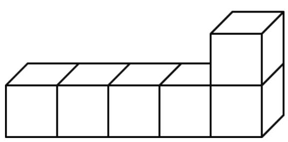


The object has 6 cubes. So, the number of faces is: $6 \times 6 = 36$
There are 7 places where the faces overlap. So, $2 \times 7 = 14$
The surface area, in square centimetres, is: $36 - 14 = 22$
The surface area of my composite object is 22 cm^2.

b) For example:



The object has 6 cubes. So, the number of faces is: $6 \times 6 = 36$
There are 6 places where the faces overlap. So, $2 \times 6 = 12$
The surface area, in square centimetres, is: $36 - 12 = 24$
The surface area of my composite object is 24 cm^2.



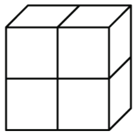
The object has 6 cubes. So, the number of faces is: $6 \times 6 = 36$
There are 5 places where the faces overlap. So, $2 \times 5 = 10$
The surface area, in square centimetres, is: $36 - 10 = 26$
The surface area of my composite object is 26 cm^2.

c) 22 cm^2 (2 rows of 3)
24 cm^2 (1 row of 4 and 1 row of 2, or 1 row of 3, 1 row of 2, and 1 row of 1)
26 cm^2 (1 row of 6, or 1 row of 5 and 1 row of 1, or 1 row of 4, 1 row of 1, and 1 row of 1)

d) Greatest surface area: 26 cm^2
The composite objects with greatest surface area have 5 places where the faces overlap (for example, 1 row of 6, or 1 row of 5 and 1 row of 1, or 1 row of 4, 1 row of 1, and 1 row of 1)

Least surface area: 22 cm^2
The composite object with least surface area have 7 places where the faces overlap (for example, 2 rows of 3)

15. There is only one composite object possible, using centimetre cubes, that has a surface area of 16 cm^2:
a composite object made from 4 centimetre cubes arranged in 2 rows of 2 cubes.



16. Volume = area × height
Since the height of each block is 1 m, the side length of each layer is equal to the square root of the number that represents the volume.

The dimensions of the top surface of the layers are:
5 m by 5 m, 4 m by 4 m, 3 m by 3 m, 2 m by 2 m, and 1 m by 1 m.
Surface area of bottom layer: 2(5 × 5) + 4(5 × 1) = 70
Surface area of 2nd layer: 2(4 × 4) + 4(4 × 1) = 48
Surface area of 3rd layer: 2(3 × 3) + 4(3 × 1) = 30
Surface area of 4th layer: 2(2 × 2) + 4(2 × 1) = 16
Surface area of 5th layer: 6(1 × 1) = 6

The difference between the surface area of each layer, going from top to bottom, follows this pattern:
10, 14, 18, 22, … It increases by 4 with each new layer.

In square metres, total surface area – overlap
= 70 + 48 + 30 + 16 + 6 – 2(1 × 1) – 2(2 × 2) – 2(3 × 3) – 2(4 × 4)
= 110

17. a) In square units, the piece made from 3 cubes has surface area:
(6 × 3) – (2 × 2) = 14
In square units, the pieces made from 4 cubes have surface area:
(6 × 4) – (3 × 2) = 18

b) The 7 pieces can be combined to form a 3 by 3 by 3 cube.

c) Determine the surface area, in square units, of the original 7 pieces.
Surface area = 1 × 14 + 6 × 18
= 122

The surface area of the larger cube, in square units:
Surface area larger cube = 6 × 3 × 3
= 54
So, 54 faces be painted.

To determine the number of faces that will not be painted, subtract:
122 – 54 = 68
So, 68 faces will not be painted.

Lesson 1.4 Surface Areas of Other Composite Objects Practice (pages 40–43)

Check

3. a) The overlap is the area of the base of the cylinder. So, instead of calculating the area of the top of the cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the cylinder.

Surface area = curved surface area of cylinder + surface area of cube
= circumference of base × height of cylinder + 6 × area of cube face
= $(2 \times \pi \times 1 \times 4) + (6 \times 4 \times 4)$
$\doteq 121$
The surface area of the composite object is about 121 cm^2.

b) The overlap is the area of the base of the cylinder. So, instead of calculating the area of the top of the cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the cylinder.

Surface area = curved surface area of the cylinder + surface area of the rectangular prism
= circumference of base × height of cylinder + 2 × area of top rectangular face + 2 × area of front rectangular face + 2 × area of side rectangular face
= $(2 \times \pi \times 0.5 \times 3) + (2 \times 4 \times 6) + (2 \times 3 \times 6) + (2 \times 3 \times 4)$
$\doteq 117$
The surface area of the composite object is about 117 cm^2.

c) The overlap is the area of one base of the bottom cylinder. So, instead of calculating the area of the other base of the bottom cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the bottom cylinder.

Surface area = curved surface area of bottom cylinder + surface area of top cylinder
= circumference of base × height of bottom cylinder + circumference of base × height of top cylinder + 2 × area of one circular base of top cylinder
= $(2 \times \pi \times 1 \times 10) + (2 \times \pi \times 5 \times 2) + 2(\pi \times 5^2)$
$\doteq 283$
The surface area of the composite object is about 283 cm^2.

d) The area of overlap is 2 times the area of one face of the cube.

Surface area = surface area of triangular prism + area of 6 faces of cube – area of 2 faces of cube
= surface area of triangular prism + area of 4 faces of cube
= $(2 \times \frac{1}{2} \times 9 \times 12) + (15 \times 6) + (9 \times 6) + (12 \times 6) + (4 \times 3 \times 3)$
= 360
The surface area of the composite object is 360 cm^2.

e) The area of overlap is 2 times the area of one face of the cube.

Surface area = surface area of triangular prism + area of 6 faces of cube – area of 2 faces of cube
= surface area of triangular prism + area of 4 faces of cube
= 2 × area of triangular base + area of 3 rectangular faces + area of 4 faces of cube
= $(2 \times \frac{1}{2} \times 5 \times 12) + (6 \times 13) + (6 \times 5) + (12 \times 6) + (4 \times 2 \times 2)$
= 256
The surface are of the composite object is 256 cm^2.

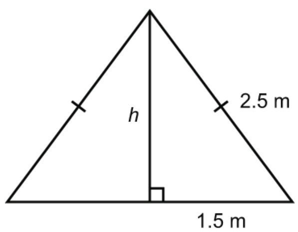
4. a) The overlap is the area of one base of the narrow cylinder. So, instead of calculating the area of the other base of the narrow cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the narrow cylinder.

Surface area = curved surface area of narrow cylinder + surface area of wider cylinder
= circumference of base × height of narrow cylinder + circumference of base × height of wider cylinder + 2 × area of one circular base of wider cylinder
$= (2 \times \pi \times 0.5 \times 4.5) + (2 \times \pi \times 2 \times 1.5) + (2 \times \pi \times 2^2)$
$\doteq 58.1$
The surface area of the composite object is about 58.1 cm^2.

b) The overlap is 2 times the area of one base of the cylinder.

Surface area = surface area of cube + surface area of rectangular prism + curved surface area of cylinder – 2 × area of cylinder base overlap
$= (6 \times 2.5 \times 2.5) + (4 \times 2.5 \times 1.5) + (2 \times 1.5 \times 1.5) + (2 \times \pi \times 0.25 \times 3.5) - (2 \times \pi \times 0.25^2)$
$\doteq 62.1$
The surface area of the composite object is about 62.1 m^2.

5. a)

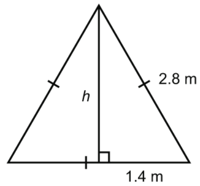


Height, h, of the triangular base:
$h^2 + 1.5^2 = 2.5^2$
$h^2 = 2.5^2 - 1.5^2$
$= 4$
$h = \sqrt{4}$
$= 2$

The overlap is the area of the top of the cylinder. So, instead of calculating the area of the base of the cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the cylinder.

