| **Lesson 2.1** | **What Is a Power?** | **Practice (pages 55–57)** |
|---|---|---|

### *Check*

**4. a)** There are 4 unit squares.

$4 = 2^2$

**b)** There are 9 unit squares.

$9 = 3^2$

**c)** There are 25 unit squares.

$25 = 5^2$

**5. a)** There are 27 unit cubes.

$27 = 3^3$

**b)** There are 8 unit cubes.

$8 = 2^3$

**c)** There are 125 unit cubes.
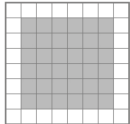
$125 = 5^3$

**6. a)** $4^2 = 4 \times 4$

$\quad\quad = 16$

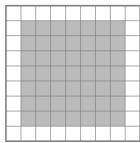Each side length of the square is 4 units, and the area of the square is 16.



**b)** $6 \times 6 = 36$

Each side length of the square is 6 units, and the area of the square is 36.
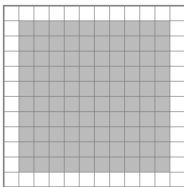


**c)** $49 = 7 \times 7$

Each side length of the square is 7 units, and the area of the square is 49.
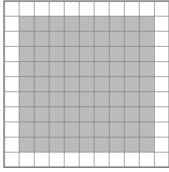


**d)** $10^2 = 10 \times 10$

$\quad\quad = 100$

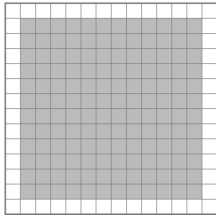Each side length of the square is 10 units, and the area of the square is 100.

**e)** 81 = 9 × 9

Each side length of the square is 9 units, and the area of the square is 81.

**f)** 12 × 12 = 144

Each side length of the square is 12 units, and the area of the square is 144.

7. **a)** The base of $2^7$ is 2.

**b)** The base of $4^3$ is 4.

**c)** The base of $8^2$ is 8.

**d)** The base of $(-10)^5$ is –10.

**e)** The base of $(-6)^7$ is –6.

**f)** The base of $8^3$ is 8.

8. **a)** The exponent of $2^5$ is 5.

**b)** The exponent of $6^4$ is 4.

**c)** The exponent of $9^1$ is 1.

**d)** The exponent of $-3^2$ is 2.

**e)** The exponent of $(-2)^9$ is 9.

**f)** The exponent of $(-8)^3$ is 3.

9. **a)** $3^2 = 3 \times 3$

**b)** $10^4 = 10 \times 10 \times 10 \times 10$

**c)** $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

**d)** $(-6)^5 = (-6)(-6)(-6)(-6)(-6)$

**e)** $-6^5 = -(6^5)$
$= -(6 \times 6 \times 6 \times 6 \times 6)$

**f)** $-4^2 = -(4^2)$
$\quad\quad = -(4 \times 4)$

**10. a)** $3^2 = 3 \times 3$
$\quad\quad = 9$

This can be shown with 9 unit square tiles arranged in a 3 by 3 square:
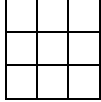
$2^3 = 2 \times 2 \times 2$
$\quad\quad = 8$

This can be shown with 8 unit cubes arranged in a 2 by 2 by 2 cube.

**b)** $3^2$ is a square number because it is a power with exponent 2, and can be represented by a square with side length equal to the base of the power.

$2^3$ is a cube number because it is a power with exponent 3, and can be represented by a cube with edge length equal to the base of the power.

**11.** $6^4 = 6 \times 6 \times 6 \times 6$
$\quad\quad = 1296$
$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$
$\quad\quad = 4096$

**12. a)** $4 \times 4 \times 4 \times 4$
The base is 4. There are 4 equal factors, so the exponent is 4.
So, $4 \times 4 \times 4 \times 4 = 4^4$

**b)** $2 \times 2 \times 2$
The base is 2. There are 3 equal factors, so the exponent is 3.
So, $2 \times 2 \times 2 = 2^3$

**c)** $5 \times 5 \times 5 \times 5 \times 5 \times 5$
The base is 5. There are 6 equal factors, so the exponent is 6.
So, $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

**d)** $10 \times 10 \times 10$
The base is 10. There are 3 equal factors, so the exponent is 3.
So, $10 \times 10 \times 10 = 10^3$

**e)** $(-79)(-79)$
The base is $(-79)$. There are 2 equal factors, so the exponent is 2.
So, $(-79)(-79) = (-79)^2$

**f)** $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$
The base is $(-2)$. There are 8 equal factors, so the exponent is 8.
So, $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2) = -(-2)^8$

## *Apply*

**13. a)** 5 × 5

The base is 5. There are 2 equal factors, so the exponent is 2.

So, $5 \times 5 = 5^2$
$= 25$

**b)** 3 × 3 × 3 × 3

The base is 3. There are 4 equal factors, so the exponent is 4.

So, $3 \times 3 \times 3 \times 3 = 3^4$
$= 81$

**c)** 10 × 10 × 10 × 10 × 10

The base is 10. There are 5 equal factors, so the exponent is 5.

So, $10 \times 10 \times 10 \times 10 \times 10 = 10^5$
$= 100\ 000$

**d)** –(9 × 9 × 9)

The base is 9. There are 3 equal factors, so the exponent is 3.

So, $-(9 \times 9 \times 9) = -9^3$
$= -729$

**e)** (–2)(–2)(–2)

The base is (–2). There are 3 equal factors, so the exponent is 3.

So, $(-2)(-2)(-2) = (-2)^3$
$= -8$

**f)** –(–4)(–4)(–4)

the base is (–4). There are 3 equal factors, so the exponent is 3.

So, $-(-4)(-4)(-4) = -(-4)^3$
$= 64$

**g)** (–5)(–5)(–5)(–5)

The base is (–5). There are 4 equal factors, so the exponent is 4.

So, $(-5)(-5)(-5)(-5) = (-5)^4$
$= 625$

**h)** –(5)(5)(5)(5)

The base is 5. There are 4 equal factors, so the exponent is 4.

So, $-(5)(5)(5)(5) = -5^4$
$= -625$

**i)** –(–5)(–5)(–5)(–5)

The base is (–5). There are 4 equal factors, so the exponent is 4.

So, $-(-5)(-5)(-5)(-5) = -(-5)^4$
$= -625$

**14. a)** I predict the answer is positive, because the base is positive.

$2^3 = 2 \times 2 \times 2$
$= 8$

**b)** I predict the answer is positive, because the base is positive.

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 1\ 000\ 000$

**c)** I predict the answer is positive, because the base is positive.
$3^1 = 3$

**d)** The exponent applies only to the base 7, and not to the negative sign.
I predict the answer is negative, because there is a negative sign in front of the base.
$-7^3 = -(7^3)$
$\quad\quad = -(7 \times 7 \times 7)$
$\quad\quad = -343$

**e)** The sign of a product with an odd number of negative factors is negative.
So, I predict the answer is negative, because the base is negative, and the exponent is an odd number.
$(-7)^3 = (-7)(-7)(-7)$
$\quad\quad\quad = -343$

**f)** The sign of a product with an even number of negative factors is positive.
So, I predict the answer is positive, because the exponent is an even number.
$(-2)^8 = (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$
$\quad\quad\quad = 256$

**g)** The exponent applies only to the base 2, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-2^8 = -(2^8)$
$\quad\quad = -(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
$\quad\quad = -256$

**h)** The exponent applies only to the base 6, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-6^4 = -(6^4)$
$\quad\quad = -(6 \times 6 \times 6 \times 6)$
$\quad\quad = -1296$

**i)** The sign of a product with an even number of negative factors is positive.
So, I predict the answer is positive, because the exponent is an even number.
$(-6)^4 = (-6)(-6)(-6)(-6)$
$\quad\quad\quad = 1296$

**j)** The sign of a product with an even number of negative factors is positive.
But, there is another negative sign in front of the base, so I predict the answer is negative.
$-(-6)^4 = -((-6)(-6)(-6)(-6))$
$\quad\quad\quad\quad = -(1296)$
$\quad\quad\quad\quad = -1296$

**k)** The sign of a product with an odd number of negative factors is negative.
So, I predict the answer is negative, because the base is negative, and the exponent is an odd number.
$(-5)^3 = (-5)(-5)(-5)$
$\quad\quad\quad = -125$

**l)** The exponent applies only to the base 4, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-4^4 = -(4^4)$
$\quad\quad = -(4 \times 4 \times 4 \times 4)$
$\quad\quad = -256$

**15. a) i)** $3 \times 3 = 3^2$

$\qquad\qquad = 9$

There are 9, or $3^2$, stamps in a 3 by 3 block.

    **ii)** Each stamp is $1.55, so the value of 9 stamps is: $9 \times \$1.55 = \$13.95$

    The value of a 3 by 3 block of stamps is $13.95.

  **b) i)** $4 \times 4 = 4^2$

$\qquad\qquad = 16$

There are 16, or $4^2$, stamps in a 4 by 4 block.

    **ii)** Each stamp is $0.52, so the value of 16 stamps is: $16 \times \$0.52 = \$8.32$

    The value of a 4 by 4 block of stamps is $8.32.

**16. a)** $3^{12} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\qquad\quad = 531\ 441$

  **b)** $-7^7 = -(7^7)$

$\qquad\quad = -(7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7)$

$\qquad\quad = -823\ 543$

  **c)** $5^{11} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$\qquad\quad = 48\ 828\ 125$

  **d)** $-(-4)^{10} = -((-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4))$

$\qquad\qquad\quad = -(1\ 048\ 576)$

$\qquad\qquad\quad = -1\ 048\ 576$

  **e)** $(-9)^8 = (-9)(-9)(-9)(-9)(-9)(-9)(-9)(-9)$

$\qquad\qquad = 43\ 046\ 721$

  **f)** $2^{23} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\qquad\quad = 8\ 388\ 608$

**17. a) i)** $4^3 = 4 \times 4 \times 4$           As repeated multiplication

$\qquad\quad = 64$               Standard form

    **ii)** $-4^3 = -(4^3)$

$\qquad\qquad = -(4 \times 4 \times 4)$     As repeated multiplication

$\qquad\qquad = -64$             Standard form

    **iii)** $-(-4^3) = -(-(4^3))$

$\qquad\qquad\quad = -(-(4 \times 4 \times 4))$    As repeated multiplication

$\qquad\qquad\quad = -(-64)$          $-(-64)$ is the opposite of $-64$, which is 64

$\qquad\qquad\quad = 64$               Standard form

    **iv)** $(-4^3) = (-(4^3))$

$\qquad\qquad\quad = (-(4 \times 4 \times 4))$    As repeated multiplication

$\qquad\qquad\quad = (-64)$

$\qquad\qquad\quad = -64$            Standard form

**b)** $4^3$ is positive because the base 4 is positive.

$-(-4^3)$ is positive because $-4^3$ is negative, and $-(-4^3)$ is the opposite of $-4^3$.

$-4^3$ is negative because the negative sign is not part of the base of the power.

$(-4^3)$ is negative because the exponent is inside the brackets and has no effect on the negative sign.

**c) i)** $4^2 = 4 \times 4$                  As repeated multiplication

     $= 16$                      Standard form

**ii)** $-4^2 = -(4^2)$

         $= -(4 \times 4)$            As repeated multiplication

         $= -16$               Standard form

**iii)** $-(-4^2) = -(-(4^2))$

            $= -(-(4 \times 4))$        As repeated multiplication

            $= -(-16)$           $-(-16)$ is the opposite of $-16$, which is 16

            $= 16$               Standard form

**iv)** $(-4^2) = (-(4 \times 4))$       As repeated multiplication

       $= -16$               Standard form

**d)** $4^2$ is positive because the base 4 is positive.

$-(-4^2)$ is positive because $-4^2$ is negative, and $-(-4^2)$ is the opposite of $-4^2$.

$-4^2$ is negative because the negative sign is not part of the base of the power.

$(-4^2)$ is negative because the exponent is inside the brackets and has no effect on the negative sign.

**e)** $5^4, -5^4, -(-5^4), (-5^4)$

$9^1, -9^1, -(-9^1), (-9^1)$

$10^9, -10^9, -(-10^9), (-10^9)$

If a power has a positive base and the power does not have a negative sign in front of it, the product will be positive.

If the power has a positive base, and there are two negative signs in front of the power, the product will be positive because one negative sign produces a negative product, then the second negative sign indicates that the product will be the opposite number.

If there is a negative sign in front of the power and the power has a positive base, the product will be negative.

**18. a)** All three expressions are equal.

For $-3^5$, the negative sign is not part of the base of the power:

$-3^5 = -(3^5)$

     $= -(3 \times 3 \times 3 \times 3 \times 3)$

     $= -243$

For $(-3)^5$, the brackets indicate that the negative sign is part of the base of the power. The product is negative because the exponent is odd, and the product of an odd number of negative integers is negative:

$(-3)^5 = (-3)(-3)(-3)(-3)(-3)$

     $= -243$

For $(-3^5)$, the brackets serve no purpose and $(-3^5)$ is the same as $-3^5$.

So, $(-3^5) = -(3^5)$

       $= -(3 \times 3 \times 3 \times 3 \times 3)$

       $= -243$

**b)** The expressions $-4^6$ and $(-4^6)$ are equal.

For $-4^6$, the negative sign is not part of the base of the power:

$-4^6 = -(4^6)$

$\quad\quad = -(4 \times 4 \times 4 \times 4 \times 4 \times 4)$

$\quad\quad = -4096$

For $(-4^6)$, the brackets serve no purpose and $(-4^6)$ means the same as $-4^6$.

So, $(-4^6) = -(4^6)$

$\quad\quad\quad\quad = -(4 \times 4 \times 4 \times 4 \times 4 \times 4)$

$\quad\quad\quad\quad = -4096$

For $(-4)^6$, the brackets indicate that the negative sign is part of the base of the power. The product is positive because the exponent is even, and the product of an even number of negative integers is positive:

$(-4)^6 = (-4)(-4)(-4)(-4)(-4)(-4)$

$\quad\quad = 4096$

**19. a)** A negative base in a power produces a negative product when the exponent is an odd number.
Some examples are:

$(-3)^3 = (-3)(-3)(-3)$

$\quad\quad = -27$

$(-2)^5 = (-2)(-2)(-2)(-2)\,(-2)$

$\quad\quad = -32$

$(-10)^7 = (-10)(-10)(-10)(-10)(-10)(-10)(-10)$

$\quad\quad\quad = -10\,000\,000$

**b)** A negative base in a power produces a positive product when the exponent is an even number.
Some examples are:

$(-3)^2 = (-3)(-3)$

$\quad\quad = 9$

$(-2)^4 = (-2)(-2)(-2)(-2)$

$\quad\quad = 16$

$(-10)^6 = (-10)(-10)(-10)(-10)(-10)(-10)$

$\quad\quad\quad = 1\,000\,000$

## *Take It Further*

**20. a)** $4 = 2 \times 2$

$\quad\quad = 2^2$

**b)** $16 = 2 \times 2 \times 2 \times 2$

$\quad\quad = 2^4$

**c)** $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\quad\quad = 2^6$

**d)** $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\quad\quad = 2^8$

**e)** $32 = 2 \times 2 \times 2 \times 2 \times 2$

$\quad\quad = 2^5$

**f)** $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\quad\quad = 2^7$

I multiplied 2 by 2 repeatedly until I reached the number. I could have also divided the number by 2 repeatedly and counted how many times I had to do this until I reached 1.

**21.** I tried dividing each number by 2, by 3, by 4, and so on.

I know that 2, 4, 8, and 16 are all multiples of 2, so when I wrote the power with a base of 2, I then checked to see if it could be written with a different base that is a multiple of 2.

When the exponent of a power is even and greater than 2, it can be written a different way with a base that is the square of the current base.

The same is true for 9 as a multiple of 3.