Surface area = 2 × area of triangular base + area of 3 rectangular faces of triangular prism + curved surface area of cylinder
$= (2 \times \frac{1}{2} \times 3 \times 2) + (3 \times 1) + (2.5 \times 1) + (2.5 \times 1) + (2 \times \pi \times 0.5 \times 2.5)$
$\doteq 21.9$
The surface area of the composite object is about 21.9 m^2.

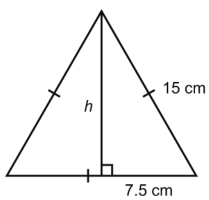
b)



Height, h, of the triangular base:
$h^2 + 1.4^2 = 2.8^2$
$h^2 = 2.8^2 - 1.4^2$
$= 5.88$
$h = \sqrt{5.88}$
$\doteq 2.42$
The overlap is two times the area of one rectangular face of the triangular prism.

Surface area = 2 × area of triangular base + area of one rectangular face of triangular prism + surface area of rectangular prism
$\doteq (2 \times \frac{1}{2} \times 2.8 \times 2.42) + (1.2 \times 2.8) + (2 \times 1.4 \times 4.8) + (2 \times 1.4 \times 2.8) + (2 \times 4.8 \times 2.8)$
$\doteq 58.3$
The surface area of the composite object is about 58.3 cm^2.

6.



Height, h, of the triangular base:
$h^2 + 7.5^2 = 15^2$
$h^2 = 15^2 - 7.5^2$
$= 168.75$
$h = \sqrt{168.75}$
$\doteq 13$
The overlap is the area of the base of the cylinder. So, instead of calculating the area of the top of the cylinder, then subtracting that area as the overlap, calculate only the curved surface area of the cylinder.

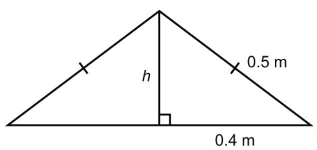
Surface area of stand (including bottom of the base)
= 2 × area of triangular base + area of 3 rectangular faces of triangular prism + curved surface area of cylinder
$\doteq (2 \times \frac{1}{2} \times 15 \times 13) + (3 \times 15 \times 3) + (2 \times \pi \times 2 \times 30)$
$\doteq 707$
The area that will be painted is about 707 cm^2.

7. a) Surface area = 2 × area of triangular base + area of 2 rectangular faces of triangular prism + 2 × area of front face of rectangular prism + 2 × area of side face of rectangular prism + area of base of rectangular prism
$= (2 \times \frac{1}{2} \times 2.0 \times 0.75) + (2 \times 1.25 \times 3.0) + (2 \times 3.0 \times 2.0) + (2 \times 2.0 \times 2.0) + (3.0 \times 2.0)$
$= 35$
The surface area of the playhouse is 35 m^2.
Note: The surface area does not need to include the base. In this case, the surface area of the playhouse is 29 m^2.

b) Answers will vary.
One possible answer is to have a door with width 0.75 m and height 1.5 m, and two square windows, 0.7 m by 0.7 m. If the playhouse was being painted or covered with siding, then the surface area would be reduced by the sum of the areas of the door and windows.

c) Answers will vary.
Surface area = total surface area of the playhouse – area of door – area of windows
$= 35 - (0.75 \times 1.5) - (2 \times 0.7 \times 0.7)$
$\doteq 32.9$
The surface area would be about 32.9 m^2.
Note: If we start with a total surface area of 29 m^2 (which does not include the base), then the surface area is about 26.9 m^2.

8. a) The doghouse is composed of a triangular prism atop of a rectangular prism.
Height, *h*, of the triangular base:

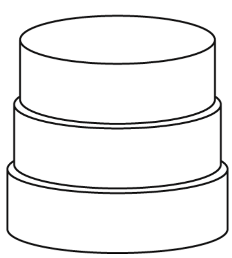


$h^2 + 0.4^2 = 0.5^2$
$h^2 = 0.5^2 - 0.4^2$
$= 0.09$
$h = \sqrt{0.09}$
$= 0.3$
Since this is a house, do not include the base in the calculation. Also, include 4 times the area of the overhang, since there are 2 overhangs, and each one has a top side and an underside.

Surface area = (surface area of sides of rectangular prism – area of doorway) + 2 × area of triangular base + area of 2 rectangular faces of triangular prism + 4 × area of overhang
$= ((2 \times 0.8 \times 1.5) + (2 \times 0.8 \times 0.8) - (0.6 \times 0.5)) + (2 \times \frac{1}{2} \times 0.8 \times 0.3) + (2 \times 0.5 \times 1.5) + (4 \times 1.5 \times 0.1)$
$= 5.72$
The surface area of the doghouse is 5.72 m^2.

b) Enough stain is needed to cover twice the surface area.
$2 \times 5.72\ m^2 = 11.44\ m^2$
Since a 1-L can covers 6 m^2, 2 cans of 1-L wood stain are needed.

9. a)



b) Do not include the base of the cake in the calculation because this will not be frosted.
The overlap is the area of the base of the top layer, and the area of the base of the middle layer.

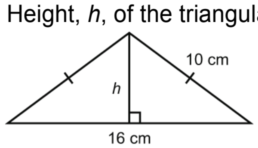
Surface area = curved surface area of top layer + area of top of cake + curved surface area of middle layer + (area of top of middle layer – area of base of top layer) + curved surface of bottom layer + (area of one base of bottom layer – area of base of middle layer)
$= (2 \times \pi \times 10 \times 7.5) + (\pi \times 10^2) + (2 \times \pi \times 11.25 \times 7.5) + (\pi \times 11.25^2) - (\pi \times 10^2) + (2 \times \pi \times 12.5 \times 7.5) + (\pi \times 12.5^2) - (\pi \times 11.25^2)$
$= (2 \times \pi \times 10 \times 7.5) + (2 \times \pi \times 11.25 \times 7.5) + (2 \times \pi \times 12.5 \times 7.5) + (\pi \times 12.5^2)$
$\doteq 2081.3$
The frosted area of the cake is about 2081.3 cm^2.

10. a) Surface area = curved surface area of top 3 layers + curved surface area of bottom layer + area of one base of bottom layer
$= (2 \times \pi \times 10 \times 7.5) + (2 \times \pi \times 11.25 \times 7.5) + (2 \times \pi \times 12.5 \times 7.5) + (2 \times \pi \times 13.75 \times 7.5) + (\pi \times 13.75^2)$
$\doteq 2832.3$
The frosted area of the cake is about 2832.3 cm^2.

b) Surface area = curved surface area of top 4 layers + curved surface area of bottom layer + area of one base of bottom layer
$= (2 \times \pi \times 10 \times 7.5) + (2 \times \pi \times 11.25 \times 7.5) + (2 \times \pi \times 12.5 \times 7.5) + (2 \times \pi \times 13.75 \times 7.5) + (2 \times \pi \times 15 \times 7.5) + (\pi \times 15^2)$
$\doteq 3652.1$
The frosted area of the cake is about 3652.1 cm^2.

c) The surface area increases each time a new layer is added. It increases by the curved surface area of the new layer + the area of one base of the new layer – the area of the base of the previous layer.

11. Think of the birdhouse as a triangular prism atop of a rectangular prism.
Height, *h*, of the triangular base:

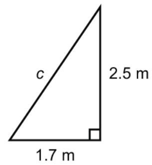


$h^2 + 8^2 = 10^2$
$h^2 = 10^2 - 8^2$
$= 36$
$h = \sqrt{36}$
$= 6$

Assume Rory does not paint the base of the birdhouse. The overlap of the perch is the area of one base of the perch. So, instead of calculating the area of one base of the perch, then subtracting that area as the overlap, calculate only the curved surface area of the perch.

Surface area = 2 × area of triangular base + area of 2 rectangular faces of triangular prism + 2 × area of front face of rectangular prism + 2 × area of side face of rectangular prism + curved surface area of perch – area of entrance
$= (2 \times \frac{1}{2} \times 16 \times 6) + (2 \times 10 \times 15) + (2 \times 12 \times 16) + (2 \times 12 \times 15) + (2 \times \pi \times 0.5 \times 7) - (\pi \times 1.5^2)$
$\doteq 1155$
The area that needs to be painted is about 1155 cm^2.

12. a) The canvas of the trailer is composed of two triangular prisms and one rectangular prism.
Determine the unknown length, c, of the triangle:



$c^2 = 2.5^2 + 1.7^2$
$c^2 = 9.14$
$c = \sqrt{9.14}$
$\doteq 3.02$

Assume the top of the trailer is canvas.
Surface area = 4 × area of triangle base + 2 × area of rectangular face of triangular prism + 2 × area of front face of rectangular prism + area of top face of rectangular prism

$\doteq (4 \times \frac{1}{2} \times 1.7 \times 2.5) + (2 \times 3.02 \times 2.5) + (2 \times 5 \times 2.5) + (5 \times 2.5)$

$= 61.1$

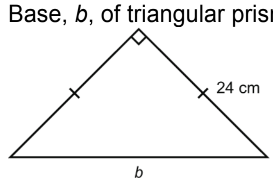
The surface area of the canvas on the trailer is 61.1 m^2.

b) Yes, the surface area of canvas will increase. The material must be elastic and stretch. The distance from the top of the tent trailer to the bottom of the overhang was approximately 3.0 m.
It was a straight line segment. With the insertion of the bars it is now two lines segments. The shortest distance between two points is a straight line segment. Since it is now two line segments, the distance between the two points must be longer so there must be more area.

13. a) Surface area = 6 × area of one cube face
= 6 × 24 × 24
= 3456

The surface area of the cube is 3456 cm^2.

b) Base, b, of triangular prism:



$b^2 = 24^2 + 24^2$
$b^2 = 1152$
$b = \sqrt{1152}$
$\doteq 33.94$

Surface area of triangular prism = 2 × area of base + area of 3 rectangular faces

$= (2 \times \frac{1}{2} \times 24 \times 24) + (2 \times 24 \times 48) + (33.94 \times 48)$

$\doteq 4509$

The surface area of this prism is about 4509 cm^2.

c) The dimensions of the cube are 24 cm × 24 cm × 24 cm. The triangular prism has a base with edge lengths 24 cm, 24 cm, and 33.9 cm, and length 48 cm.

14. Do not include the base of the bottom cylinder in the calculation.

Surface area = curved surface area of top cylinder + curved surface area of middle cylinder + curved surface area of bottom cylinder + 3 × area of larger cylinder base + curved surface area of the bath – 2 × area of smaller cylinder base

$= (2 \times \pi \times 22 \times 13) + (2 \times \pi \times 15 \times 40) + (2 \times \pi \times 22 \times 13) + (3 \times \pi \times 22^2) + (2 \times \pi \times 15 \times 2) - (2 \times \pi \times 15^2)$

$\doteq 10\ 700.3$

The area to be tiled is about 10 700 cm^2.

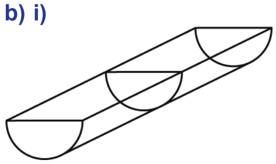
15. a) Surface area = curved surface area of cylinder + 2 × area of cylinder base

$= (2 \times \pi \times 9 \times 50) + (2 \times \pi \times 9^2)$

$\doteq 3336$

The surface area of the cylinder is about 3336 cm^2.

b) i)



ii) Surface area = $\frac{1}{2}$ × curved surface area of cylinder + area of base of cylinder + area of rectangular face

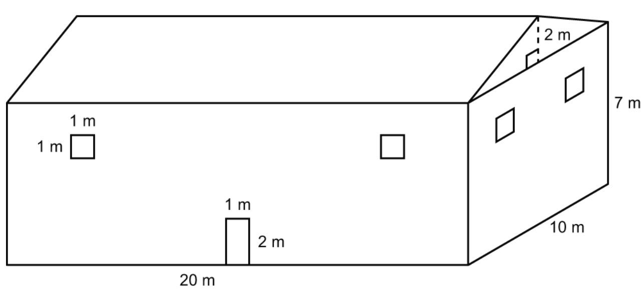
$= (\frac{1}{2} \times 2 \times \pi \times 9 \times 100) + (\pi \times 9^2) + (100 \times 18)$

$\doteq 4882$

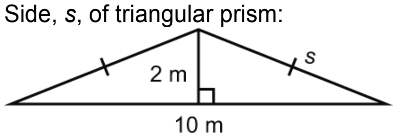
The surface area of the composite object is about 4882 cm^2.

16. a) Answers will vary. Ensure students' measurements are reasonable and that their houses have at least one door and one window.

For example:



b) The house has one 1-m by 2-m door and four 1-m by 1-m windows.
Side, *s*, of triangular prism:



$s^2 = 5^2 + 2^2$

$s^2 = 29$

$s = \sqrt{29}$

$\doteq 5.4$

Surface area = 2 × area of triangular base + area of 2 rectangular faces of triangular prism + 2 × area of front face of rectangular prism + 2 × area of side face of rectangular prism – area of door – area of windows

$$\doteq (2 \times \frac{1}{2} \times 10 \times 2) + (2 \times 20 \times 5.4) + (2 \times 7 \times 20) + (2 \times 7 \times 10) - (1 \times 2) - (4 \times 1 \times 1)$$

$= 650$

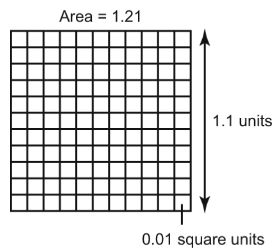
The surface area of this home is 650 m^2.

c) Multiply the insulation cost by the surface area of the house.
650 m^2 × \$4.25/m^2 = \$2762.50
It will cost \$2762.50 to insulate this home.

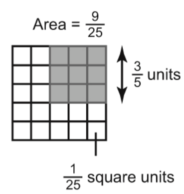
Review (pages 45–47)