**a) i)** $16 = 16^1$

$16 = 4 \times 4$

$= 4^2$

$16 = 2 \times 2 \times 2 \times 2$

$= 2^4$

**ii)** $81 = 81^1$

$81 = 9 \times 9$

$= 9^2$

$81 = 3 \times 3 \times 3 \times 3$

$= 3^4$

**iii)** $256 = 256^1$

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8$

$256 = 4 \times 4 \times 4 \times 4$

$= 4^4$

$256 = 16 \times 16$

$= 16^2$

**b)** To find more numbers that can be written as a power in more than one way, I chose a number, squared it, then squared the product again.

I chose 10, squared it to get $10 \times 10 = 100$, then squared 100 to get $100 \times 100 = 10\ 000$.

So, one number that can be written as a power in more than one way is 10 000 because:

$10\ 000 = 100^2$

$= 100 \times 100$

and also:

$10\ 000 = 10^4$

$= 10 \times 10 \times 10 \times 10$

I chose 5, squared it to get $5 \times 5 = 25$, then squared 25 to get $25 \times 25 = 625$.

So, another number that can be written as a power in more than one way is 625 because:

$625 = 25^2$

$= 25 \times 25$

and also:

$625 = 5^4$

$= 5 \times 5 \times 5 \times 5$

**22. a)** The powers in each pair have the same numbers in them.

The powers are different because the bases and the exponents are interchanged.

**b) i)** $3^2$ is greater than $2^3$ because $3^2 = 9$ and $2^3 = 8$

**ii)** $2^5$ is greater than $5^2$ because $2^5 = 32$ and $5^2 = 25$

**iii)** $3^4$ is greater than $4^3$ because $3^4 = 81$ and $4^3 = 64$

**iv)** $4^5$ is greater than $5^4$ because $4^5 = 1024$ and $5^4 = 625$

**23.** I know that $3^5$ is greater than $3^4$, because the powers have the same base, but $3^5$ has a greater exponent.
I know that $6^3$ is greater than $5^2$ because $6^3$ has a greater base and a greater exponent than $5^2$.
$3^5$ can be written as $3 \times 3 \times 3 \times 3 \times 3$
$6^3$ can be written as $6 \times 6 \times 6 = 3 \times 3 \times 3 \times 2 \times 2 \times 2$
$3^4$ can be written as $3 \times 3 \times 3 \times 3$
The product of the first three factors in each repeated multiplication is the same, so look at the remaining factors.
$3 \times 3 = 9$, and $2 \times 2 \times 2 = 8$; $9 > 8$, so $3^5 > 6^3$
The remaining factor in $3^4$ is 3, so $6^3 > 3^4$
And $3^4 > 5^2$ since $3 \times 3 \times 3 \times 3 = 81$, $5 \times 5 = 25$ and $81 > 25$.
So, from greatest to least:
$3^5, 6^3, 3^4, 5^2$

**24. a)** There are 64 1-unit squares on the checkerboard.
     $64 = 8 \times 8$
     $\quad = 8^2$

   **b)** There are 49 2-unit squares on the checkerboard.
     $49 = 7 \times 7$
     $\quad = 7^2$

   **c)** There are 36 3-unit squares on the checkerboard.
     $36 = 6 \times 6$
     $\quad = 6^2$

   **d)** There are 25 4-unit squares on the checkerboard.
     $25 = 5 \times 5$
     $\quad = 5^2$

   **e)** There are 16 5-unit squares on the checkerboard.
     $16 = 4 \times 4$
     $\quad = 4^2$

   **f)** There are nine 6-unit squares on the checkerboard.
     $9 = 3 \times 3$
     $\quad = 3^2$

   **g)** There are four 7-unit squares on the checkerboard.
     $4 = 2 \times 2$
     $\quad = 2^2$

   **h)** There is one 8-unit square on the checkerboard.
     $1 = 1 \times 1$
     $\quad = 1^2$

   Each number of squares is a square number that decreases as the size of the square increases.

**25.** You can tell if a number is a square number by trying to determine if there is a number that, when multiplied by itself, has a product equal to the number being investigated.
For example, 2096;  try $40 \times 40 = 1600$ and $50 \times 50 = 2500$, determine $44 \times 44 = 1936$, then
$46 \times 46 = 2116$, so 2096 is not a square number because $45 \times 45$ would have a product with 5 in the ones place. A similar method works for a cube number, but multiply a number by itself twice. For example, 2096; try $10 \times 10 \times 10 = 1000$, and $12 \times 12 \times 12 = 1728$, and $13 \times 13 \times 13 = 2197$, so 2096 is not a cube number either.

### *Check*

4. A power with exponent 0 is equal to 1.
   a) $50^0 = 1$

   b) $9^0 = 1$

   c) $1^0 = 1$

   d) $17^0 = 1$

5. A power with exponent 0 is equal to 1.
   a) $(-6)^0 = 1$

   b) $-11^0 = -1$

   c) $-8^0 = -1$

   d) $(-24)^0 = 1$

6. a) $1000 = 10^3$

   b) $100\ 000 = 10^5$

   c) $1\ 000\ 000\ 000 = 10^9$

   d) Ten thousand $= 10\ 000$
   $$= 10^4$$

   e) One hundred billion $= 100\ 000\ 000\ 000$
   $$= 10^{11}$$

### *Apply*

7. Answers may vary.
   $10^0 = 1$
   $1^4 = 1$
   $(-6)^0 = 1$

8. a) $10^7 = 10\ 000\ 000$

   b) $10^2 = 100$

   c) $10^0 = 1$

   d) $10^{10} = 10\ 000\ 000\ 000$

   e) $10^1 = 10$

   f) $10^6 = 1\ 000\ 000$

9.  a) 6 000 000 000 = 6 × 1 000 000 000
    $\qquad\qquad\qquad$ = $6 \times 10^9$

    b) 200 = 2 × 100
    $\qquad\quad$ = $2 \times 10^2$

    c) Use a place-value chart.

| Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|
| 5 | 1 | 4 | 1 | 5 |

    51 415 = 50 000 + 1000 + 400 + 10 + 5
    $\qquad$ = (5 × 10 000) + (1 × 1000) + (4 × 100) + (1 × 10) + (5 × 1)
    $\qquad$ = $(5 \times 10^4) + (1 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (5 \times 10^0)$

    d) Use a place-value chart.

| Ten millions | Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|---|
| 6 | 0 | 7 | 0 | 2 | 0 | 0 | 8 |

    60 702 008 = 60 000 000 + 700 000 + 2000 + 8
    $\qquad\qquad$ = (6 × 10 000 000) + (7 × 100 000) + (2 × 1000) + (8 × 1)
    $\qquad\qquad$ = $(6 \times 10^7) + (7 \times 10^5) + (2 \times 10^3) + (8 \times 10^0)$

    e) Use a place-value chart.

| Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|
| 3 | 0 | 2 | 4 | 1 | 1 |

    302 411 = 300 000 + 2000 + 400 + 10 + 1
    $\qquad$ = (3 × 100 000) + (2 × 1000) + (4 × 100) + (1 × 10) + (1 × 1)
    $\qquad$ = $(3 \times 10^5) + (2 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (1 \times 10^0)$

    f) Use a place-value chart.

| Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| 2 | 0 | 0 | 0 | 0 | 0 | 8 |

    2 000 008 = 2 000 000 + 8
    $\qquad$ = (2 × 1 000 000) + (8 × 1)
    $\qquad$ = $(2 \times 10^6) + (8 \times 10^0)$

10. a) $7 \times 10^7$ = 7 × 10 000 000
    $\qquad\qquad$ = 70 000 000

    b) $(3 \times 10^4) + (9 \times 10^3) + (5 \times 10^1) + (7 \times 10^0)$
    = (3 × 10 000) + (9 × 1000) + (5 × 10) + (7 × 1)
    = 30 000 + 9000 + 50 + 7
    = 39 057

    c) $(8 \times 10^8) + (5 \times 10^5) + (2 \times 10^2)$
    = (8 × 100 000 000) + (5 × 100 000) + (2 × 100)
    = 800 000 000 + 500 000 + 200
    = 800 500 200

**d)** $(9 \times 10^{10}) + (8 \times 10^9) + (1 \times 10^0)$
  $= (9 \times 10\ 000\ 000\ 000) + (8 \times 1\ 000\ 000\ 000) + (1 \times 1)$
  $= 90\ 000\ 000\ 000 + 8\ 000\ 000\ 000 + 1$
  $= 98\ 000\ 000\ 001$

**e)** $1 \times 10^{15} = 1 \times 1\ 000\ 000\ 000\ 000\ 000$
      $= 1\ 000\ 000\ 000\ 000\ 000$

**f)** $(4 \times 10^3) + (1 \times 10^0) + (9 \times 10^5) + (3 \times 10^1)$
  $= (4 \times 1000) + (1 \times 1) + (9 \times 100\ 000) + (3 \times 10)$
  $= 4000 + 1 + 900\ 000 + 30$
  $= 900\ 000 + 4000 + 30 + 1$
  $= 904\ 031$

**11.** 500 million = $500\ 000\ 000$
          $= 5 \times 100\ 000\ 000$
          $= 5 \times 10^8$

  40 000 = $4 \times 10\ 000$
          $= 4 \times 10^4$

  3 million = $3\ 000\ 000$
          $= 3 \times 1\ 000\ 000$
          $= 3 \times 10^6$

  17 000 = $(1 \times 10\ 000) + (7 \times 1000)$
          $= (1 \times 10^4) + (7 \times 10^3)$

  130 000 = $(1 \times 100\ 000) + (3 \times 10\ 000)$
          $= (1 \times 10^5) + (3 \times 10^4)$

  600 = $6 \times 100$
      $= 6 \times 10^2$

**12.** Negative bases may vary.

  For example: I chose $(-3)$.

| Exponent | Power | Standard Form |
|---|---|---|
| 5 | $(-3)^5$ | −243 |
| 4 | $(-3)^4$ | 81 |
| 3 | $(-3)^3$ | −27 |
| 2 | $(-3)^2$ | 9 |
| 1 | $(-3)^1$ | −3 |
| 0 | $(-3)^0$ | 1 |

Each number in standard form is $-\dfrac{1}{3}$ of the number before it; that is, divide each number by −3 to get the next number. The standard form of $(-3)^1$ is −3 so for the next power, $(-3)^0$, we divide −3 by −3:

$(-3)^0 = \dfrac{-3}{-3} = 1.$

**13.** Write each number in standard form and compare.

   **a)** $(4 \times 10^3) + (6 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$
   $= (4 \times 1000) + (6 \times 100) + (6 \times 10) + (7 \times 1)$
   $= 4000 + 600 + 60 + 7$
   $= 4667$
   4667 is greater than 4327

   **b)** $(2 \times 10^4) + (4 \times 10^3) + (2 \times 10^2) + (4 \times 10^1)$
   $= (2 \times 10\ 000) + (4 \times 1000) + (2 \times 100) + (4 \times 10)$
   $= 20\ 000 + 4000 + 200 + 40$
   $= 24\ 240$
   24 240 is greater than 2432

   **c)** $(7 \times 10^7) + (7 \times 10^3)$
   $= (7 \times 10\ 000\ 000) + (7 \times 1000)$
   $= 70\ 000\ 000 + 7000$
   $= 70\ 007\ 000$
   70 007 000 is greater than 777 777

**14. a)** One billion $= 1\ 000\ 000\ 000$
   $= 10^9$

   $100\ 000 = 10^5$

   $1000 = 10^3$

   $1 = 10^0$

   $100 = 10^2$

   10 million $= 10\ 000\ 000$
   $= 10^7$

   **b)** $1\ (10^0)$, $100\ (10^2)$, $1000\ (10^3)$, $100\ 000\ (10^5)$, 10 million $(10^7)$, one billion $(10^9)$

   **c)** To compare powers of 10, I need only compare their exponents; the least number has the least exponent.

### *Take It Further*

**15.** Strategies may vary. For example:

I noticed earlier that the exponent of a power of 10 is equal to the number of zeros in the number. I also realized that trillion, quadrillion, and quintillion are the next three units after one billion, so I know that their exponents are multiples of 3.

I know that one million is one thousand thousands, or $10^6$; one billion is one thousand millions, or $10^9$. So following that pattern, one trillion is one thousand billions, or $10^{12}$, which is 1 followed by 12 zeros; one quadrillion is one thousand trillions, or $10^{15}$, which is 1 followed by 15 zeros; and one quintillion is one thousand quadrillions, or $10^{18}$, which is 1 followed by 18 zeros.

| **Lesson 2.3** | **Order of Operations with Powers** | **Practice (pages 66–68)** |

### *Check*

**3. a)** $3^2 + 1 = (3)(3) + 1$     Evaluate the power, then add.
$\qquad = 9 + 1$
$\qquad = 10$

**b)** $3^2 - 1 = (3)(3) - 1$     Evaluate the power, then subtract.
$\qquad = 9 - 1$
$\qquad = 8$

**c)** $(3 + 1)^2 = 4^2$          Add first, since this operation is within the brackets, then evaluate the power.
$\qquad = 16$

**d)** $(3 - 1)^2 = 2^2$          Subtract first, since this operation is within the brackets, then evaluate the power.
$\qquad = 4$

**e)** $2^2 + 4 = (2)(2) + 4$     Evaluate the power, then add.
$\qquad = 4 + 4$
$\qquad = 8$

**f)** $2^2 - 4 = (4)(4) - 4$     Evaluate the power, then subtract.
$\qquad = 4 - 4$
$\qquad = 0$

**g)** $(2 + 4)^2 = 6^2$          Add first, since this operation is within the brackets, then evaluate the power.
$\qquad = 36$

**h)** $(2 - 4)^2 = (-2)^2$       Subtract first, since this operation is within the brackets, then evaluate the power.
$\qquad = 4$

**i)** $2 - 4^2 = 2 - (2)(2)$     Evaluate the power, then subtract.
$\qquad = 2 - 16$
$\qquad = -14$

**j)** $2^2 - 4^2 = (2)(2) - (4)(4)$     Evaluate the powers, then subtract.
$\qquad = 4 - 16$
$\qquad = -12$

**4. a)** $2^3 \times 5 = (2)(2)(2) \times 5$     Evaluate the power, then multiply.
$\qquad = 8 \times 5$
$\qquad = 40$

**b)** $2 \times 5^2 = 2 \times (5)(5)$     Evaluate the power, then multiply.
$\qquad = 2 \times 25$
$\qquad = 50$

**c)** $(2 \times 5)^3 = 10^3$     Multiply first, since this operation is within the brackets, then evaluate
$\qquad = (10)(10)(10)$     the power.
$\qquad = 1000$

**d)** $(2 \times 5)^2 = 10^2$

$= (10)(10)$

$= 100$

Multiply first, since this operation is within the brackets, then evaluate the power.

**e)** $(-10)^3 \div 5 = (-10)(-10)(-10) \div 5$

$= -1000 \div 5$

$= -200$

Evaluate the power, then divide.

**f)** $(-10) \div 5^0 = (-10) \div 1$

$= -10$

Evaluate the power, then divide.

**g)** $[(-10) \div 5]^3 = [-2]^3$

$= (-2)(-2)(-2)$

$= -8$

Divide first, since this operation is within the brackets, then evaluate the power.

**h)** $[(-10) \div 5]^0 = [-2]^0$

$= 1$

Divide first, since this operation is within the brackets, then evaluate the power.

**5.  a)** $2^3 + (-2)^3 = (2)(2)(2) + (-2)(-2)(-2)$

$= 8 + (-8)$

$= 0$

Evaluate the powers, then add.

**b)** $(2 - 3)^3 = (-1)^3$

$= (-1)(-1)(-1)$

$= -1$

Subtract first, since this operation is within the brackets, then evaluate the power.

**c)** $2^3 - (-3)^3 = (2)(2)(2) - (-3)(-3)(-3)$

$= 8 - (-27)$

$= 8 + 27$

$= 35$

Evaluate the powers, then subtract.

**d)** $(2 + 3)^3 = 5^3$

$= (5)(5)(5)$

$= 125$

Add first, since this operation is within the brackets, then evaluate the power.

**e)** $2^3 \div (-1)^3 = (2)(2)(2) \div (-1)(-1)(-1)$

$= 8 \div (-1)$

$= -8$

Evaluate the powers, then divide.

**f)** $(2 \div 2)^3 = 1^3$

$= 1$

Divide first, since this operation is within the brackets, then evaluate the power.

**g)** $2^3 \times (-2)^3 = 8 \times (-8)$

$= -64$

Evaluate the powers, then multiply.

**h)** $(2 \times 1)^3 = 2^3$

$= 8$

Multiply first, since this operation is within the brackets, then evaluate the power.

*Apply*

6.  **a) i)**  $4^2 + 4^3 = (4)(4) + (4)(4)(4)$        Evaluate the powers, then add.
                $= 16 + 64$
                $= 80$

    **ii)** $5^3 + 5^6 = (5)(5)(5) + (5)(5)(5)(5)(5)(5)$        Evaluate the powers, then add.
                $= 125 + 15\ 625$
                $= 15\ 750$

    **b) i)** $6^3 - 6^2 = (6)(6)(6) - (6)(6)$        Evaluate the powers, then subtract.
                $= 216 - 36$
                $= 180$

    **ii)** $6^3 - 6^5 = (6)(6)(6) - (6)(6)(6)(6)(6)$    Evaluate the powers, then subtract.
                $= 216 - 7776$
                $= -7560$

7.  In the 2nd row, 36 should be positive, because $(-6)^2 = (-6)(-6)$
                                                    $= 36$
    Then, the student should have multiplied first $4 \times 16$, not added $9 + 4$.
    The correct solution:
    $3^2 + 2^2 \times 2^4 + (-6)^2 = 9 + 4 \times 16 + 36$
                        $= 9 + 64 + 36$
                        $= 109$

8.  **a)** First, I would evaluate the power: $(5)^2 = 25$
        $(7)(4) - (5)^2 = (7)(4) - 25$        Then I would multiply.
                        $= 28 - 25$        Then subtract.
                        $= 3$

    **b)** First, I would evaluate the expression in brackets: $(2 - 5) = -3$
        $6(2 - 5)^2 = 6(-3)^2$                Then I would evaluate the power.
                    $= 6(9)$                Then multiply.
                    $= 54$

    **c)** First, I would evaluate the power: $(-3)^2 = 9$
        $(-3)^2 + (4)(7) = 9 + (4)(7)$        Then I would multiply.
                        $= 9 + 28$        Then add.
                        $= 37$

    **d)** First, I would evaluate the power: $4^0 = 1$
        $(-6) + 4^0 \times (-2) = (-6) + 1 \times (-2)$        Then I would multiply.
                        $= (-6) + (-2)$        Then add.
                        $= -8$

    **e)** First, I would evaluate the expression in brackets: $[10 \div (-2)] = -5$
        $10^2 \div [10 \div (-2)]^2 = 10^2 \div (-5)^2$        Then I would evaluate the powers.
                        $= 100 \div 25$        Then divide.
                        $= 4$

**f)** First, I would evaluate the expression in brackets: $[18 ÷ (–6)] = –3$

$[18 ÷ (–6)]^3 × 2 = (–3)^3 × 2$          Then I would evaluate the power.

$= –27 × 2$          Then multiply.

$= –54$

9. Acronyms may vary. For example:

The order of operations is <u>b</u>rackets first, then <u>e</u>valuate the powers, then <u>m</u>ultiply and <u>d</u>ivide, then <u>a</u>dd and <u>s</u>ubtract. That makes the acronym BEDMAS, which I can remember by:

<u>B</u>ring <u>E</u>very <u>M</u>other <u>D</u>aisies <u>A</u>nd <u>S</u>unflowers

**10. a)** $(3 + 4)^2 × (4 – 6)^3 = 7^2 × (–2)^3$          Do the operations in the brackets first.

$= (7)(7) × (–2)(–2)(–2)$          Then evaluate the powers.

$= 49 × (–8)$          Then multiply.

$= –392$

**b)** $(8 ÷ 2^2 + 1)^3 – 3^5$          Evaluate the power in the brackets first.

$= (8 ÷ 4 + 1)^3 – 3^5$          Then divide in the brackets.

$= (2 + 1)^3 – 3^5$          Then add in the brackets.

$= 3^3 – 3^5$          Then evaluate the powers.

$= (3)(3)(3) – (3)(3)(3)(3)(3)$

$= 27 – 243$          Then subtract.

$= –216$

**c)** $4^3 ÷ [8(6^0 – 2^1)]$          Evaluate the powers in the brackets first.

$= 4^3 ÷ [8(1 – 2)]$          Then subtract in the brackets.

$= 4^3 ÷ [8(–1)]$          Then multiply in the square brackets.

$= 4^3 ÷ (–8)$          Then evaluate the power.

$= 64 ÷ (–8)$          Then divide.

$= –8$

**d)** $9^2 ÷ [9 ÷ (–3)]^2$          Divide in the brackets first.

$= 9^2 ÷ (–3)^2$          Then evaluate the powers.

$= 81 ÷ 9$          Then divide.

$= 9$

**e)** $(2^2 × 1^3)^2$          Evaluate the powers in the brackets first.

$= (4 × 1)^2$          Then multiply in the brackets.

$= 4^2$          Then evaluate the power.

$= 16$

**f)** $(11^3 + 5^2)^0 + (4^2 – 2^4)$          Evaluate the powers first.

$= 1 + (16 – 16)$          Then subtract in the brackets.

$= 1 + 0$          Then add.

$= 1$

**11.** $(–4^3 × 10) – (6 ÷ 2)$

The brackets are not necessary because the order of operations matches the order in which the multiplication and division are written and should be carried out.

$–4^3 × 10 – 6 ÷ 2 = –64 × 10 – 6 ÷ 2$          Evaluate the power first, then multiply and divide.

$= –640 – 3$          Then subtract.

$= –643$

**12.** Evaluate the expression to determine the cost.

$70 \times 3^2 + 60 \times 3^2 \times 0.75 = 70 \times 9 + 60 \times 9 \times 0.75$      Evaluate the powers first, then multiply.

$\qquad\qquad\qquad\qquad\qquad\quad = 630 + 405$      Then add.

$\qquad\qquad\qquad\qquad\qquad\quad = 1035$

It costs $1035 to tile the floor.

**13.** $2^3 + (3 \times 4)^2 - 6 = 2^3 + 12^2 - 6$      Multiply in the brackets first, then evaluate the powers.

$\qquad\qquad\qquad\quad = 8 + 144 - 6$      Then add and subtract.

$\qquad\qquad\qquad\quad = 146$

$(2^3 + 3) \times 4^2 - 6 = (8 + 3) \times 4^2 - 6$      Evaluate the power in the brackets first, then add.

$\qquad\qquad\qquad\quad = 11 \times 4^2 - 6$      Then evaluate the power.

$\qquad\qquad\qquad\quad = 11 \times 16 - 6$      Then multiply.

$\qquad\qquad\qquad\quad = 176 - 6$      Then subtract.

$\qquad\qquad\qquad\quad = 170$

$2^3 + 3 \times (4^2 - 6) = 2^3 + 3 \times (16 - 6)$      Evaluate the power in the brackets first, then subtract.

$\qquad\qquad\qquad\quad = 2^3 + 3 \times 10$      Then evaluate the power.

$\qquad\qquad\qquad\quad = 8 + 3 \times 10$      Then multiply.

$\qquad\qquad\qquad\quad = 8 + 30$      Then add.

$\qquad\qquad\qquad\quad = 38$

$(2^3 + 3 \times 4^2) - 6 = (8 + 3 \times 16) - 6$      Evaluate the powers in the brackets first, the multiply.

$\qquad\qquad\qquad\quad = (8 + 48) - 6$      Then add in the brackets.

$\qquad\qquad\qquad\quad = 56 - 6$      Then subtract.

$\qquad\qquad\qquad\quad = 50$

$(2^3 + 3 \times 4)^2 - 6 = (8 + 3 \times 4)^2 - 6$      Evaluate the power in the brackets first, then multiply.

$\qquad\qquad\qquad\quad = (8 + 12)^2 - 6$      Then add in the brackets.

$\qquad\qquad\qquad\quad = 20^2 - 6$      Then evaluate the power.

$\qquad\qquad\qquad\quad = 400 - 6$      Then subtract.

$\qquad\qquad\qquad\quad = 394$

$2^3 + (3 \times 4^2 - 6) = 2^3 + (3 \times 16 - 6)$      Evaluate the power in the brackets first, then multiply.

$\qquad\qquad\qquad\quad = 2^3 + (48 - 6)$      Then subtract in the brackets.

$\qquad\qquad\qquad\quad = 2^3 + 42$      Then evaluate the power.

$\qquad\qquad\qquad\quad = 8 + 42$      Then add.

$\qquad\qquad\qquad\quad = 50$

$(2^3 + 3 \times 4^2 - 6) = 8 + 3 \times 16 - 6$      Evaluate the powers first, then multiply.

$\qquad\qquad\qquad\quad = 8 + 48 - 6$      Then add and subtract.

$\qquad\qquad\qquad\quad = 50$

I can get 5 different answers by changing the position of the brackets.

**14. a)** $3 + 5 \times 8 = 3 + 40$

$\qquad\qquad\quad = 43$

$\quad 5 \times 8 + 3 = 40 + 3$

$\qquad\qquad\quad = 43$

Both expressions are equal to 43; even though the numbers are written in a different order I still multiply before I add.

**b)** $3^2 + 2^2 = 9 + 4$
$\qquad\qquad = 13$
$(3 + 2)^2 = 5^2$
$\qquad\qquad = 25$

The answers are different; in the first expression, the squares of the numbers are added. In the second expression, the brackets indicate that the sum of the numbers should be squared.

**c)** $3^3 \times 2^3 - 5^2 = 27 \times 8 - 25$
$\qquad\qquad\qquad = 216 - 25$
$\qquad\qquad\qquad = 191$
$(3 \times 2)^3 - 5 \times 5 = 6^3 - 5 \times 5$
$\qquad\qquad\qquad\quad = 216 - 25$
$\qquad\qquad\qquad\quad = 191$

Both expressions are equal to 191. The answers are the same because the exponents indicate repeated multiplication and the brackets do not change that.

**d)** $2^3 \times 3^2 = 8 \times 9$
$\qquad\qquad = 72$
$(2 \times 3)^5 = 6^5$
$\qquad\qquad = 7776$

The answers are different because the brackets indicate that the product should be found first. The exponents are also different.

**e)** $5 \times 3 - 3^2 \times 4 + 20 \times 7 = 5 \times 3 - 9 \times 4 + 20 \times 7$
$\qquad\qquad\qquad\qquad\qquad = 15 - 36 + 140$
$\qquad\qquad\qquad\qquad\qquad = 119$
$5 \times (3 - 3^2) \times 4 + 20 \times 7 = 5 \times (3 - 9) \times 4 + 20 \times 7$
$\qquad\qquad\qquad\qquad\qquad\quad = 5 \times (-6) \times 4 + 20 \times 7$
$\qquad\qquad\qquad\qquad\qquad\quad = -120 + 140$
$\qquad\qquad\qquad\qquad\qquad\quad = 20$

The answers are different because including the brackets changes the order in which the operations are done.

**15.** The student made a mistake evaluating the expression in the first set of brackets: she multiplied 3 by 4 instead of squaring 4 first then multiplying by 3. This mistake did not affect the answer because everything inside the first set of brackets is a power with an exponent of 0, which equals 1. A more efficient solution is:
$-(24 - 3 \times 4^2)^0 \div (-2)^3 = -(1) \div (-8)$
$$= \frac{1}{8}$$

**16. a)** $(14 + 10)^2 \times (21 - 28)^3$
I used mental math to evaluate the brackets: 14 + 10 = 24, and 21 − 28 = −7
Then I used the calculator:
24∧2×(−7)∧3=
−197 568

**b)** $(36 \div 2^2 + 11)^3 - 10^5$
I used mental math to evaluate the power inside the brackets: $2^2 = 4$
Then I used the calculator:
(36÷4+11)∧3−10∧5=
−92 000

c) $\dfrac{12^3}{36(12^0 - 13^1)}$

I used mental math to evaluate $12^0 = 1$ and $13^1 = 13$.
Then I used the calculator:
12∧3÷(36(1–13))=
–4

d) $\dfrac{81^2}{9^2 + (-9)^2}$

81∧2÷(9∧2+(–9)∧2)=
40.5

e) $(14^2 + 6^3)^2$
(14∧2+6∧3)∧2=
169 744

f) $(11^3 + 25^2)^0 + (27^2 - 33^4)$
I know that a power with exponent 0 is equal to 1, so the first bracket is equal to 1.
1+27∧2–33∧4=
–1 185 191

**17.** I know that a power with exponent 0 is equal to 1. The first expression has exponent 0, so it is equal to 1.
The second expression will be less than 0 since $-3 \times 6$ is less than $4^2$.
The third expression is the addition of 3 positive terms, so this will be greater than 0.
I predict that the first expression has a value closest to 0.
$(30 + 9 \times 11 \div 3)^0 = 1$
$(-3 \times 6) + 4^2 = -18 + 16$
$\qquad\qquad\qquad = -2$
$1 + (1 \div 1)^2 + 1^0 = 1 + 1 + 1$
$\qquad\qquad\qquad\quad = 3$
My prediction was correct.

**18. a)** $(-6)^2 - 2[(-8) \div 2]^2 = 36 - 2(-4)^2$
$\qquad\qquad\qquad\qquad\quad = 36 - 2(16)$
$\qquad\qquad\qquad\qquad\quad = 36 - 32$
$\qquad\qquad\qquad\qquad\quad = 4$
Marcia's answer is correct.

**b)** I think Robbie forgot that the square of a negative number is positive when he evaluated $(-4)^2$.
$(-6)^2 - 2[(-8) \div 2]^2 = 36 - 2(-4)^2$
$\qquad\qquad\qquad\qquad\quad = 36 - 2(-16)$
$\qquad\qquad\qquad\qquad\quad = 36 + 32$
$\qquad\qquad\qquad\qquad\quad = 68$
I think Nick forgot that the square of a negative number is positive when he evaluated $(-6)^2$.
$(-6)^2 - 2[(-8) \div 2]^2 = -36 - 2(-4)^2$
$\qquad\qquad\qquad\qquad\quad = -36 - 2(16)$
$\qquad\qquad\qquad\qquad\quad = -36 - 32$
$\qquad\qquad\qquad\qquad\quad = -68$

**19.** Evaluate the expression to determine the cost.

$\dfrac{10 \times 130}{25} + 25^3 \div 10^6 \times 14 \times 150$        Evaluate the powers first.

$= \dfrac{10 \times 130}{25} + 15625 \div 1\,000\,000 \times 14 \times 150$        Then multiply and divide.

$= 52 + 32.8125$
$= 84.8125$
The customer pays $84.81.