Lesson 1.1

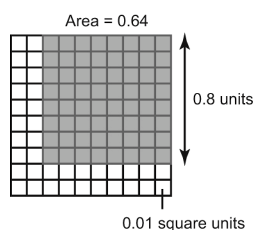
1. a) $\sqrt{1.21} = 1.1$



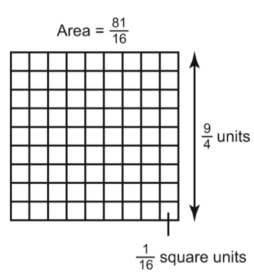
b) $\sqrt{\frac{9}{25}} = \frac{3}{5}$



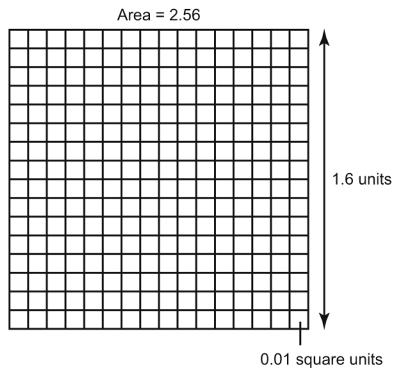
c) $\sqrt{0.64} = 0.8$



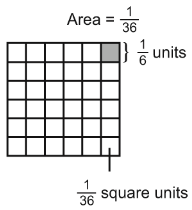
d) $\sqrt{\frac{81}{16}} = \frac{9}{4}$



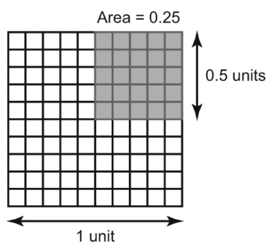
e) $\sqrt{2.56} = 1.6$



f) $\sqrt{\frac{1}{36}} = \frac{1}{6}$

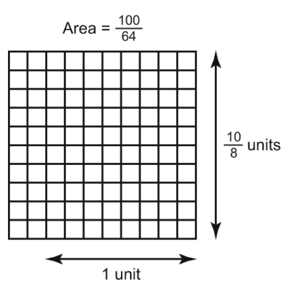


g) $\sqrt{0.25} = 0.5$

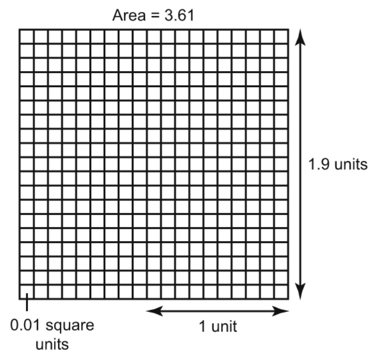


h) $\sqrt{\frac{100}{64}} = \frac{10}{8}$

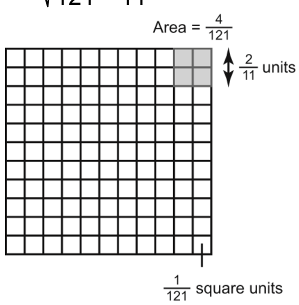
$= \frac{5}{4}$



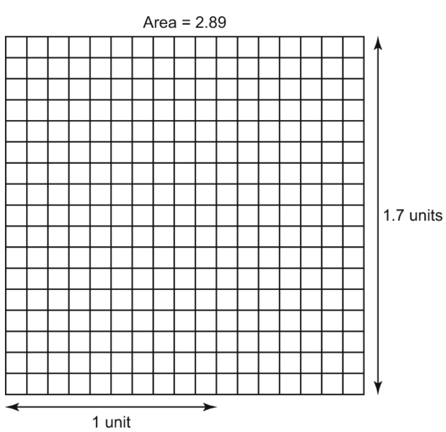
i) $\sqrt{3.61} = 1.9$



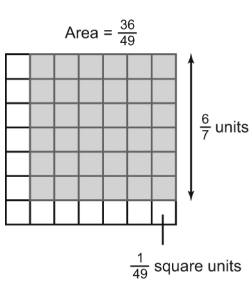
j) $\sqrt{\frac{4}{121}} = \frac{2}{11}$



k) $\sqrt{2.89} = 1.7$



l) $\sqrt{\frac{36}{49}} = \frac{6}{7}$



2. a) $\sqrt{\frac{144}{25}} = \sqrt{\frac{12}{5} \times \frac{12}{5}}$
$= \frac{12}{5}$

b $\sqrt{\frac{225}{64}} = \sqrt{\frac{15}{8} \times \frac{15}{8}}$
$= \frac{15}{8}$

c) $\sqrt{\frac{196}{81}} = \sqrt{\frac{14}{9} \times \frac{14}{9}}$
$= \frac{14}{9}$

d) $\sqrt{\frac{324}{121}} = \sqrt{\frac{18}{11} \times \frac{18}{11}}$
$= \frac{18}{11}$

e) $\sqrt{0.0196} = \sqrt{\frac{196}{10\,000}}$
$= \sqrt{\frac{14}{100} \times \frac{14}{100}}$
$= \frac{14}{100}$, or 0.14

f) $\sqrt{0.0289} = \sqrt{\frac{289}{10\,000}}$
$= \sqrt{\frac{17}{100} \times \frac{17}{100}}$
$= \frac{17}{100}$, or 0.17

g) $\sqrt{1.69} = \sqrt{\frac{169}{100}}$
$= \sqrt{\frac{13}{10} \times \frac{13}{10}}$
$= \frac{13}{10}$, or 1.3

h) $\sqrt{4.41} = \sqrt{\frac{441}{100}}$
$= \sqrt{\frac{21}{10} \times \frac{21}{10}}$
$= \frac{21}{10}$, or 2.1

3. a) $\frac{48}{120}$ simplifies to $\frac{2}{5}$; this fraction cannot be written as a product of two equal fractions, so it is not a perfect square.

b) $1.6 = \frac{16}{10}$

The numerator can be written as 16 = 4 × 4, but the denominator cannot be written as a product of two equal factors. So, 1.6 is not a perfect square.

c) $\frac{49}{100} = \frac{7}{10} \times \frac{7}{10}$

$\frac{49}{100}$ is a perfect square, because it can be written as a product of two equal factors.

d) $0.04 = \frac{4}{100}$

$= \frac{2}{10} \times \frac{2}{10}$

0.04 is a perfect square, because it can be written as a product of two equal factors.

e) $\frac{144}{24} = 6$

$\frac{144}{24}$ is not a perfect square; 6 cannot be written as a product of two equal factors.

f) $2.5 = \frac{25}{10}$

The numerator can be written as 25 = 5 × 5, but the denominator cannot be written as a product of two equal factors. So, 2.5 is not a perfect square.

g) $\frac{50}{225}$ simplifies to $\frac{2}{9}$.

The denominator can be written as 9 = 3 × 3, but the numerator cannot be written as a product of two equal factors. So, $\frac{50}{225}$ is not a perfect square.

h) $1.96 = \frac{196}{100}$

$= \frac{14}{10} \times \frac{14}{10}$

1.96 is a perfect square, since it can be written as a product of two equal factors.

i) $\frac{63}{28}$ simplifies to $\frac{9}{4}$.

$\frac{9}{4} = \frac{3}{2} \times \frac{3}{2}$

$\frac{63}{28}$ is a perfect square, since it can be written as a product of two equal fractions.

4. a) $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

So, $\sqrt{\frac{9}{25}} = \frac{3}{5}$

b) 1.6 × 1.6 = 2.56

So, $\sqrt{2.56} = 1.6$

c) $\frac{9}{7} \times \frac{9}{7} = \frac{81}{49}$

So, $\sqrt{\frac{81}{49}} = \frac{9}{7}$

d) $0.8 \times 0.8 = 0.64$

So, $\sqrt{0.64} = 0.8$

5. Take the square root of each area to determine the side length of each square.

a) $\sqrt{0.81} = 0.9$

The side length of the square is 0.9 m.

b) $\sqrt{0.01} = 0.1$

The side length of the square is 0.1 m.

c) $\sqrt{4.84} = 2.2$

The side length of the square is 2.2 cm.

d) $\sqrt{6.25} = 2.5$

The side length of the square is 2.5 cm.

e) $\sqrt{0.16} = 0.4$

The side length of the square is 0.4 km.

f) $\sqrt{1.44} = 1.2$

The side length of the square is 1.2 km.

Lesson 1.2

6. Estimates will vary; for example:

a) 3.8 is between the perfect squares 1 and 4, and closer to 4.

$\sqrt{1} = 1$ and $\sqrt{4} = 2$

So, $\sqrt{3.8}$ is between 1 and 2, and closer to 2.

Estimate $\sqrt{3.8}$ as 1.9.

To check, evaluate: $1.9^2 = 3.61$, which is close to 3.8.

So, $\sqrt{3.8} \doteq 1.9$

b) 33.8 is between the perfect squares 25 and 36, and closer to 36.