**20. a)** $(10 + 2) \times 3^2 - 2 = 12 \times 3^2 - 2$        Add within the brackets first, then evaluate the power.
$\qquad\qquad\qquad\qquad = 12 \times 9 - 2$        Then multiply.
$\qquad\qquad\qquad\qquad = 108 - 2$        Then subtract.
$\qquad\qquad\qquad\qquad = 106$

**b)** $10 + 2 \times (3^2 - 2) = 10 + 2 \times (9 - 2)$        Evaluate the power and subtract within the brackets first.
$\qquad\qquad\qquad\qquad = 10 + 2 \times 7$        Then multiply.
$\qquad\qquad\qquad\qquad = 10 + 14$        Then add.
$\qquad\qquad\qquad\qquad = 24$

**c)** $(10 + 2) \times (3^2 - 2) = 12 \times (9 - 2)$        Evaluate the brackets first.
$\qquad\qquad\qquad\qquad = 12 \times 7$        Then multiply.
$\qquad\qquad\qquad\qquad = 84$

**d)** $(10 + 2 \times 3)^2 - 2 = (10 + 6)^2 - 2$        Multiply, then add within the brackets first.
$\qquad\qquad\qquad\qquad = 16^2 - 2$        Then evaluate the power.
$\qquad\qquad\qquad\qquad = 256 - 2$        Then subtract.
$\qquad\qquad\qquad\qquad = 254$

**21. a)** $20 \div (2 + 2) \times 2^2 + 6 = 20 \div 4 \times 4 + 6$        Add within the brackets and evaluate the power first, then divide.
$\qquad\qquad\qquad\qquad = 5 \times 4 + 6$        Then multiply.
$\qquad\qquad\qquad\qquad = 20 + 6$        Then add.
$\qquad\qquad\qquad\qquad = 26$

**b)** $20 \div 2 + 2 \times (2^2 + 6) = 20 \div 2 + 2 \times (4 + 6)$        Evaluate the power, then add within the brackets first.
$\qquad\qquad\qquad\qquad = 20 \div 2 + 2 \times 10$        Then divide and multiply.
$\qquad\qquad\qquad\qquad = 10 + 20$        Then add.
$\qquad\qquad\qquad\qquad = 30$

**c)** $20 \div (2 + 2 \times 2^2) + 6 = 20 \div (2 + 2 \times 4) + 6$        Evaluate the power, then multiply within the brackets.
$\qquad\qquad\qquad\qquad = 20 \div (2 + 8) + 6$        Then add within the brackets.
$\qquad\qquad\qquad\qquad = 20 \div 10 + 6$        Then divide.
$\qquad\qquad\qquad\qquad = 2 + 6$        Then add.
$\qquad\qquad\qquad\qquad = 8$

**d)** $(20 \div 2 + 2) \times (2^2 + 6) = (10 + 2) \times (4 + 6)$        Evaluate the brackets first.
$\qquad\qquad\qquad\qquad = 12 \times 10$        Then multiply.
$\qquad\qquad\qquad\qquad = 120$

**22.** $5 \times 4^2 - (2^3 + 3^3) \div 5 = 5 \times 4^2 - (8 + 27) \div 5$          Evaluate the expression in the brackets first.

$\quad\quad\quad\quad\quad\quad\quad = 5 \times 4^2 - 35 \div 5$          Then evaluate the power.

$\quad\quad\quad\quad\quad\quad\quad = 5 \times 16 - 35 \div 5$          Then multiply and divide.

$\quad\quad\quad\quad\quad\quad\quad = 80 - 7$          Then subtract.

$\quad\quad\quad\quad\quad\quad\quad = 73$

No, Blake did not win the prize, since his answer was 11.

**23.** Expressions will vary. For example:

$[(-2)^3 - 2]^2 + 3 \times 6 \div 3^2 = [(-8) - 2]^2 + 18 \div 9$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad = (-10)^2 + 2$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad = 100 + 2$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad = 102$

Both of us followed the same order of operations. It might be possible to get the same answer if one of us divided before multiplying or combined steps but if both of us follow the same order then we should always get the same answer.

## *Take It Further*

**24. a)** $1^3 = 1^2$

$\quad\quad 1^3 + 2^3 = 3^2$

$\quad\quad 1^3 + 2^3 + 3^3 = 6^2$

$\quad\quad 1^3 + 2^3 + 3^3 + 4^3 = 10^2$

$\quad\quad 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2$

$\quad\quad 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 21^2$

$\quad\quad 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 28^2$

The pattern on the left side of the equation:
Start with a base of 1 raised to the power of 3, then add the next consecutive number raised to the power of 3 each time.

The pattern on the right side of the equation:
The answer is the sum of the bases on the left side of the equation, raised to the power of 2.

**b)** $3^2 - 1^2 = 2^3$

$\quad\quad 6^2 - 3^2 = 3^3$

$\quad\quad 10^2 - 6^2 = 4^3$

$\quad\quad 15^2 - 10^2 = 5^3$

$\quad\quad 21^2 - 15^2 = 6^3$

$\quad\quad 28^2 - 21^2 = 7^3$

$\quad\quad 36^2 - 28^2 = 8^3$

The patterns on the left side of the equation:
For the base of the first term, start at 3, then add 3; increase the amount added by 1 each time. Write the base to the power of 2 each time.
For the base of the second term, start at 1, then add 2; increase the amount added by 1 each time. Write the base to the power of 2 each time.

The pattern on the right side of the equation:
The answer is the difference between the bases on the left side of the equation, raised to the power of 3.

**25.** Numbers will vary. For example: I choose (−2) and 3.

**a)** $(-2)^2 + 3^2 = 4 + 9$

$\quad\quad\quad\quad\quad\quad = 13$

**b)** $(-2 + 3)^2 = 1^2$
$\phantom{(-2 + 3)^2} = 1$

**c)** The answers are different.

**d)** I do not agree. "The sum of the squares of two numbers" means that you square the numbers first then add them, as in part a. "The square of the sum of two numbers" means you add the numbers first then square their sum, as in part b. These expressions are not equal because the operations are performed in different orders.

26. Expressions will vary. For example:
$4 \div 4 + 4 - 4 = 1 + 4 - 4$
$\phantom{4 \div 4 + 4 - 4} = 1$

$4 \div 4 + 4 \div 4 = 1 + 1$
$\phantom{4 \div 4 + 4 \div 4} = 2$

$4 - 4 + 4 - 4^0 = 4 - 4 + 4 - 1$
$\phantom{4 - 4 + 4 - 4^0} = 3$

$4^0 + 4^0 + 4^0 + 4^0 = 1 + 1 + 1 + 1$
$\phantom{4^0 + 4^0 + 4^0 + 4^0} = 4$

$4 - 4 + 4 + 4^0 = 4 - 4 + 4 + 1$
$\phantom{4 - 4 + 4 + 4^0} = 5$

$4 + 4 - 4^0 - 4^0 = 4 + 4 - 1 - 1$
$\phantom{4 + 4 - 4^0 - 4^0} = 6$

$4 + 4^0 + 4^0 + 4^0 = 4 + 1 + 1 + 1$
$\phantom{4 + 4^0 + 4^0 + 4^0} = 7$

$(4 + 4) \times 4 \div 4 = 8 \times 4 \div 4$
$\phantom{(4 + 4) \times 4 \div 4} = 8$

$4 \div 4 + 4 + 4 = 1 + 4 + 4$
$\phantom{4 \div 4 + 4 + 4} = 9$

27. **a) i)** $2 \times 2 \times 2 \times 2 = 2^4$        As a power of 2
$\phantom{2 \times 2 \times 2 \times 2} = 16$        Standard form

  **ii)** $2 \times 2 = 2^2$        As a power of 2
$\phantom{2 \times 2} = 4$        Standard form

  **iii)** $2 \times 2 \times 2 \times 2 \times 2 = 2^5$        As a power of 2
$\phantom{2 \times 2 \times 2 \times 2 \times 2} = 32$        Standard form

  **iv)** $2 \times 2 \times 2 = 2^3$        As a power of 2
$\phantom{2 \times 2 \times 2} = 8$        Standard form

 **b) i)** $28 = 16 + 8 + 4$
$\phantom{28} = 2^4 + 2^3 + 2^2$

**ii)** $12 = 8 + 4$
$\quad\quad = 2^3 + 2^2$

**iii)** $25 = 16 + 8 + 1$
$\quad\quad = 2^4 + 2^3 + 2^0$

**iv)** $31 = 16 + 8 + 4 + 2 + 1$
$\quad\quad = 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

**v)** $50 = 32 + 16 + 2$
$\quad\quad = 2^5 + 2^4 + 2^1$

**vi)** $75 = 64 + 8 + 2 + 1$
$\quad\quad = 2^6 + 2^3 + 2^1 + 2^0$

**c)** I chose base 3.
  **i)** $28 = 27 + 1$
$\quad\quad = 3^3 + 3^0$

  **ii)** $12 = 9 + 3$
$\quad\quad = 3^2 + 3^1$

  **iii)** $25 = 9 + 9 + 3 + 3 + 1$
$\quad\quad = 3^2 + 3^2 + 3^1 + 3^1 + 3^0$

  **iv)** $31 = 27 + 3 + 1$
$\quad\quad = 3^3 + 3^1 + 3^0$

  **v)** $50 = 27 + 9 + 9 + 3 + 1 + 1$
$\quad\quad = 3^3 + 3^2 + 3^2 + 3^1 + 3^0 + 3^0$

  **vi)** $75 = 27 + 27 + 9 + 9 + 3$
$\quad\quad = 3^3 + 3^3 + 3^2 + 3^2 + 3^1$

**Mid-Unit Review**                                                              **(page 69)**

*Lesson 2.1*

**1. a)** $14^2 = 196$

  **b)** $5^1 = 5$

  **c)** $-8^3 = -512$

  **d)** $-(-4)^4 = -256$

  **e)** $(-6)^3 = -216$

  **f)** $(-2)^8 = 256$

**2.**

|     | Power | Base | Exponent | Repeated Multiplication | Standard Form |
|-----|-------|------|----------|-------------------------|---------------|
| **a)** | $4^3$ | 4 | 3 | 4 × 4 × 4 | 64 |
| **b)** | $2^5$ | 2 | 5 | 2 × 2 × 2 × 2 × 2 | 32 |
| **c)** | $8^6$ | 8 | 6 | 8 × 8 × 8 × 8 × 8 × 8 | 262 144 |
| **d)** | $7^2$ | 7 | 2 | 7 × 7 | 49 |
| **e)** | $3^4$ | 3 | 4 | 3 × 3 × 3 × 3 | 81 |

**3. a)**

| Power of 7 | Standard Form |
|------------|---------------|
| $7^1$ | 7 |
| $7^2$ | 49 |
| $7^3$ | 343 |
| $7^4$ | 2401 |
| $7^5$ | 16 807 |
| $7^6$ | 117 649 |
| $7^7$ | 823 543 |
| $7^8$ | 5 764 801 |

  **b)** The pattern in the ones digits is: 7, 9, 3, 1, 7, 9, 3, 1, …
  Powers that have a multiple of 4 as an exponent end with 1.
  Powers that have an exponent that is 1 more than a multiple of 4 end with 7.
  Powers that have an exponent that is 2 more than a multiple of 4 end with 9.
  Powers that have an exponent that is 1 less than a multiple of 4 end with 3.

  **c)**

| Power of 7 | Standard Form |
|------------|---------------|
| $7^9$ | 40 353 607 |
| $7^{10}$ | 282 475 249 |
| $7^{11}$ | 1 977 326 743 |

  **d) i)** $7^{12}$ will end in a 1 because 12 is a multiple of 4.
    **ii)** $7^{14}$ will end in a 9 because 14 is 2 more than a multiple of 4.
    **iii)** $7^{17}$ will end in a 7 because 17 is 1 more than a multiple of 4.
    **iv)** $7^{22}$ will end in a 9 because 22 is 2 more than a multiple of 4.

## Lesson 2.2

4.  **a)** $10^6$ = 1 000 000

    **b)** $10^0$ = 1

    **c)** $10^8$ = 100 000 000

    **d)** $10^4$ = 10 000

5.  **a)** one billion = 1 000 000 000
                      = $10^9$

    **b)** one = 1
              = $10^0$

    **c)** 100 = $10^2$

    **d)** 100 000 = $10^5$

6.  A power with exponent 0 is equal to 1.
    **a)** $(-5)^0$ = 1

    **b)** $25^0$ = 1

    **c)** $-6^0 = -(6^0)$
              = $-1$

    **d)** $9^0$ = 1

7.  1 ha = 100 m × 100 m
          = 10 000 $m^2$
          = $10^4$ $m^2$

## Lesson 2.3

8.  **a)** Subtract in the brackets first: $(-21 - 6) = -27$
    $(-21 - 6)^2 + 14 = (-27)^2 + 14$          Then evaluate the power.
                      $= 729 + 14$          Then add.
                      $= 743$

    **b)** Multiply in the brackets first: $(2 × 3) = 6$
    $6 ÷ (-2) + (2 × 3)^2 = 6 ÷ (-2) + 6^2$          Then evaluate the power.
                        $= 6 ÷ (-2) + 36$          Then divide.
                        $= -3 + 36$          Then add.
                        $= 33$

    **c)** Evaluate the expressions in the brackets first: $[5 - (-4)] = 9$ and $(21 ÷ 7) = 3$
    $[5 - (-4)]^3 - (21 ÷ 7)^4 = 9^3 - 3^4$          Then evaluate the powers.
                        $= 729 - 81$          Then subtract.
                        $= 648$

    **d)** Evaluate the 0 exponent first.
    $[(6 - 21)^3 × (2 + 2)^6]^0 = 1$

**e)** Subtract in the brackets first: (3 – 5) = –2

$(3 - 5)^5 \div (-4) = (-2)^5 \div (-4)$          Then evaluate the power.

$= -32 \div (-4)$          Then divide.

$= 8$

**f)** Subtract in the brackets first: (7 – 4) = 3

$-30 - (7 - 4)^3 = -30 - 3^3$          Then evaluate the power.

$= -30 - 27$          Then subtract.

$= -57$

**9.** $-2^4 \times 5 + 16 \div (-2)^3 = -16 \times 5 + 16 \div (-8)$          Evaluate the powers, then multiply and divide.

$= -80 + (-2)$          Then add.

$= -82$

The answer is –82. Sophia is correct.

Victor likely made the mistake of including the negative sign in the power $-2^4$ and evaluated it as 16.

Then, he would have evaluated $16 \times 5 + 16 \div (-8) = 80 + (-2)$

$= 78$

**10.** The term $(-3)^3 = -27$, not 27

The term $(-9)^0 = 1$, not –1

Then in the 2nd line, the student evaluated 16 – 27 before dividing; this is the wrong order of operations.

The correct solution:

$(-2)^4 - (-3)^3 \div (-9)^0 \times 2^3 = 16 - (-27) \div 1 \times 8$

$= 16 - (-27) \times 8$

$= 16 - (-216)$

$= 232$

I think the errors occurred because the student was not careful evaluating powers with negative bases.

The student also did not follow the correct order of operations.

### *Check*

4. When the powers have the same base, use the exponent law for products and add the exponents.

   a) $5^5 \times 5^4 = 5^{(5+4)}$
   $= 5^9$

   b) $10^2 \times 10^{11} = 10^{(2+11)}$
   $= 10^{13}$

   c) $(-3)^3 \times (-3)^3 = (-3)^{(3+3)}$
   $= (-3)^6$

   d) $21^6 \times 21^4 = 21^{(6+4)}$
   $= 21^{10}$

   e) $(-4)^1 \times (-4)^3 = (-4)^{(1+3)}$
   $= (-4)^4$

   f) $6^{12} \times 6^3 = 6^{(12+3)}$
   $= 6^{15}$

   g) $2^0 \times 2^4 = 2^{(0+4)}$
   $= 2^4$

   h) $(-7)^3 \times (-7)^0 = (-7)^{(3+0)}$
   $= (-7)^3$

5. When the powers have the same base, use the exponent law for quotients and subtract the exponents.

   a) $4^5 \div 4^3 = 4^{(5-3)}$
   $= 4^2$

   b) $8^9 \div 8^6 = 8^{(9-6)}$
   $= 8^3$

   c) $15^{10} \div 15^0 = 15^{(10-0)}$
   $= 15^{10}$

   d) $(-6)^8 \div (-6)^3 = (-6)^{(8-3)}$
   $= (-6)^5$

   e) $\dfrac{2^{12}}{2^{10}} = 2^{(12-10)}$
   $= 2^2$

   f) $\dfrac{(-10)^{12}}{(-10)^6} = (-10)^{(12-6)}$
   $= (-10)^6$

   g) $\dfrac{6^5}{6^1} = 6^{(5-1)}$
   $= 6^4$

**h)** $\dfrac{(-1)^5}{(-1)^4} = (-1)^{(5-4)}$

$= (-1)^1$

## *Apply*

**6.  a) i)**  $3^4 \div 3^4 = 3^{(4-4)}$

$= 3^0$

$= 1$

**ii)** $(-4)^6 \div (-4)^6 = (-4)^{(6-6)}$

$= (-4)^0$

$= 1$

**iii)** $\dfrac{5^8}{5^8} = 5^{(8-8)}$

$= 5^0$

$= 1$

**iv)** $\dfrac{(-6)^3}{(-6)^3} = (-6)^{(3-3)}$

$= (-6)^0$

$= 1$

**b)** When I use the exponent law for a quotient of powers to evaluate the expressions in part a, each expression has an exponent of 0.
When I use the order of operations to evaluate the expressions, each expression simplifies to a number divided by itself, which is equal to 1.
This verifies that a power with exponent 0 is equal to 1.