$\sqrt{25} = 5$ and $\sqrt{36} = 6$

So, $\sqrt{33.8}$ is between 5 and 6, and closer to 6.

Estimate $\sqrt{33.8}$ as 5.8.

To check, evaluate: $5.8^2 = 33.64$, which is close to 33.8.

So, $\sqrt{33.8} \doteq 5.8$

c) 133.8 is between the perfect squares 121 and 144, and closer to 144.

$\sqrt{121} = 11$ and $\sqrt{144} = 12$

So, $\sqrt{133.8}$ is between 11 and 12, and closer to 12.

Estimate $\sqrt{133.8}$ as 11.6.

To check, evaluate: $11.6^2 = 134.56$, which is close to 133.8.

So, $\sqrt{133.8} \doteq 11.6$

d) 233.8 is between the perfect squares 225 and 256, and closer to 225.
$\sqrt{225} = 15$ and $\sqrt{256} = 16$
So, $\sqrt{233.8}$ is between 15 and 16, and closer to 15.
Estimate $\sqrt{233.8}$ as 15.3.
To check, evaluate: $15.3^2 = 234.09$, which is close to 233.8.
So, $\sqrt{233.8} \doteq 15.3$

7. Estimates will vary; for example:

a) $\frac{77}{10} = 7.7$, which is between the perfect squares 4 and 9, and closer to 9.
$\sqrt{4} = 2$ and $\sqrt{9} = 3$
So, $\sqrt{\frac{77}{10}}$ is between 2 and 3, and closer to 3.
Estimate $\sqrt{\frac{77}{10}}$ as 2.8.
To check, evaluate: $2.8^2 = 7.84$, which is close to 7.7.
So, $\sqrt{\frac{77}{10}} \doteq \frac{28}{10}$, or $\frac{14}{5}$

b) In the fraction $\frac{18}{11}$, 18 is close to the perfect square 16, and 11 is close to the perfect square 9.
$\sqrt{\frac{16}{9}} = \frac{4}{3}$
So, $\sqrt{\frac{18}{11}} \doteq \frac{4}{3}$
Or, another way:
$\frac{18}{11} = \frac{198}{121}$, which is close to the perfect square $\frac{196}{121}$
$\sqrt{\frac{196}{121}} = \frac{14}{11}$
So, $\sqrt{\frac{18}{11}} \doteq \frac{14}{11}$

c) In the fraction $\frac{15}{39}$, 15 is close to the perfect square 16, and 39 is close to the perfect square 36.
$\sqrt{\frac{16}{36}} = \frac{4}{6}$, or $\frac{2}{3}$
So, $\sqrt{\frac{15}{39}} \doteq \frac{2}{3}$
Or, another way:
$\frac{15}{39} = \frac{225}{585}$, which is close to the perfect square $\frac{225}{576}$
$\sqrt{\frac{225}{576}} = \frac{15}{24}$, or $\frac{5}{8}$
So, $\sqrt{\frac{15}{39}} \doteq \frac{5}{8}$

d) In the fraction $\frac{83}{19}$, 83 is close to the perfect square 81, and 19 is close to the perfect square 16.

$\sqrt{\frac{81}{16}} = \frac{9}{4}$

So, $\sqrt{\frac{83}{19}} \doteq \frac{9}{4}$

e) In the fraction $\frac{28}{103}$, 28 is close to the perfect square 25, and 103 is close to the perfect square 100.

$\sqrt{\frac{25}{100}} = \frac{1}{2}$

So, $\sqrt{\frac{28}{103}} \doteq \frac{1}{2}$

f) In the fraction $\frac{50}{63}$, 50 is close to the perfect square 49, and 63 is close to the perfect square 64.

$\sqrt{\frac{49}{64}} = \frac{7}{8}$

So, $\sqrt{\frac{50}{63}} \doteq \frac{7}{8}$

8. Estimates and explanations will vary. For example: I used benchmarks to estimate each square root, because they provided a small range of numbers to work with.

a) 5.9 is between the perfect squares 4 and 9, and closer to 4.
So, $\sqrt{5.9}$ is between 2 and 3, and closer to 2.
Estimate $\sqrt{5.9}$ as 2.4.
To check, evaluate: $2.4^2 = 5.76$, which is close to 5.9.
So, $\sqrt{5.9}$ is about 2.4.

b) $\frac{7}{20} = \frac{49}{140}$, which is close to the perfect square $\frac{49}{144}$

$\sqrt{\frac{49}{144}} = \frac{7}{12}$

So, $\sqrt{\frac{7}{20}} \doteq \frac{7}{12}$

c) 0.65 is between the perfect squares 0.64 and 0.81, but closer to 0.64.
So, $\sqrt{0.65}$ is between 0.8 and 0.9, but closer to 0.8.
Estimate $\sqrt{0.65}$ as 0.81.
To check, evaluate: $0.81^2 = 0.6561$, which is close to 0.65.
So, $\sqrt{0.65} \doteq 0.81$

d) In the fraction $\frac{21}{51}$, 21 is close to the perfect square 25, and 51 is close to the perfect square 49.

$\sqrt{\frac{25}{49}} = \frac{5}{7}$

So, $\sqrt{\frac{21}{51}} \doteq \frac{5}{7}$

e) 23.2 is between the perfect squares 16 and 25, and closer to 25.
$\sqrt{16} = 4$ and $\sqrt{25} = 5$
So, $\sqrt{23.2}$ is between 4 and 5, and closer to 5.
Estimate $\sqrt{23.2}$ as 4.8. To check, evaluate: $4.8^2 = 23.04$, which is close to 23.2.
So, $\sqrt{23.2}$ is about 4.8.

f) In the fraction $\frac{88}{10}$, 88 is close to the perfect square 81, and 10 is close to the perfect square 9.

$$\sqrt{\frac{81}{9}} = 3$$

So, $\sqrt{\frac{88}{10}} \doteq 3$

9. a) Correct; $1.5^2 = 2.25$, which is close to 2.4.

b) Incorrect; $0.4^2 = 0.16$, not 1.6.
$\sqrt{1.6} \doteq 1.3$

c) Incorrect; $15.6^2 = 243.36$
$\sqrt{156.8} \doteq 12.5$

d) Correct; $6.9^2 = 47.61$, which is close to 47.8.

e) Correct; $0.7^2 = 0.49$, which is close to 0.5.

f) Incorrect; $0.5^2 = 0.25$
$\sqrt{0.7} \doteq 0.8$

10. $\sqrt{27.4} \doteq 5.2$ is correctly placed on the number line.
$\sqrt{25.3} \doteq 5.03$, so it should be just to the right of 5 on the number line.
$\sqrt{37.2} \doteq 6.1$, so it should be just to the right of 6 on the number line.
$\sqrt{60.8} \doteq 7.8$ is correctly placed on the number line.
$\sqrt{82.8} \doteq 9.1$, so it should be just to the right of 9 on the number line.
$\sqrt{98.1} \doteq 9.9$, so it should be just to the left of 10 on the number line.

11. a) $1^2 = 1$ and $2^2 = 4$, so all square roots between 1 and 4 are between 1 and 2:
$\sqrt{3.2}$, $\sqrt{2.3}$, $\sqrt{2.8}$, $\sqrt{1.2}$

b) $11^2 = 121$ and $12^2 = 144$, so all square roots between 121 and 144 are between 11 and 12:
$\sqrt{125.4}$, $\sqrt{134.5}$, $\sqrt{129.9}$

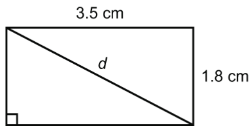
c) $3.5^2 = 12.25$ and $4.5^2 = 20.25$, so all square roots between 12.25 and 20.25 are between 3.5 and 4.5:
$\sqrt{12.9}$, $\sqrt{15.2}$

d) $1.5^2 = 2.25$ and $2.5^2 = 6.25$, so all square roots between 2.25 and 6.25 are between 1.5 and 2.5:
$\sqrt{5.7}$, $\sqrt{4.8}$, $\sqrt{3.2}$, $\sqrt{2.3}$, $\sqrt{2.8}$

e) $4.5^2 = 20.25$ and $5.5^2 = 30.25$, so all square roots between 20.25 and 30.25 are between 4.5 and 5.5:
$\sqrt{21.2}$, $\sqrt{23.1}$, $\sqrt{29.1}$

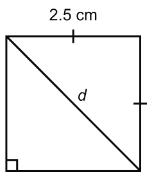
f) $14.5^2 = 210.25$ and $15.5^2 = 240.25$, so all square roots between 210.25 and 240.25 are between 14.5 and 15.5:
$\sqrt{237.1}$, $\sqrt{222.1}$, $\sqrt{213.1}$

12. a)



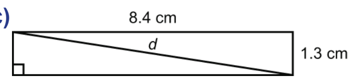
$d^2 = 3.5^2 + 1.8^2$
$= 15.49$
$d = \sqrt{15.49}$
$\doteq 3.94$
A diagonal is about 3.9 cm.

b)



$d^2 = 2.5^2 + 2.5^2$
$= 12.5$
$d = \sqrt{12.5}$
$\doteq 3.5$
A diagonal is about 3.5 cm.

c)



$d^2 = 8.4^2 + 1.3^2$
$= 72.25$
$d = \sqrt{72.25}$
$= 8.5$
A diagonal is 8.5 cm.

13. Answers will vary. For example:

a) The number with a square root of $\frac{1}{3}$ is $\frac{1}{9}$.
The number with a square root of 1 is 1.
So, any number between $\frac{1}{9}$ and 1 will have a square root between $\frac{1}{3}$ and 1.
For example: $\frac{1}{2}$
$\sqrt{\frac{1}{2}} \doteq 0.707$, which is between $\frac{1}{3}$ and 1.

b) The number with a square root of 0.2 is 0.04.
The number with a square root of 0.3 is 0.09.
So, any number between 0.04 and 0.09 will have a square root between 0.2 and 0.3.
For example: 0.0625
$\sqrt{0.0625} = 0.25$, which is between 0.2 and 0.3.

c) The number with a square root of 1.4 is 1.96.
The number with a square root of 1.41 is 1.9881.
So, any number between 1.96 and 1.9881 will have a square root between 1.4 and 1.41.
For example: 1.97
$\sqrt{1.97} \doteq 1.404$, which is between 1.4 and 1.41.

d) $\frac{1}{10} = 0.1$; $\frac{3}{10} = 0.3$
The number with a square root of 0.1 is 0.01.
The number with a square root of 0.3 is 0.09.
So, any number between 0.01 and 0.09 will have a square root between 0.1 and 0.3.
For example: 0.04, or $\frac{4}{100} = \frac{1}{25}$
$\sqrt{\frac{1}{25}} = \frac{1}{5}$, which is between $\frac{1}{10}$ and $\frac{3}{10}$.

14. a) i) $\sqrt{0.0015} \doteq 0.0387$

ii) $\sqrt{0.15} \doteq 0.3873$

iii) $\sqrt{15} \doteq 3.8730$

iv) $\sqrt{1500} \doteq 38.7298$

v) $\sqrt{150\,000} \doteq 387.2983$

b) As the numbers increase by factors of 100, their square roots increase by factors of 10.
The previous two square roots are $\sqrt{0.000\,015} \doteq 0.003\,87$ and $\sqrt{0.000\,000\,15} \doteq 0.000\,387$.
The next two square roots are $\sqrt{15\,000\,000} \doteq 3872.983$ and $\sqrt{1\,500\,000\,000} \doteq 38\,729.83$.

Lesson 1.3

15. a) The composite object has 4 cubes.
The total number of faces is: $4 \times 6 = 24$
The faces overlap in 3 places.
Surface area = total number of faces – overlaps
$= 24 - 2 \times 3$
$= 18$
The surface area of the composite object is 18 cm^2.

b) The composite object has 5 cubes.
The total number of faces is: $6 \times 5 = 30$
The faces overlap in 4 places.
Surface area = total number of faces – overlaps
$= 30 - 2 \times 4$
$= 22$
The surface area of the composite object is 22 cm^2.

c) The composite object has 6 cubes. The total number of faces is: $6 \times 6 = 36$
The faces overlap in 5 places.
Surface area = total number of faces – overlaps
$= 36 - 2 \times 5$
$= 26$
The surface area of the composite object is 26 cm^2.

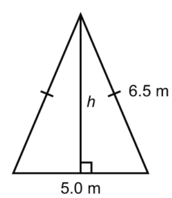
16. Overlapping area must be subtracted from the total surface area of the objects.

a) Surface area = area of 5 faces of cube + 2 × area of top of rectangular prism + 2 × area of side of rectangular prism + 2 × area of front of rectangular prism – area of 1 face of cube
$= (5 \times 1.5 \times 1.5) + (2 \times 3.5 \times 4.5) + (2 \times 0.7 \times 3.5) + (2 \times 0.7 \times 4.5) - (1.5 \times 1.5)$
$= 51.7$
The surface area of the composite object is 51.7 cm^2.

b) Surface area = 2 × area of top of large rectangular prism + 2 × area of front of large rectangular prism + 2 × area of side of large rectangular prism + 2 × area of front of small rectangular prism + 2 × area of top of small rectangular prism + area of side of small rectangular prism – area of side of small rectangular prism
$= (2 \times 7.6 \times 10.5) + (2 \times 10.5 \times 8.4) + (2 \times 8.4 \times 7.6) + (2 \times 3.5 \times 4.2) + (2 \times 3.5 \times 3.2)$
$= 515.48$
The surface area of the composite object is 515.48 m^2.

c) Surface area = surface area triangular prism + surface area rectangular prism + surface area cube – area overlap

The height, h, of the triangular base is:
$h^2 + 2.5^2 = 6.5^2$
$h^2 = 6.5^2 - 2.5^2$
$h = \sqrt{6.5^2 - 2.5^2}$
$= \sqrt{36}$
$= 6$



Surface area triangular prism = 2 × area triangular base + 2 × area side rectangular faces + area bottom rectangular face
$= (2 \times \frac{1}{2} \times 5 \times 6) + (2 \times 6.5 \times 3.6) + (5 \times 3.6)$
$= 94.8$

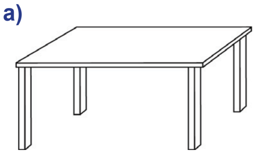
Surface area rectangular prism = 2 × area front face + 2 × area top face + 2 × area side face
$= 2 \times 6.2 \times 5 + 2 \times 5 \times 3.6 + 2 \times 6.2 \times 3.6$
$= 142.64$

Surface area cube $= 6 \times 3.6 \times 3.6$
$= 77.76$

Area overlap $= 2(5 \times 3.6) + 2(3.6 \times 3.6)$
$= 61.92$

Surface area = surface area triangular prism + surface area rectangular prism + surface area cube – area overlap
$= 94.8 + 142.64 + 77.76 - 61.92$
$= 253.28$
The surface area of the composite object is 253.28 m^2.