**7.  a) i)**  $3^4 \times 3^9 = 3^{(4+9)}$

$= 3^{13}$

$= 1\ 594\ 323$

**ii)** $3^9 \times 3^4 = 3^{(9+4)}$

$= 3^{13}$

$= 1\ 594\ 323$

Both expressions have the same product.

**b)** When I use the exponent law for a product of powers to evaluate each expression, each product is $3^{13}$.
The order in which I multiply two powers does not affect the answer.

**8.  a)**  $3^4 \times 3^9 \div 3^{11} = 3^{(4+9)} \div 3^{11}$

$= 3^{13} \div 3^{11}$

$= 3^{(13-11)}$

$= 3^2$

**b)** $(-4)^3 \div (-4)^2 \times (-4)^{10} = (-4)^{(3-2)} \times (-4)^{10}$

$= (-4)^1 \times (-4)^{10}$

$= (-4)^{(1+10)}$

$= (-4)^{11}$

**c)** $6^0 \times 6^3 \div 6^2 = 6^{(0 + 3)} \div 6^2$

$$= 6^3 \div 6^2$$
$$= 6^{(3 - 2)}$$
$$= 6^1$$

**d)** $\dfrac{4^3 \times 4^5}{4^2 \times 4^6} = \dfrac{4^{(3+5)}}{4^{(2+6)}}$

$$= \dfrac{4^8}{4^8}$$
$$= 4^{(8 - 8)}$$
$$= 4^0$$

**e)** $\dfrac{(-3)^4 \times (-3)^4}{(-3)^4} = \dfrac{(-3)^{(4+4)}}{(-3)^4}$

$$= \dfrac{(-3)^8}{(-3)^4}$$
$$= (-3)^{(8 - 4)}$$
$$= (-3)^4$$

**9. a) i)** $(-6)^1 \times (-6)^7 \div (-6)^7 = (-6)^{(1 + 7)} \div (-6)^7$

$$= (-6)^8 \div (-6)^7$$
$$= (-6)^{(8 - 7)}$$
$$= (-6)^1$$
$$= -6$$

**ii)** $(-6)^7 \div (-6)^7 \times (-6)^1 = (-6)^{(7 - 7)} \times (-6)^1$

$$= (-6)^0 \times (-6)^1$$
$$= (-6)^{(0 + 1)}$$
$$= (-6)^1$$
$$= -6$$

**b)** Changing the order of the terms does not affect the answer because, in each case, the same terms are multiplied, and the terms that are divided are equal, so their quotient is 1.

**10. a)** $10^2 \times 10^2 + 10^4 = 10^{(2 + 2)} + 10^4$

$$= 10^4 + 10^4$$
$$= 10\ 000 + 10\ 000$$
$$= 20\ 000$$

**b)** $10^3 \times 10^3 - 10^3 = 10^{(3 + 3)} - 10^3$

$$= 10^6 - 10^3$$
$$= 1\ 000\ 000 - 1000$$
$$= 999\ 000$$

**c)** $10^{11} - 10^3 \times 10^6 = 10^{11} - 10^{(3 + 6)}$

$$= 10^{11} - 10^9$$
$$= 100\ 000\ 000\ 000 - 1\ 000\ 000\ 000$$
$$= 99\ 000\ 000\ 000$$

**d)** $10^1 + 10^5 \times 10^2 = 10^1 + 10^{(5 + 2)}$

$$= 10^1 + 10^7$$
$$= 10 + 10\ 000\ 000$$
$$= 10\ 000\ 010$$

**e)** $10^6 \div 10^2 \times 10^2 = 10^{(6-2)} \times 10^2$
$$= 10^4 \times 10^2$$
$$= 10^{(4+2)}$$
$$= 10^6$$
$$= 1\ 000\ 000$$

**f)** $10^9 \div 10^9 = 10^{(9-9)}$
$$= 10^0$$
$$= 1$$

**g)** $\dfrac{10^{12}}{10^6} = 10^{(12-6)}$
$$= 10^6$$
$$= 1\ 000\ 000$$

**h)** $\dfrac{10^4 \times 10^3}{10^2} = \dfrac{10^{(4+3)}}{10^2}$
$$= \dfrac{10^7}{10^2}$$
$$= 10^{(7-2)}$$
$$= 10^5$$
$$= 100\ 000$$

**i)** $\dfrac{10^{11}}{10^4 \times 10^2} = \dfrac{10^{11}}{10^{(4+2)}}$
$$= \dfrac{10^{11}}{10^6}$$
$$= 10^{(11-6)}$$
$$= 10^5$$
$$= 100\ 000$$

**j)** $\dfrac{10^5}{10^3} + 10^2 = 10^{(5-3)} + 10^2$
$$= 10^2 + 10^2$$
$$= 100 + 100$$
$$= 200$$

**11. a)** $2^6 - 2^2 \times 2^3 = 2^6 - 2^{(2+3)}$
$$= 2^6 - 2^5$$
$$= 64 - 32$$
$$= 32$$

I multiplied using the exponent law for the product of powers. Then I evaluated each power. Then I subtracted.

**b)** $2^6 \times 2^2 - 2^3 = 2^{(6+2)} - 2^3$
$$= 2^8 - 2^3$$
$$= 256 - 8$$
$$= 248$$

I multiplied using the exponent law for the product of powers. Then I evaluated each power. Then I subtracted.

**c)** The steps were not different. In each case, I used the order of operations and the exponent law.

**12. a)** In square metres, the area of the field is

$10^4 \times 10^3 = 10^{(4 + 3)}$

$\qquad\qquad = 10^7$

$\qquad\qquad = 10\ 000\ 000$

**b)** In metres, the perimeter of the field is

$2(10^4 + 10^3) = 2(10\ 000 + 1000)$

$\qquad\qquad = 2(11\ 000)$

$\qquad\qquad = 22\ 000$

**c) i)** All possible dimensions are all the pairs of powers of 10 that have a product of $10^7$, so find all possible exponents that have a sum of 7.

$10^7$ m by $10^0$ m

since $10^7 \times 10^0 = 10^{(7 + 0)}$

$\qquad\qquad\qquad = 10^7$

$10^6$ m by $10^1$ m

since $10^6 \times 10^1 = 10^{(6 + 1)}$

$\qquad\qquad\qquad = 10^7$

$10^5$ m by $10^2$ m

since $10^5 \times 10^2 = 10^{(5 + 2)}$

$\qquad\qquad\qquad = 10^7$

$10^4$ m by $10^3$ m

since $10^4 \times 10^3 = 10^{(4 + 3)}$

$\qquad\qquad\qquad = 10^7$

**ii)** $2(10^7$ m $+ 10^0$ m$) = 2(10\ 000\ 000$ m $+ 1$ m$)$

$\qquad\qquad\qquad = 2(10\ 000\ 001$ m$)$

$\qquad\qquad\qquad = 20\ 000\ 002$ m

$2(10^6$ m $+ 10^1$ m$) = 2(1\ 000\ 000$ m $+ 10$ m$)$

$\qquad\qquad\qquad = 2(1\ 000\ 010$ m$)$

$\qquad\qquad\qquad = 2\ 000\ 020$ m

$2(10^5$ m $+ 10^2$ m$) = 2(100\ 000$ m $+ 100$ m$)$

$\qquad\qquad\qquad = 2(100\ 100$ m$)$

$\qquad\qquad\qquad = 200\ 200$ m

$2(10^4$ m $+ 10^3$ m$) = 2(10\ 000$ m $+ 1000$ m$)$

$\qquad\qquad\qquad = 2(11\ 000$ m$)$

$\qquad\qquad\qquad = 22\ 000$ m

**d)** The exponent laws are helpful for solving area problems because they are used to simplify expressions that include products, and we multiply to determine area. To determine perimeter, we add and there are no exponent laws for determining the sums of powers.

**13. a)** $2^3 \times 2^2 - 2^5 \times 2 = 2^{(3 + 2)} - 2^{(5 + 1)}$

$\qquad\qquad\qquad = 2^5 - 2^6$

$\qquad\qquad\qquad = 32 - 64$

$\qquad\qquad\qquad = -32$

**b)** $3^2 \times 3 + 2^2 \times 2^4 = 3^{(2+1)} + 2^{(2+4)}$
$$= 3^3 + 2^6$$
$$= 27 + 64$$
$$= 91$$

**c)** $4^2 - 3^0 \times 3 + 2^3 = 4^2 - 3^{(0+1)} + 2^3$
$$= 4^2 - 3^1 + 2^3$$
$$= 16 - 3 + 8$$
$$= 21$$

**d)** $(-3)^6 \div (-3)^5 - (-3)^5 \div (-3)^3 = (-3)^{(6-5)} - (-3)^{(5-3)}$
$$= (-3)^1 - (-3)^2$$
$$= -3 - 9$$
$$= -12$$

**e)** $(-2)^4[(-2)^5 \div (-2)^3] + (-2)^4 = (-2)^4 \times (-2)^{(5-3)} + (-2)^4$
$$= (-2)^4 \times (-2)^2 + (-2)^4$$
$$= (-2)^{(4+2)} + (-2)^4$$
$$= (-2)^6 + (-2)^4$$
$$= 64 + 16$$
$$= 80$$

**f)** $-2^4(2^6 \div 2^2) - 2^4 = -2^4 \times 2^{(6-2)} - 2^4$
$$= -2^4 \times 2^4 - 2^4$$
$$= -2^{(4+4)} - 2^4$$
$$= -2^8 - 2^4$$
$$= -256 - 16$$
$$= -272$$

**g)** $(-5)^3 \div (-5)^2 \times (-5)^0 + (-5)^2 \div (-5) = (-5)^{(3-2)} \times (-5)^0 + (-5)^{(2-1)}$
$$= (-5)^1 \times (-5)^0 + (-5)^1$$
$$= (-5)^{(1+0)} + (-5)^1$$
$$= (-5)^1 + (-5)^1$$
$$= -5 + (-5)$$
$$= -10$$

**14.** Examples will vary.
$4^3 \times 3^2 = (4)(4)(4)(3)(3) = (64)(9) = 576$
We cannot apply the exponent law for products, because we would have to choose a base, either 4 or 3, raised to the power of the sum of the exponents, and that does not give a correct answer.

$$\frac{5^3}{2^4} = \frac{(5)(5)(5)}{(2)(2)(2)(2)} = \frac{125}{16} = 7.8125$$

We cannot apply the exponent law for quotients, because we would have to choose a base, either 5 or 2, and raise it to the power of the difference of the exponents, which does not give a correct answer.

**15. a)** The student multiplied the exponents instead of adding them.
Correct answer:
$4^3 \times 4^4 = 4^{(3+4)}$
$$= 4^7$$

**b)** The student divided the exponents instead of subtracting them.
Note: the exponent is inside the brackets, so the negative sign is not part of the base of the power.
Correct answer:

$$\frac{\left(-7^6\right)}{\left(-7^3\right)} = 7^{(6-3)}$$

$$= 7^3$$

**c)** The student used the exponent laws even though the bases are different.
Correct answer:
$3^2 \times 2^3 = 9 \times 8$
$\qquad = 72$

**d)** The student multiplied the exponents in the divisor instead of adding them.
Correct answer:

$$\frac{5^8}{5^4 \times 5^2} = \frac{5^8}{5^{(4+2)}}$$

$$= \frac{5^8}{5^6}$$

$$= 5^2$$

$$= 25$$

**e)** The student added all the exponents even though only 2 of them were parts of products of powers.
Correct answer:
$1^2 + 1^3 \times 1^2 = 1^2 + 1^{(3+2)}$
$\qquad\qquad = 1^2 + 1^5$
$\qquad\qquad = 1 + 1$
$\qquad\qquad = 2$

**16. a)** Total magnification is: $10^2 \times 10^1$

**b)** $10^2 \times 10^1 = 10^{(2+1)}$
$\qquad\qquad = 10^3$
$\qquad\qquad = 1000$
The image is 1000 times as large as the bacteria.

**17. a) i)** $5^2 + 5^3 = 25 + 125$
$\qquad\qquad = 150$
**ii)** $5^2 \times 5^3 = 5^{(2+3)}$
$\qquad\qquad = 5^5$
$\qquad\qquad = 3125$

**b)** I could not use an exponent law to simplify the expression in part i because it is the sum of two powers. I could use an exponent law to simplify the expression in part ii because it is the product of two powers.

**18. a) i)** $4^3 - 4^2 = 64 - 16$
$\qquad\qquad = 48$
**ii)** $4^3 \div 4^2 = 4^{(3-2)}$
$\qquad\qquad = 4^1$
$\qquad\qquad = 4$

**b)** I could not use an exponent law to simplify the expression in part i because it is the difference of two powers. I could use an exponent law to simplify the expression in part ii because it is the quotient of two powers.

**19.** I used mental math to simplify the expressions. Those that simplified to a single power with an even number as the exponent had a positive value, because the sign of a product with an even number of negative factors is positive.

Those that simplified to a single power with an odd number as the exponent had a negative value, because the sign of a product with an odd number of negative factors is negative.

**a)** $(-2)^2 \times (-2)^3$ simplifies to $(-2)^{(2 + 3)} = (-2)^5$
The answer is negative because $(-2)^5$ is negative.

**b)** $(-2)^0 \times (-2)^5$ simplifies to $(-2)^{(0 + 5)} = (-2)^5$
The answer is negative because $(-2)^5$ is negative.

**c)** $(-2)^5 \div (-2)^3$ simplifies to $(-2)^{(5 - 3)} = (-2)^2$
The answer is positive because $(-2)^2$ is positive.
$(-2)^2 = 4$

**d)** $(-2)^6 \div (-2)^6$ simplifies to $(-2)^{(6 - 6)} = (-2)^0$
The answer is positive because $(-2)^0 = 1$, and this is positive.

**e)** $\dfrac{(-2)^3 \times (-2)^4}{(-2)^3 \times (-2)^2}$   Divide the numerator and denominator of the fraction by their common factor: $(-2)^3$

The fraction simplifies to $\dfrac{(-2)^4}{(-2)^2} = (-2)^{(4 - 2)}$
$$= (-2)^2$$
The answer is positive because $(-2)^2$ is positive.
$(-2)^2 = 4$

**f)** $\dfrac{(-2)^6}{(-2)^3 \times (-2)^2}$ simplifies to $\dfrac{(-2)^6}{(-2)^{(3+2)}} = \dfrac{(-2)^6}{(-2)^5}$
$$= (-2)^{(6 - 5)}$$
$$= (-2)^1$$
The answer is negative because $(-2)^1$ is negative.

## *Take It Further*

**20.** I found 8 pairs of powers with the same base that have a product of 64:
$8^1 \times 8^1 = 64$
$8^0 \times 8^2 = 64$
$4^3 \times 4^0 = 64$
$4^2 \times 4^1 = 64$
$2^6 \times 2^0 = 64$
$2^5 \times 2^1 = 64$
$2^4 \times 2^2 = 64$
$2^3 \times 2^3 = 64$

I found 8 pairs of powers with different bases that have a product of 64:

$8^1 \times 2^3 = 64$

$8^0 \times 4^3 = 64$

$8^0 \times 2^6 = 64$

$2^6 \times 4^0 = 64$

$4^3 \times 2^0 = 64$

$8^2 \times 2^0 = 64$

$4^2 \times 2^2 = 64$

$2^4 \times 4^1 = 64$

There are many more pairs of powers, because I could pair any power with a value of 64 with any power with a 0 exponent (because a power with a 0 exponent is equal to 1).