17. a)

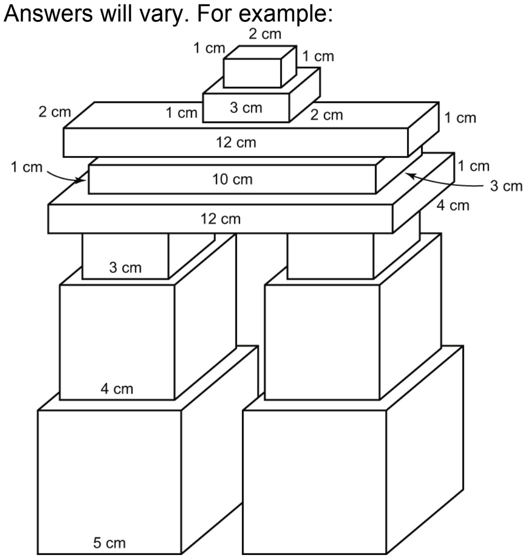


b) The overlap is 4 times the area of the top of the table leg. So, instead of calculating 4 times the area of the bottom of the table leg, then subtracting that area as the overlap, calculate only the area of the sides of the table legs.

Surface area = 2 × area of tabletop + 2 × area of front of table + 2 × area of side of table + 16 × area of side of table leg
= (2 × 106 × 50) + (2 × 2 × 106) + (2 × 2 × 50) + (16 × 75 × 3)
= 14 824

The surface area of the desk is 14 824 cm^2.

18. Answers will vary. For example:



Surface area = surface area of 2 legs + surface area of body + surface area of head – overlaps

Surface area of 2 legs = 2(surface area top cube) + 2(surface area middle cube) + 2(surface area bottom cube)
= 2(6 × 3 × 3) + 2(6 × 4 × 4) + 2 (6 × 3 × 3)
= 600

Surface area of body = surface area bottom rectangular prism + surface area middle rectangular prism + surface area top rectangular prism
= [2(12 × 1) + 2(12 × 4) + 2(4 × 1)] + [2(10 × 1) + 2(10 × 3) + 2(1 × 3)] + [2(12 × 1) + 2(12 × 2) + 2(1 × 2)]
= 290

Surface area of head = surface area top rectangular prism + surface area bottom rectangular prism
= [4(2 × 1) + 2(1 × 1)] + [2(3 × 1) + 2(3 × 2) + 2(2 × 1)]
= 32

Overlap = 2(2 × 1) + 2(3 × 2) + 2(10 × 2) + 2(10 × 3) + 2(3 × 3) + 2(3 × 3) + 2(4 × 4)
= 184

So, the surface area of my Inukshuk, in square centimetres, is:

Surface area = 600 + 290 + 32 – 184
= 738

Lesson 1.4

19. a) Surface area = curved surface area of cylinder + 2 × area of top of cylinder + 4 × area of side of rectangular prism

$= (2 \times \pi \times 9.6 \times 3.5) + (2 \times \pi \times 9.6^2) + (4 \times 2.5 \times 15)$

$\doteq 940.2$

The surface area of the composite object is about 940.2 cm^2.

b) Surface area = curved surface area of middle cylinder + 2 × curved surface area of end cylinders + 4 × area of base of end cylinders – (2 × area of base of middle cylinder)

$= (2 \times \pi \times 3.6 \times 10.4) + (2 \times 2 \times \pi \times 7.8 \times 2.8) + (4 \times \pi \times 7.8^2) - (2 \times \pi \times 3.6^2)$

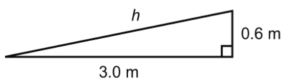
$\doteq 1192.8$

The surface area of the composite object is about 1192.8 cm^2.

20. a) Surface area = 2 × surface area of triangular prism + surface area of rectangular prism

The overlap is the side of each triangular prism, and the sides of the rectangular prism, so do not include these in the calculation.

Calculate the hypotenuse, *h*, of the right triangle in the triangular prism:



$h^2 = 3^2 + 0.6^2$

$= 9.36$

$h = \sqrt{9.36}$

$\doteq 3.06$

Surface area $= (4 \times \frac{1}{2} \times 3 \times 0.6) + (2 \times 3.06 \times 1.5) + (2 \times 3 \times 1.5) + (2 \times 2 \times 0.6) + (2 \times 2 \times 1.5)$

$= 30.18$ m^2

To the nearest tenth, the surface area to be painted is about 30.2 m^2.

b) The amount of paint needed is 2 times the surface area:

2×30.2 $m^2 = 60.4$ m^2

Since one container covers 35 m^2, 2 containers of paint are needed to cover 60.4 m^2.

2 × \$19.95 = \$39.90

The paint will cost \$39.90.

Practice Test (page 48)

1. a) i) $\sqrt{\frac{49}{4}} = \sqrt{\frac{7}{2} \times \frac{7}{2}}$

$= \frac{7}{2}$, or 3.5

ii)

$\sqrt{6.25} = \sqrt{\frac{625}{100}}$

$= \sqrt{\frac{25}{10} \times \frac{25}{10}}$

$= \frac{25}{10}$, or 2.5

iii) $\sqrt{\frac{64}{9}} = \sqrt{\frac{8}{3} \times \frac{8}{3}}$

$= \frac{8}{3}$

iv) 98.5 is close to the perfect square 100, and $\sqrt{100} = 10$

So, $\sqrt{98.5} \doteq 9.9$

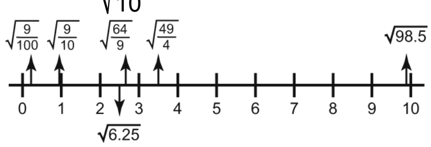
v) $\sqrt{\frac{9}{100}} = \sqrt{\frac{3}{10} \times \frac{3}{10}}$

$= \frac{3}{10}$, or 0.3

vi) $\frac{9}{10} = 0.9$, which is between the perfect squares 0.81 and 1, and slightly closer to 0.81.

So, $\sqrt{\frac{9}{10}}$ is between 0.9 and 1, and slightly closer to 0.9.

Estimate $\sqrt{\frac{9}{10}}$ as 0.95.



b) I can use the closest perfect squares as benchmarks to help estimate the square root of a non-perfect square.

2. a) i) $\sqrt{\frac{3}{7}} \doteq 0.65$

ii) $\sqrt{52.5625} = 7.25$

iii) $\sqrt{\frac{576}{25}} = 4.8$

iv) $\sqrt{213.16} = 14.6$

v) $\sqrt{135.4} \doteq 11.64$

b) ii, iii, and iv are exact; i and v are approximate.

c) A square root shown on a calculator may be approximate. For example, in a non-terminating or repeating decimal, there are many more numbers after the decimal point that cannot be displayed on the screen.

3. Answers will vary. For example:

a) A perfect square between 0 and 0.5 is 0.25.
$0.5 \times 0.5 = 0.25$, or $\sqrt{0.25} = 0.5$
I know 0.25 is a perfect square because it can be written as a product of two equal factors.

b) The number with a square root of 0.5 is 0.25. So, any number between 0 and 0.25 will have a square root between 0 and 0.5. For example:
0.04 is a number whose square root is between 0 and 0.5.

$$\begin{aligned}\sqrt{0.04} &= \sqrt{\frac{4}{100}} \\ &= \frac{2}{10} \\ &= 0.2\end{aligned}$$

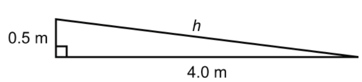
So, 0.04 has a square root between 0 and 0.5.

4. The island and the canoes form the three vertices of a right triangle. I used the Pythagorean Theorem to determine the distance between the canoes.
The distance between the canoes, d, is:

$$\begin{aligned}d^2 &= 2.56^2 + 8.28^2 \\ &= 75.112 \\ d &= \sqrt{75.112} \\ &\doteq 8.67\end{aligned}$$

The canoes are about 8.67 km apart.

5. **a)** Treat the roof as a triangular prism, with a right triangle base, sitting on top of a rectangular prism.
The hypotenuse, h, of the triangular base is:



$$\begin{aligned}h^2 &= 0.5^2 + 4^2 \\ &= 16.25 \\ h &= \sqrt{16.25} \\ &\doteq 4.03\end{aligned}$$

Do not include the floor of the shed in the surface area.
Surface area of shed = 2 × area of triangular front face + area of roof + 2 × area of front of rectangular prism + area of side of rectangular prism – area of door – area of window

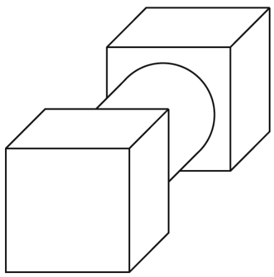
$$= \left(2 \times \frac{1}{2} \times 4 \times 0.5\right) + (4.03 \times 6) + (2 \times 4 \times 3) + (3 \times 6) - (1 \times 2) - (1 \times 1)$$

$$= 65.18$$

The surface area of the shed, not including the door and window, is about 65.2 m^2.

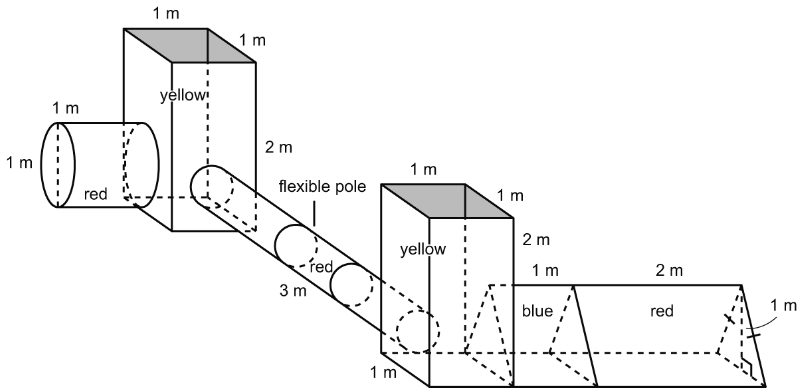
b) The amount of paint needed is 2 times the surface area.
$2 \times 65.2\ m^2 = 130.4\ m^2$
Since one litre covers 10 m^2, 14 litres are needed to cover 130.4 m^2.
14 × \$3.56 = \$49.84
It will cost \$49.84 to paint the shed.

6. a)



b) Volume of cube = 64 = side length × side length × side length
64 = 4 × 4 × 4; so, side length of cube is 4 cm
Surface area = curved surface area of cylinder + 12 × area of cube face – 2 × area of base of cylinder
$= (2 \times \pi \times 2 \times 5) + (12 \times 4 \times 4) - (2 \times \pi \times 2^2)$
$\doteq 229.7$
The surface area of this object is about 229.7 cm^2.

Unit Problem Design a Play Structure (page 49)



Assumptions
The cylinders and the triangular prisms are open at both ends.
The rectangular prisms each have one square base made of material, and are open at the top.
Both prisms have poles running along all edges, for better support.

➤Cylinders
Surface area of short cylinder = $2 \times \pi \times 0.5 \times 1$
$\doteq 3.14$
The surface area of the short cylinder is about 3.14 m^2.
Red material costs: \$10/m^2
So, cost of material for short cylinder: \$10 × 3.14 = \$31.40

Circumference of cylinder = $2\pi r$
$= 2 \times \pi \times 0.5$
$\doteq 3.14$
There are 2 flexible poles in a cylinder that is 1 m long; 2 × 3.14 m = 6.28 m
Cost of flexible poles for short cylinder: \$4 × 6.28 = \$25.13

Surface area of long cylinder = $2 \times \pi \times 0.5 \times 3$
$\doteq 9.42$
The surface area of the long cylinder is about 9.42 m^2.

Cost of material for long cylinder: \$10 × 9.42 = \$94.2

There are 4 flexible poles in a cylinder that is 3 m long; 4 × 3.14 m = 12.56 m
Cost of flexible poles for long cylinder: \$4 × 12.56 = \$50.24

➤Rectangular prisms
Surface area of rectangular prism = area of square base + 4 × area of rectangular side
$= (1)(1) + 4(2)(1)$
$= 9$
For the first rectangular prism, the overlap is 2 times the area of circular base of cylinder.
Subtract the overlap from the surface area.
So, $9 - 2 \times \pi \times 0.5^2 \doteq 7.43$
The surface area of the first rectangular prism is about 7.43 m^2.

Yellow material costs: \$11/m^2
So, cost of material for first rectangular prism: \$11 × 7.43 = \$81.73
There are 16 m of poles in this rectangular prism. The poles cost \$3/m, so \$3 × 16 = \$48

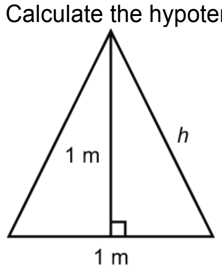
For the second rectangular prism, the overlap is the area of one circular base of cylinder and the area of on triangular base of triangular prism. Subtract this overlap from the surface area.

So, $9 - \pi \times 0.5^2 - \frac{1}{2} \times 1 \times 1 \doteq 7.71$

The surface area of the second rectangular prism is about 7.71 m^2.
Cost of material for second rectangular prism: \$11 × 7.71 = \$84.81

➤Triangular prisms

Calculate the hypotenuse, *h*, of the triangular base.



$h^2 = 0.5^2 + 1^2$
$= 1.25$
$h = \sqrt{1.25}$
$\doteq 1.12$

Surface area of blue triangular prism = 2 × area of rectangular face + area of bottom rectangular face
= 2 × 1.12 × 1 + 1 × 1
= 3.24

The surface area of the blue triangular prism is 3.24 m^2.
Blue material: \$12/m^2
So, cost of material for blue triangular prism: \$12 × 3.24 = \$36.88
Perimeter of triangle = 1 + 1.12 + 1.12
= 3.24
2 × 3.24 + 3 = 9.48
There are of 9.48 m of poles in the blue triangular prism. The poles cost \$3/m, so \$3 × 9.48 = \$28.44

Surface area of red triangular prism = 2 × area of rectangular face + area of bottom rectangular face
= 2 × 1.12 × 2 + 1 × 2
= 6.48

The surface area of the red triangular prism is 6.48 m^2.
Cost of material for red triangular prism: \$10 × 6.48 = \$64.80

2 × 3.24 + 6 = 12.48
There are 12.48 m of poles in the red triangular prism. The poles cost \$3/m, so \$3 × 12.48 = \$37.44

Total surface area = curved surface area of short cylinder + curved surface area of long cylinder + surface area of first rectangular prism + surface area of second rectangular prism + surface area of blue triangular prism + surface area of red triangular prism
= 3.14 + 9.42 + 7.43 + 7.71 + 3.24 + 6.48
= 37.42

The total surface area is 37.42 m^2.

Total cost of the project = cost of short cylinder + cost of poles + cost of long cylinder + cost of poles + cost of first rectangular prism + cost of poles + cost of second rectangular prism + cost of poles + cost of blue triangular prism + cost of poles + cost of red triangular prism + cost of poles

= 31.41 + 25.13 + 94.20 + 50.24 + 81.73 + 48 + 84.81 + 48 + 36.88 + 28.44 + 64.80 + 37.44

= 631.08

The total cost of the project is $631.08, plus an additional $125 as a donation for the sewing of the fabric.
$631.08 + $125 = $756.08, which is within a budget of $800.

Unique features

I cut out some flaps that can be rolled up, to provide a window.
One flap is cut out in the middle section of the long cylinder, and each rectangular prism has 2 cut-outs.
This way, children can look out of the structure at different points within the structure.
The cut-outs do not affect the total surface area, or the cost of the project.
Since I have some extra money in my budget, I can buy Velcro fasteners to keep the cut-outs rolled up.