**21. a)** 1 km = 1000 m

$= 10^3$ m

1 m = 100 cm, so

$10^3$ m = $10^3 \times 100$ cm

$= 10^3 \times 10^2$ cm

$= 10^5$ cm

There are $10^5$ cm, or 100 000 cm in 1 km.

**b)** 1 km = $10^5$ cm

1 cm = 10 mm, so

$10^5$ cm = $10^5 \times 10^1$ mm

$= 10^6$ mm

There are $10^6$ mm, or 1 000 000 mm in 1 km.

**c)** 1 km = 1000 m

$= 10^3$ m

$\dfrac{1}{10^3}$ km = $\dfrac{10^3}{10^3}$ m

$= 1$ m

$10^5 \left(\dfrac{1}{10^3}\right)$ km = $10^5 \times 1$ m

$= 10^5$ m

$10^5$ m = $\dfrac{10^5}{10^3}$ km

$= 10^2$ km

There are $10^2$ km, or 100 km in $10^5$ m.

**d)** 1 m = 1000 mm

$= 10^3$ mm

$\dfrac{1}{10^3}$ m = $\dfrac{10^3}{10^3}$ mm

$= 1$ mm

$10^9 \left(\dfrac{1}{10^3}\right)$ m = $10^9 \times 1$ mm

$= 10^9$ mm

$10^9$ mm = $\dfrac{10^9}{10^3}$ m

$= 10^6$ m

There are $10^6$ m, 1 000 000 m in $10^9$ mm.

**22. a)** $1 \text{ km}^2 = (1000 \text{ m})^2$

$\qquad\qquad = 1000 \text{ m} \times 1000 \text{ m}$

$\qquad\qquad = 1\,000\,000 \text{ m}^2$

$\qquad\qquad = 10^6 \text{ m}^2$

$\quad 10^2 \text{ km}^2 = 10^2 \times 10^6 \text{ m}^2$

$\qquad\qquad = 10^8 \text{ m}^2$

There are $10^8 \text{ m}^2$, or $100\,000\,000 \text{ m}^2$ in $10^2 \text{ km}^2$.

**b)** $1 \text{ m}^2 = (100 \text{ cm})^2$

$\qquad\qquad = 10\,000 \text{ cm}^2$

$\qquad\qquad = 10^4 \text{ cm}^2$

$\dfrac{1}{10^4} \text{ m}^2 = \dfrac{10^4}{10^4} \text{ cm}^2$

$\qquad\qquad = 1 \text{ cm}^2$

$10^6 \left(\dfrac{1}{10^4}\right) \text{ m}^2 = 10^6 \times 1 \text{ cm}^2$

$10^6 \text{ cm}^2 = \dfrac{10^6}{10^4} \text{ m}^2$

$\qquad\qquad = 10^2 \text{ m}^2$

There are $10^2 \text{ m}^2$, or $100 \text{ m}^2$ in $10^6 \text{ cm}^2$.

**c)** $1 \text{ cm}^2 = (10 \text{ mm})^2$

$\qquad\qquad = 100 \text{ mm}^2$

$\qquad\qquad = 10^2 \text{ mm}^2$

$\quad 10^6 \text{ cm}^2 = 10^6 \times 10^2 \text{ mm}^2$

$\qquad\qquad = 10^8 \text{ mm}^2$

There are $10^8 \text{ mm}^2$, or $100\,000\,000 \text{ mm}^2$ in $10^6 \text{ cm}^2$.

**d)** $1 \text{ km}^2 = (100\,000 \text{ cm})^2$

$\qquad\qquad = 10\,000\,000\,000 \text{ cm}^2$

$\qquad\qquad = 10^{10} \text{ cm}^2$

There are $10^{10} \text{ cm}^2$, or $10\,000\,000\,000 \text{ cm}^2$ in $1 \text{ km}^2$.

**23.** All the units of measure in the metric system are related by powers of 10, so I could use the exponent laws to help me multiply and divide the units. When I work with square units I use the powers of 10 and the exponent laws for the product of powers to square the numbers.

| **Lesson 2.5** | **Exponent Laws II** | **Practice (pages 84–85)** |

### *Check*

**4.** Use the exponent law for a power of a product.

   **a)** $(6 \times 4)^3 = 6^3 \times 4^3$

   **b)** $(2 \times 5)^4 = 2^4 \times 5^4$

   **c)** $[(-2) \times 3]^5 = (-2)^5 \times 3^5$

   **d)** $(25 \times 4)^2 = 25^2 \times 4^2$

   **e)** $(11 \times 3)^1 = 11^1 \times 3^1$

   **f)** $[(-3) \times (-2)]^3 = (-3)^3 \times (-2)^3$

**5.** Use the exponent law for a power of a quotient.

   **a)** $(8 \div 5)^3 = 8^3 \div 5^3$

   **b)** $(21 \div 5)^4 = 21^4 \div 5^4$

   **c)** $[(-12) \div (-7)]^5 = (-12)^5 \div (-7)^5$

   **d)** $\left(\dfrac{10}{3}\right)^3 = \dfrac{10^3}{3^3}$

   **e)** $\left(\dfrac{1}{3}\right)^2 = \dfrac{1^2}{3^2}$

   **f)** $\left(\dfrac{27}{100}\right)^4 = \dfrac{27^4}{100^4}$

**6.** Use the exponent law for a power of a power.

   **a)** $(3^2)^4 = 3^{2 \times 4}$
   $= 3^8$

   **b)** $(6^3)^3 = 6^{3 \times 3}$
   $= 6^9$

   **c)** $(5^3)^1 = 5^{3 \times 1}$
   $= 5^3$

   **d)** $(7^0)^6 = 7^{0 \times 6}$
   $= 7^0$

   **e)** $-(8^2)^2 = -(8^{2 \times 2})$
   $= -8^4$

   **f)** $[(-3)^4]^2 = (-3)^{4 \times 2}$
   $= (-3)^8$

7. The expression $(2^4)^2$ means $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$.
   The expression $(2^2)^4$ means $(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$.
   Each expression is the product of eight factors of 2. Both expressions are equal to $2^8$.

8. Use the exponent laws for a power of a product and for a power of a quotient.
   **a)** $[3 \times (-5)]^3 = 3^3 \times (-5)^3$

   **b)** $-(2 \times 4)^5 = -(2^5 \times 4^5)$
   $$= -2^5 \times 4^5$$

   **c)** $\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4}$

   **d)** $\left(\dfrac{-7}{-2}\right)^2 = \dfrac{(-7)^2}{(-2)^2}$

   **e)** $-[(-10) \times 3]^3 = -[(-10)^3 \times 3^3]$
   $$= -(-10)^3 \times 3^3$$

   **f)** $(16 \div 9)^2 = 16^2 \div 9^2$

## *Apply*

9. The expression $(-5^2)^3$ is negative, because $-5^2 = -(5^2)$
   $$= -25,$$
   and $(-25)^3$ is the product of three factors of $-25$, and the product of an odd number of negative factors is negative.

10. **a)** I multiplied in the brackets first, because it was easier than using the power of a product law. Then I evaluated the power:
   $$(3 \times 2)^3 = 6^3$$
   $$= 216$$

   **b)** I multiplied in the brackets first, because it was easier than using the power of a product law. Then I evaluated the power:
   $$[(-2) \times 4]^2 = (-8)^2$$
   $$= 64$$

   **c)** I divided in the brackets first, because it was easier than using the power of a quotient law. Then I evaluated the power:
   $$\left(\dfrac{9}{-3}\right)^3 = (-3)^3$$
   $$= -27$$

   **d)** I divided in the brackets first, because it was easier than using the power of a quotient law. Then I evaluated the power:
   $$\left(\dfrac{8}{2}\right)^2 = 4^2$$
   $$= 16$$

   **e)** I used the zero exponent law:
   $$(12^8)^0 = 1$$

**f)** I used the power of a power law, then a calculator:

$[(-4)^2]^2 = (-4)^4$

$\qquad = 256$

**11.** The expression $[(-2)^3]^4$ is positive because its exponent is even after the power of a power law is applied:

$[(-2)^3]^4 = (-2)^{12}$

The sign of a product with an even number of negative factors is positive.

The expression $[(-2)^3]^5$ is negative because its exponent is odd after the power of a power law is applied:

$[(-2)^3]^5 = (-2)^{15}$

The sign of a product with an odd number of negative factors is negative.

**12.** The expression $-(4^2)^3 = -(4^{2 \times 3})$

$\qquad\qquad\qquad = -4^6$

$\qquad\qquad\qquad = -4096$

This is negative because the negative sign in front of the expression is not affected when the power of a power is calculated.

The expression $(-4^2)^3 = (-16)^3$

$\qquad\qquad\qquad = -4096$

This is negative because $-4^2 = -16$, and $(-16)^3$ is negative.

The expression $[(-4)^2]^3 = 16^3$

$\qquad\qquad\qquad = 4096$

This is positive because $(-4^2) = 16$, and $16^3$ is positive.

**13. a) i)** $(4 \times 3)^3 = 12^3$

$\qquad\qquad\qquad = 1728$

$\qquad (4 \times 3)^3 = 4^3 \times 3^3$

$\qquad\qquad\qquad = 64 \times 27$

$\qquad\qquad\qquad = 1728$

**ii)** The results are the same. I preferred doing the operation in brackets first because I only had to use my calculator once, to evaluate $12^3$.

**b) i)** $[(-2) \times (-5)]^2 = 10^2$

$\qquad\qquad\qquad = 100$

$\qquad [(-2) \times (-5)]^2 = (-2)^2 \times (-5)^2$

$\qquad\qquad\qquad = 4 \times 25$

$\qquad\qquad\qquad = 100$

**ii)** The results are the same. I preferred doing the operation in brackets first because I know the product of $-2$ and $-5$ is 10, and it is easy to square 10.

**c) i)** $\left(\dfrac{6}{2}\right)^4 = 3^4$

$\qquad\qquad = 81$

$\qquad \left(\dfrac{6}{2}\right)^4 = \dfrac{6^4}{2^4}$

$\qquad\qquad = \dfrac{1296}{16}$

$\qquad\qquad = 81$

**ii)** The results are the same. I preferred doing the operation in brackets first because I did not need to use a calculator.

**d) i)** $\left(\dfrac{14}{2}\right)^0 = 7^0$

$\qquad\qquad = 1$

$\left(\dfrac{14}{2}\right)^0 = 1$

**ii)** The results are the same. I preferred to use the zero exponent law because I only needed one step in the solution.

**e) i)** $[(-5)^2]^2 = 25^2$

$\qquad\qquad = 625$

$[(-5)^2]^2 = (-5)^4$

$\qquad\qquad = 625$

**ii)** The results are the same. I preferred the exponent law method because I just had to enter $5^4$ in a calculator.

**f) i)** $(2^5)^3 = 32^3$

$\qquad\qquad = 32\ 768$

$(2^5)^3 = 2^{15}$

$\qquad\qquad = 32\ 768$

**ii)** The results are the same. I preferred the exponent law method because I just had to enter $2^{15}$ in my calculator.

The method I preferred was not always the same. Sometimes I could simplify the expression using the order of operations and evaluate the exponent using mental math. I preferred to use the exponent law when evaluating powers of powers because I could multiply the exponents in my head and then evaluate with a calculator.

**14.** Use the exponent laws to simplify first.

**a)** $(3^2 \times 3^1)^2 = (3^{2+1})^2$       Use the exponent law for products.

$\qquad\qquad = (3^3)^2$       Use the power of a power law.

$\qquad\qquad = 3^{3 \times 2}$       Multiply the exponents.

$\qquad\qquad = 3^6$

$\qquad\qquad = 729$

**b)** $(4^6 \div 4^4)^2 = (4^{6-4})^2$       Use the exponent law for quotients.

$\qquad\qquad = (4^2)^2$       Use the power of a power law.

$\qquad\qquad = 4^{2 \times 2}$       Multiply the exponents.

$\qquad\qquad = 4^4$

$\qquad\qquad = 256$

**c)** $[(-2)^0 \times (-2)^3]^2 = [(-2)^{0+3}]^2$       Use the exponent law for products.

$\qquad\qquad = [(-2)^3]^2$       Use the power of a power law.

$\qquad\qquad = (-2)^{3 \times 2}$       Multiply the exponents.

$\qquad\qquad = (-2)^6$

$\qquad\qquad = 64$

**d)** $(10^6 \div 10^4)^3 = (10^{6-4})^3$                    Use the exponent law for quotients.
$\qquad\qquad\qquad = (10^2)^3$                           Use the power of a power law.
$\qquad\qquad\qquad = 10^{2 \times 3}$                        Multiply the exponents.
$\qquad\qquad\qquad = 10^6$
$\qquad\qquad\qquad = 1\ 000\ 000$

**e)** $(10^3)^2 \times (10^2)^3 = 10^{3 \times 2} \times 10^{2 \times 3}$          Use the power of a power law.
$\qquad\qquad\qquad\quad = 10^6 \times 10^6$                Use the exponent law for products.
$\qquad\qquad\qquad\quad = 10^{6+6}$                      Add the exponents.
$\qquad\qquad\qquad\quad = 10^{12}$
$\qquad\qquad\qquad\quad = 1\ 000\ 000\ 000\ 000$

**f)** $(12^2)^4 \div (12^3)^2 = 12^{2 \times 4} \div 12^{3 \times 2}$          Use the power of a power law.
$\qquad\qquad\qquad\quad = 12^8 \div 12^6$                  Use the exponent law for quotients.
$\qquad\qquad\qquad\quad = 12^{8-6}$                      Subtract the exponents.
$\qquad\qquad\qquad\quad = 12^2$
$\qquad\qquad\qquad\quad = 144$

**g)** $(5^2)^6 \div (5^3)^4 = 5^{2 \times 6} \div 5^{3 \times 4}$            Use the power of a power law.
$\qquad\qquad\qquad\quad = 5^{12} \div 5^{12}$                 Use the exponent law for quotients.
$\qquad\qquad\qquad\quad = 5^{12-12}$                     Subtract the exponents.
$\qquad\qquad\qquad\quad = 5^0$
$\qquad\qquad\qquad\quad = 1$

**h)** $[(-2)^2]^3 \times (-2)^3 = (-2)^{2 \times 3} \times (-2)^3$          Use the power of a power law.
$\qquad\qquad\qquad\quad = (-2)^6 \times (-2)^3$             Use the exponent law for products.
$\qquad\qquad\qquad\quad = (-2)^{6+3}$                    Add the exponents.
$\qquad\qquad\qquad\quad = (-2)^9$
$\qquad\qquad\qquad\quad = -512$

**15. a)** $(3^2 \times 2^2)^3 = (6^4)^3$      The student multiplied the bases and multiplied the powers.
Correct solution:
$(3^2 \times 2^2)^3 = 3^{2 \times 3} \times 2^{2 \times 3}$
$\qquad\qquad\quad = 3^6 \times 2^6$
$\qquad\qquad\quad = 729 \times 64$
$\qquad\qquad\quad = 46\ 656$

**b)** $[(-3)^2]^3 = (-3)^5$      The student added the exponents instead of multiplying them.
Correct solution:
$[(-3)^2]^3 = (-3)^{2 \times 3}$
$\qquad\qquad = (-3)^6$
$\qquad\qquad = 729$

**c)** $\left(\dfrac{6^2}{6^1}\right)^2 = 6^4$      The student may have thought that $6^1$ is 1.

Correct solution:

$\left(\dfrac{6^2}{6^1}\right)^2 = (6^{2-1})^2$

$\qquad\quad = (6^1)^2$
$\qquad\quad = 6^{1 \times 2}$
$\qquad\quad = 6^2$
$\qquad\quad = 36$

**d)** $(2^6 \times 2^2 \div 2^4)^3 = (2^3)^3$          The student did not simplify the powers in the brackets correctly.
Correct solution:
$$(2^6 \times 2^2 \div 2^4)^3 = (2^{6+2} \div 2^4)^3$$
$$= (2^8 \div 2^4)^3$$
$$= (2^{8-4})^3$$
$$= (2^4)^3$$
$$= 2^{4 \times 3}$$
$$= 2^{12}$$
$$= 4096$$

**e)** $(10^2 + 10^3)^2 = (10^5)^2$          The student used the exponent law for the power of a product inside the brackets, instead of adding the powers.

Correct solution:
$$(10^2 + 10^3)^2 = (100 + 1000)^2$$
$$= 1100^2$$
$$= 1\ 210\ 000$$

**16.** Use the exponent laws to simplify first.
**a)** $(4^2 \times 4^3)^2 - (5^4 \div 5^2)^2 = (4^{2+3})^2 - (5^{4-2})^2$          Use the exponents laws for products and quotients.
$$= (4^5)^2 - (5^2)^2$$          Use the power of a power law.
$$= 4^{5 \times 2} - 5^{2 \times 2}$$          Multiply the exponents.
$$= 4^{10} - 5^4$$          Use a calculator.
$$= 1\ 048\ 576 - 625$$
$$= 1\ 047\ 951$$

**b)** $(3^3 \div 3^2)^3 + (8^4 \times 8^3)^0 = (3^{3-2})^3 + 8^0$          Use the exponent laws for quotients and products.
$$= (3^1)^3 + 8^0$$          Use the power of a power law.
$$= 3^{1 \times 3} + 8^0$$          Multiply the exponents.
$$= 3^3 + 1$$
$$= 27 + 1$$
$$= 28$$

**c)** $(2^3)^4 + (2^4 \div 2^3)^2 = 2^{3 \times 4} + (2^{4-3})^2$          Use the power of a power law and the exponent law for quotients.
$$= 2^{12} + (2^1)^2$$          Use the power of a power law.
$$= 2^{12} + 2^{1 \times 2}$$          Multiply the exponents.
$$= 2^{12} + 2^2$$
$$= 4096 + 4$$
$$= 4100$$

**d)** $(6^2 \times 6^0)^3 + (2^6 \div 2^4)^3 = (6^{2+0})^3 + (2^{6-4})^3$          Use the exponent laws for products and quotients.
$$= (6^2)^3 + (2^2)^3$$          Use the power of a power law.
$$= 6^{2 \times 3} + 2^{2 \times 3}$$          Multiply the exponents.
$$= 6^6 + 2^6$$          Use a calculator.
$$= 46\ 656 + 64$$
$$= 46\ 720$$

**e)** $(5^3 \times 5^3)^0 - (4^2)^2 = 5^0 - 4^{2 \times 2}$          Use the zero exponent law and the power of a power law.
$$= 1 - 256$$
$$= -255$$

f) $(10^5 \div 10^2)^2 + (3^3 \div 3^1)^4 = (10^{5-2})^2 + (3^{3-1})^4$          Use the exponent law for quotients.
$\qquad\qquad\qquad\qquad\qquad = (10^3)^2 + (3^2)^4$          Use the power of a power law.
$\qquad\qquad\qquad\qquad\qquad = 10^{3 \times 2} + 3^{2 \times 4}$          Multiply the exponents.
$\qquad\qquad\qquad\qquad\qquad = 10^6 + 3^8$          Use a calculator.
$\qquad\qquad\qquad\qquad\qquad = 1\ 000\ 000 + 6561$
$\qquad\qquad\qquad\qquad\qquad = 1\ 006\ 561$

**17.** Use the exponent laws to simplify first.

a) $\quad [(-2)^3 \times (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$          Use the exponent laws for products and quotients.
$= [(-2)^{3+2}]^2 - [(-3)^{3-2}]^2$          Add the exponents in the first bracket, subtract in the second.
$= [(-2)^5]^2 - [(-3)^1]^2$          Use the power of a power law.
$= (-2)^{5 \times 2} - (-3)^{1 \times 2}$          Multiply the exponents.
$= (-2)^{10} - (-3)^2$
$= 1024 - 9$
$= 1015$

b) $\quad [(-2)^3 \div (-2)^2]^2 - [(-3)^3 \times (-3)^2]^2$          Use the exponent laws for quotients and products.
$= [(-2)^{3-2}]^2 - [(-3)^{3+2}]^2$          Subtract the exponents in the first bracket, add in the second.
$= [(-2)^1]^2 - [(-3)^5]^2$          Use the power of a power law.
$= (-2)^{1 \times 2} - (-3)^{5 \times 2}$          Multiply the exponents.
$= (-2)^2 - (-3)^{10}$          Use a calculator.
$= 4 - 59\ 049$
$= -59\ 045$

c) $\quad [(-2)^3 \times (-2)^2]^2 + [(-3)^3 \div (-3)^2]^2$          Use the exponent laws for products and quotients.
$= [(-2)^{3+2}]^2 + [(-3)^{3-2}]^2$          Add the exponents in the firs t bracket, subtract in the second.
$= [(-2)^5]^2 + [(-3)^1]^2$          Use the power of a power law.
$= (-2)^{5 \times 2} + (-3)^{1 \times 2}$          Multiply the exponents.
$= (-2)^{10} + (-3)^2$
$= 1024 + 9$
$= 1033$
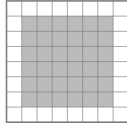
d) $\quad [(-2)^3 \div (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$          Use the exponent laws for quotients and products.
$= [(-2)^{3-2}]^2 + [(-3)^{3+2}]^2$          Subtract the exponents in the first bracket, add in the second.
$= [(-2)^1]^2 + [(-3)^5]^2$          Use the power of a power law.
$= (-2)^{1 \times 2} + (-3)^{5 \times 2}$          Multiply the exponents.
$= (-2)^2 + (-3)^{10}$          Use a calculator.
$= 4 + 59\ 049$
$= 59\ 053$

e) $\quad [(-2)^3 \div (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$          Use the exponent laws for quotients and products.
$= [(-2)^{3-2}]^2 - [(-3)^{3-2}]^2$          Subtract the exponents in the first bracket, add in the second.
$= [(-2)^1]^2 - [(-3)^1]^2$          Use the power of a power law.
$= (-2)^{1 \times 2} - (-3)^{1 \times 2}$          Multiply the exponents.
$= (-2)^2 - (-3)^2$
$= 4 - 9$
$= -5$

f)  $[(-2)^3 \times (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$     Use the exponent law for products.
    $= [(-2)^{3+2}]^2 + [(-3)^{3+2}]^2$     Add the exponents in the brackets.
    $= [(-2)^5]^2 + [(-3)^5]^2$     Use the power of a power law.
    $= (-2)^{5 \times 2} + (-3)^{5 \times 2}$     Multiply the exponents.
    $= (-2)^{10} + (-3)^{10}$     Use a calculator.
    $= 1024 + 59\ 049$
    $= 60\ 073$

**18. a) i)**  $(2 \times 3)^2 = 6^2$
              $= 6 \times 6$
              $= 36$



**ii)** $(2 \times 3)^2 = 2^2 \times 3^2$

**iii)** $2^2 \times 3^2 = 4 \times 9$
              $= 36$



**iv)** Both rectangles have an area of 36 but they have different dimensions.

**b) i)**  $(2 \times 4)^2 = 8^2$
          $= 8 \times 8$
          $= 64$
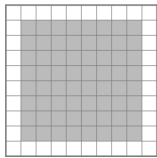


**ii)** $(2 \times 4)^2 = 2^2 \times 4^2$

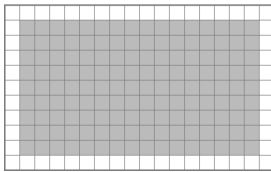**iii)** $2^2 \times 4^2 = 4 \times 16$
              $= 64$
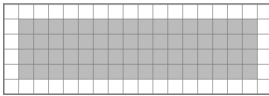


**iv)** Both rectangles have an area of 64 but they have different dimensions.

**c) i)** $(3 \times 4)^2 = 12^2$
$= 12 \times 12$
$= 144$
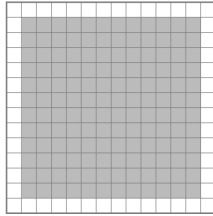
**ii)** $(3 \times 4)^2 = 3^2 \times 4^2$

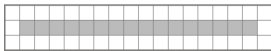**iii)** $3^2 \times 4^2 = 9 \times 16$
$= 144$

**iv)** Both rectangles have an area of 144 but they have different dimensions.

**d) i)** $(1 \times 4)^2 = 4^2$
$= 4 \times 4$
$= 16$

**ii)** $(1 \times 4)^2 = 1^2 \times 4^2$

**iii)** $1^2 \times 4^2 = 1 \times 16$
$= 16$

**iv)** Both rectangles have an area of 16 but they have different dimensions.

The relation between the square of a product and the product of squares:
Whether you square the two numbers first and then determine their product or determine the product of the two numbers first and then square it, the answer is the same.

**19.** Use the exponent laws to simplify first, where appropriate.

**a)** $(2^3 \times 2^6)^2 - (3^7 \div 3^5)^4 = (2^{3+6})^2 - (3^{7-5})^4$     Use the exponent laws for products and quotients.
$= (2^9)^2 - (3^2)^4$     Use the power of a power law.
$= 2^{9 \times 2} - 3^{2 \times 4}$     Multiply the exponents.
$= 2^{18} - 3^8$     Use a calculator.
$= 262\ 144 - 6561$
$= 255\ 583$

**b)** $(6 \times 8)^5 + (5^3)^2 = 48^5 + 5^{3 \times 2}$     Use the exponent law for products and the power of a power law.
$= 48^5 + 5^6$     Use a calculator.
$= 254\ 819\ 593$

**c)** $[(-4)^3 \times (-4)^2]^2 + (4^3 \times 4^2)^2 = [(-4)^{3+2}]^2 + (4^{3+2})^2$          Use the exponent law for products.
$= [(-4)^5]^2 + (4^5)^2$          Use the power of a power law.
$= (-4)^{5 \times 2} + 4^{5 \times 2}$          Multiply the exponents.
$= (-4)^{10} + 4^{10}$          Use a calculator.
$= 1\ 048\ 576 + 1\ 048\ 576$
$= 2\ 097\ 152$

**d)** $[(-2)^4]^3 + [(-4)^3]^2 - [(-3)^2]^4 = (-2)^{4 \times 3} + (-4)^{3 \times 2} - (-3)^{2 \times 4}$          Use the power of a power law.
$= (-2)^{12} + (-4)^6 - (-3)^8$          Use a calculator.
$= 4096 + 4096 - 6561$
$= 1631$

**e)** $[(-3)^4]^2 \times [(-4)^0]^2 - [(-3)^3]^0 = (-3)^{4 \times 2} \times (-4)^{0 \times 2} - (-3)^{3 \times 0}$          Use the power of a power law.
$= (-3)^8 \times (4)^0 - (3)^0$          Use a calculator and the zero exponent law.
$= 6561 \times 1 - 1$
$= 6560$

**f)** Multiply in the first set of brackets. Use the power of a power law in the second set of brackets and the exponent law for quotients in the third set of brackets.
$[(-5) \times (-4)]^3 + [(-6)^3]^2 - [(-3)^9 \div (-3)^8]^5$
$= 20^3 + (-6)^{3 \times 2} - [(-3)^{9-8}]^5$          Multiply and subtract the exponents.
$= 20^3 + (-6)^6 - [(-3)^1]^5$          Use a calculator.
$= 8000 + 46\ 656 - (-243)$
$= 8000 + 46\ 656 + 243$
$= 54\ 899$

## Take It Further

**20. a) i)** $81 = 9^2$
**ii)** $81 = (3 \times 3)^2$
**iii)** $81 = 3 \times 3 \times 3 \times 3$
$= 3^4$

**b) i)** $64 = 8^2$
**ii)** $64 = (2 \times 2 \times 2)^2$ or $64 = (2 \times 4)^2$
**iii)** $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

**c)** For example:
$16 = 4^2$
$= (2 \times 2)^2$
$= 2^4$
$625 = 25^2$
$= (5 \times 5)^2$
$= 5^4$
$729 = 27^2$
$= (3 \times 3 \times 3)^2$
$= 3^6$
$2401 = 49^2$
$= (7 \times 7)^2$
$= 7^4$

**21. a)**

| | |
|---|---|
| $2^0 = 1$ | $2^7 = 128$ |
| $2^1 = 2$ | $2^8 = 256$ |
| $2^2 = 4$ | $2^9 = 512$ |
| $2^3 = 8$ | $2^{10} = 1024$ |
| $2^4 = 16$ | $2^{11} = 2048$ |
| $2^5 = 32$ | $2^{12} = 4096$ |
| $2^6 = 64$ | |

**b) i)** $32 \times 64 = 2^5 \times 2^6$
$$= 2^{5+6}$$
$$= 2^{11}$$
$$= 2048$$

**ii)** $16 \times 8 \times 32 = 2^4 \times 2^3 \times 2^5$
$$= 2^{4+3+5}$$
$$= 2^{12}$$
$$= 4096$$

**iii)** $1024 \div 128 = 2^{10} \div 2^7$
$$= 2^{10-7}$$
$$= 2^3$$
$$= 8$$

**iv)** $\dfrac{16 \times 256}{1024} = \dfrac{2^4 \times 2^8}{2^{10}}$
$$= \dfrac{2^{4+8}}{2^{10}}$$
$$= \dfrac{2^{12}}{2^{10}}$$
$$= 2^{12-10}$$
$$= 2^2$$
$$= 4$$

**v)** $(8 \times 4)^3 = (2^3 \times 2^2)^3$
$$= (2^{3+2})^3$$
$$= (2^5)^3$$
$$= 2^{5 \times 3}$$
$$= 2^{15}$$
$$= 32\,768$$

**vi)** $\left(\dfrac{256}{64}\right)^4 = \left(\dfrac{2^8}{2^6}\right)^4$
$$= (2^{8-6})^4$$
$$= (2^2)^4$$
$$= 2^{2 \times 4}$$
$$= 2^8$$
$$= 256$$

### *Lesson 2.1*

1.  **a)** $4^3 = 4 \times 4 \times 4$          As repeated multiplication
       $= 64$                     Standard form

    **b)** $7^2 = 7 \times 7$          As repeated multiplication
       $= 49$                     Standard form

    **c)** $-(-2)^5 = -(-2)(-2)(-2)(-2)(-2)$          As repeated multiplication
          $= 32$                     Standard form

    **d)** $-3^4 = -(3^4)$
          $= -(3)(3)(3)(3)$     As repeated multiplication
          $= -81$                  Standard form

    **e)** $-1^8 = -(1^8)$
          $= -(1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1)$   As repeated multiplication
          $= -1$                         Standard form

    **f)** $(-1)^8 = (-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)$          As repeated multiplication
          $= 1$                     Standard form

2.  The expression $2^2 = 4$ can be modelled as the area of a square with side length 2 units.
    The expression $2^3 = 8$ can be modelled as the volume of a cube with edge length 2 units.

3.  **a)** $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$          As a power
                           $= 729$        Standard form

    **b)** $(-8)\,(-8)\,(-8) = (-8)^3$          As a power
                       $= -512$        Standard form

    **c)** $-(2 \times 2 \times 2 \times 2 \times 2 \times 2) = -2^7$          As a power
                                $= -128$      Standard form

    **d)** $12 \times 12 = 12^2$          As a power
              $= 144$        Standard form

    **e)** $4 \times 4 \times 4 \times 4 \times 4 = 4^5$          As a power
                       $= 1024$        Standard form

    **f)** $(-5)(-5)(-5)(-5) = (-5)^4$          As a power
                       $= 625$        Standard form

4.  $5^8$ means $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 390\ 625$
    $8^5$ means $8 \times 8 \times 8 \times 8 \times 8 = 32\ 768$

5.  It will take 16 min. I know this because $256 = 2^8$ so there will be 8 rows (beyond the first row) in the
    telephone tree before it has 256 people in the bottom row. Since each person calls 2 people, which takes 2
    minutes total, the tree will take $8 \times 2 = 16$ minutes.

6. **a)** Yes, the values are different; $-4^2 = -(4 \times 4)$
$$= -16$$
and $(-4)^2 = (-4) \times (-4)$
$$= 16$$
The brackets indicate that the negative sign is included in the base when it is raised to the power 2.

**b)** No, the values are the same; $-2^3 = -(2 \times 2 \times 2) = -8$ and $(-2)^3 = (-2) \times (-2) \times (-2) = -8$; the brackets indicate that the negative sign is included in the base when it is raised to the power 3, however this product is negative because an odd number of negative factors are multiplied.

7. **a) i)** $-3^2 = -(3 \times 3)$
$$= -9$$

**ii)** $-(3)^2 = -(3)(3)$
$$= -9$$

**iii)** $-(-3)^2 = -(-3)(-3)$
$$= -9$$

**iv)** $(-3)^2 = (-3)(-3)$
$$= 9$$

**b) ii)** The brackets show that the negative sign is not part of the base of the power.

**iii)** The brackets show that the first negative sign is not part of the base of the power and the second negative sign is part of the base of the power.

**iv)** The brackets show that the negative sign is part of the base of the power.

## *Lesson 2.2*

8. **a)** one hundred million = 100 000 000
$$= 10^8$$

**b)** $10 \times 10 \times 10 \times 10 = 10\ 000$
$$= 10^4$$

**c)** $1 = 10^0$

**d)** $1\ 000\ 000\ 000 = 10^9$

**e)** one thousand = 1000
$$= 10^3$$

9. **a)** $700\ 000\ 000 = 7 \times 100\ 000\ 000$
$$= 7 \times 10^8$$

**b)** $345 = 300 + 40 + 5 = (3 \times 100) + (4 \times 10) + (5 \times 1)$
$$= (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$$

**c)** $80\ 027 = 80\ 000 + 20 + 7 = (8 \times 10\ 000) + (2 \times 10) + (7 \times 1)$
$$= (8 \times 10^4) + (2 \times 10^1) + (7 \times 10^0)$$

**10. a), c)**

| Power | Repeated Multiplication | Standard Form |
|---|---|---|
| $3^5$ | 3 × 3 × 3 × 3 × 3 | 243 |
| $3^4$ | 3 × 3 × 3 × 3 | 81 |
| $3^3$ | 3 × 3 × 3 | 27 |
| $3^2$ | 3 × 3 | 9 |
| $3^1$ | 3 | 3 |
| $3^0$ | | 1 |

**b)** The exponents are decreasing by 1; the number of factors is decreasing by 1; each number in standard form is divided by 3 to get the number below it.

**c)** When the exponents decrease by 1 for the last row, the exponent is 0, and the term is $3^0$. The number of factors is decreasing by 1, so there is no factor 3. The term in the third column of the last row is $3 \div 3 = 1$. For each row, the terms in the 1st and 3rd columns are equal, so $3^0 = 1$. This will be true for whichever factor is chosen.

**11. a)** Divide the height of the mountain by the height of the tree to determine how many times as high:
$10^4 \div 10^2 = 10^2$, or 100 times as high

**b)** Divide the diameter of the star by the Earth's diameter to determine how many times as great:
$10^{12} \div 10^7 = 10^5$, or 100 000 times as great

**12. a)** $(4 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + (9 \times 10^0) = 4000 + 700 + 20 + 9$
$= 4729$

**b)** $(3 \times 10^5) + (2 \times 10^2) + (8 \times 10^0) = 300\ 000 + 200 + 8$
$= 300\ 208$

## *Lesson 2.3*

**13. a)** $3^4 + 3^2 = 81 + 9$
$= 90$

**b)** $(-4)^2 + (-4)^3 = 16 + (-64)$
$= -48$

**c)** $10^3 - 10^2 = 1000 - 100$
$= 900$

**d)** $(-5)^4 - (-5)^2 = 625 - 25$
$= 600$

**14. a)** $2^3 + (5 - 2)^4 = 8 + 3^4$
$= 8 + 81$
$= 89$

**b)** $100 \div 2 + (4 + 1)^3 = 100 \div 2 + 5^3$
$= 50 + 125$
$= 175$

**c)** $(6^2 + 7^2)^0 - (8^4 + 2^4)^0 = 1 - 1$
$= 0$

**d)** $3 \times 2^3 + 8 \div 4 = 3 \times 8 + 8 \div 4$
$$= 24 + 2$$
$$= 26$$

**e)** $(21 \div 7)^4 - 2^3 = 3^4 - 2^3$
$$= 81 - 8$$
$$= 73$$

**f)** $[(-4)^0 \times 10]^6 \div (15 - 10)^2 = [1 \times 10]^6 \div 5^2$
$$= 10^6 \div 5^2$$
$$= 1\,000\,000 \div 25$$
$$= 40\,000$$

**15. a) i)** $1000 \times 2^0 = 1000 \times 1$
$$= 1000$$

**ii)** $1000 \times 2^1 = 1000 \times 2$
$$= 2000$$

**iii)** $1000 \times 2^2 = 1000 \times 4$
$$= 4000$$

**iv)** $1000 \times 2^3 = 1000 \times 8$
$$= 8000$$

**b) i)** $1000 \times 2^4 = 1000 \times 16$
$$= 16\,000$$

**ii)** $1000 \times 2^6 = 1000 \times 64$
$$= 64\,000$$

**iii)** $1000 \times 2^9 = 1000 \times 512$
$$= 512\,000$$

**iv)** $1000 \times 2^{12} = 1000 \times 4096$
$$= 4\,096\,000$$

**16.** I can get 6 different answers by changing only the position of the brackets.
$4^3 - (2 \times 3)^4 + 11 = -1221$
$(4^3 - 2) \times 3^4 + 11 = 5033$
$(4^3 - 2 \times 3)^4 + 11 = 11\,316\,507$
$4^3 - (2 \times 3^4 + 11) = -109$
$4^3 - 2 \times (3^4 + 11) = -120$
$(4^3 - 2 \times 3^4) + 11 = -87$

**17.** The student incorrectly simplified this expression by applying the exponent law for a product of powers when the bases are not the same: –2 and 2 are not the same base. Also, $-9 \div (-3)$ is 3, not –3.
$(-2)^2 \times 2^3 - 3^2 \div (-3) + (-4)^2$
$= 4 \times 8 - 9 \div (-3) + 16$
$= 32 - (-3) + 16$
$= 51$

*Lesson 2.4*

**18. a)** $5^3 \times 5^4 = 5^{3+4}$

$\qquad\qquad = 5^7$

$\qquad\qquad = 78\ 125$

**b)** $(-2)^3 \times (-2)^2 = (-2)^{3+2}$

$\qquad\qquad\qquad = (-2)^5$

$\qquad\qquad\qquad = -32$

**c)** $3^2 \times 3^3 \times 3^1 = 3^{2+3+1}$

$\qquad\qquad\qquad = 3^6$

$\qquad\qquad\qquad = 729$

**d)** $-10^4 \times 10^0 = -10^4 \times 1$

$\qquad\qquad\qquad = -10^4$

$\qquad\qquad\qquad = -10\ 000$

**19.** $10^{11} \times 10^{11} = 10^{11+11}$

$\qquad\qquad\qquad = 10^{22}$ or $10\ 000\ 000\ 000\ 000\ 000\ 000\ 000$

There are approximately $10^{22}$ stars in the universe.

**20. a)** $7^5 \div 7^3 = 7^{5-3}$

$\qquad\qquad = 7^2$

$\qquad\qquad = 49$

**b)** $(-10)^9 \div (-10)^3 = (-10)^{9-3}$

$\qquad\qquad\qquad\qquad = (-10)^6$

$\qquad\qquad\qquad\qquad = 1\ 000\ 000$

**c)** $\dfrac{8^4}{8^2} = 8^{4-2}$

$\qquad\quad = 8^2$

$\qquad\quad = 64$

**d)** $-\dfrac{6^7}{6^4} = -6^{7-4}$

$\qquad\qquad = -6^3$

$\qquad\qquad = -216$

**21. a)** No, the laws of exponents cannot be used to simplify the expression $6^3 \times 5^5$ because the powers have different bases.

**b)** Yes, even though these powers have different bases, both bases are multiples of 3.
So, $27^2$ can be written as $(3^3)^2 = 3^6$ and $9^2$ can be rewritten as $(3^2)^2 = 3^4$. Then,
$27^2 \div 9^2 = 3^6 \div 3^4 = 3^2 = 9$

**22. a)** The student divided the exponents instead of subtracting them.
$(-3)^6 \div (-3)^2 = (-3)^{6-2}$

$\qquad\qquad\qquad = (-3)^4$

$\qquad\qquad\qquad = 81$

**b)** The student used an exponent law incorrectly to simplify an addition question. Or, the student misread the addition sign as a multiplication sign.
$(-4)^2 + (-4)^2 = 16 + 16$

$\qquad\qquad\qquad = 32$

**c)** After the first step, the student divided the exponents instead of subtracting them.

$$\frac{(-5)^2 \times (-5)^4}{(-5)^3 \times (-5)^0} = \frac{(-5)^6}{(-5)^3}$$
$$= (-5)^{6-3}$$
$$= (-5)^3$$
$$= -125$$

## Lesson 2.5

**23. a)** $(3 \times 5)^3 = 3^3 \times 5^3$
$$= 27 \times 125$$
$$= 3375$$

**b)** $(12 \div 3)^5 = 12^5 \div 3^5$
$$= 248\,832 \div 243$$
$$= 1024$$

**c)** $[(-4) \times 2]^4 = (-4)^4 \times 2^4$
$$= 256 \times 16$$
$$= 4096$$

**d)** $(63 \times 44)^0 = 63^0 \times 44^0$
$$= 1 \times 1$$
$$= 1$$

**e)** $\left(\dfrac{3}{2}\right)^5 = \dfrac{3^5}{2^5}$
$$= \frac{243}{32}, \text{ or } 7.593\,75$$

**f)** $\left(\dfrac{15}{2}\right)^2 = \dfrac{15^2}{2^2}$
$$= \frac{225}{4}, \text{ or } 56.25$$

**24.** Use the power of a power law.

**a)** $(3^2)^3 = 3^{2 \times 3}$
$$= 3^6$$

**b)** $(4^0)^6 = 4^{0 \times 6}$
$$= 4^0$$

**c)** $[(-2)^3]^3 = (-2)^{3 \times 3}$
$$= (-2)^9$$

**d)** $(5^5)^2 = 5^{5 \times 2}$
$$= 5^{10}$$

**25. a) i)** $(5 \times 3)^3 = 15^3$
$$= 3375$$

**ii)** $(5 \times 3)^3 = 5^3 \times 3^3$
$$= 125 \times 27$$
$$= 3375$$

The first method is more efficient because I can use a calculator and there are fewer steps.

**b) i)** $(3 \times 3)^4 = 9^4$
$$= 6561$$

**ii)** $(3 \times 3)^4 = 3^4 \times 3^4$
$$= 81 \times 81$$
$$= 6561$$

The first method is more efficient because I can use a calculator and there are fewer steps.

**c) i)** $(8 \div 2)^5 = 4^5$
$$= 1024$$

**ii)** $(8 \div 2)^5 = 8^5 \div 2^5$
$$= 32\ 768 \div 32$$
$$= 1024$$

Since I already know that $8 \div 2$ is 4, and since $4^5$ is easier to calculate than $8^5$, the first method is more efficient.

**d) i)** $\left(\dfrac{9}{3}\right)^2 = 3^2$
$$= 9$$

**ii)** $\left(\dfrac{9}{3}\right)^2 = \dfrac{9^2}{3^2}$
$$= \dfrac{81}{9}$$
$$= 9$$

The first method is more efficient. It is easier to simplify $\dfrac{9}{3}$ than $\dfrac{9^2}{3^2}$.

**e) i)** $(2^3)^4 = 8^4$
$$= 4096$$

**ii)** $(2^3)^4 = 2^{12}$
$$= 4096$$

Both methods are equally efficient. With a calculator, it is just as easy to calculate $8^4$ as $2^{12}$.

**f) i)** $(6^2)^0 = 36^0$
$\qquad = 1$

**ii)** $(6^2)^0 = 6^0$
$\qquad = 1$

The second method is more efficient. It is easier to use an exponent law with a zero exponent.

Students should be able to explain for each question which method they thought was more efficient. Reasons may include: could do more of the question using mental math; only had to evaluate one exponent with my calculator; fewer steps on my calculator; the simplified powers were lesser numbers; or the simplified bases were lesser numbers.

**26. a)** $6^4 \times 6^3 = 6^{4+3}$          Use the exponent law for products.
$\qquad\qquad = 6^7$
$\qquad\qquad = 279\ 936$

**b)** $(-11)^7 \div (-11)^5 = (-11)^{7-5}$          Use the exponent law for quotients.
$\qquad\qquad\qquad\quad = (-11)^2$
$\qquad\qquad\qquad\quad = 121$

**c)** $\dfrac{3^4 \times 3^5}{3^3} = \dfrac{3^{4+5}}{3^3}$          Use the exponent law for products.

$\qquad\qquad = \dfrac{3^9}{3^3}$          Use the exponent law for quotients.

$\qquad\qquad = 3^{9-3}$
$\qquad\qquad = 3^6$
$\qquad\qquad = 729$

**d)** $\dfrac{5^5}{5^3 \times 5^2} = \dfrac{5^5}{5^{3+2}}$          Use the exponent law for products.

$\qquad\qquad = \dfrac{5^5}{5^5}$          Use the exponent law for quotients.

$\qquad\qquad = 5^{5-5}$
$\qquad\qquad = 5^0$
$\qquad\qquad = 1$

**e)** $\dfrac{(-4)^3 \times (-4)^6}{(-4)^2 \times (-4)^4} = \dfrac{(-4)^{3+6}}{(-4)^{2+4}}$          Use the exponent law for products.

$\qquad\qquad\qquad = \dfrac{(-4)^9}{(-4)^6}$          Use the exponent law for quotients.

$\qquad\qquad\qquad = (-4)^{9-6}$
$\qquad\qquad\qquad = (-4)^3$
$\qquad\qquad\qquad = -64$

**f)** $\dfrac{10^6 \times 10^0}{10^3 \times 10^2} = \dfrac{10^{6+0}}{10^{3+2}}$          Use the exponent law for products.

$\qquad\qquad\qquad = \dfrac{10^6}{10^5}$          Use the exponent law for quotients.

$\qquad\qquad\qquad = 10^{6-5}$
$\qquad\qquad\qquad = 10^1$
$\qquad\qquad\qquad = 10$

**27. a)** $2^3 \times 2^2 - 2^0 + 2^4 \div 2^3 = 2^{(3+2)} - 2^0 + 2^{(4-3)}$  Use the exponents laws for products and quotients.
$$= 2^5 - 2^0 + 2^1$$  Evaluate the exponents.
$$= 32 - 1 + 2$$
$$= 33$$

**b)** $\dfrac{(-2)^3 \times (-2)^2}{(-2)^3 - (-2)^2} = \dfrac{(-2)^{(3+2)}}{(-8) - 4}$  Use the exponent law for products in the numerator. Evaluate

$$= \dfrac{(-2)^5}{(-8 - 4)}$$  the exponents in the denominator.

$$= \dfrac{-32}{-12}$$

$$= \dfrac{8}{3}$$

**c)** $12^2 \times 12^4 \div (-2)^4 - 12^0 = 12^{(2+4)} \div (-2)^4 - 12^0$  Use the exponent law for products. Then, evaluate the
$$= 12^6 \div 16 - 1$$  exponents.
$$= 186\ 624 - 1$$
$$= 186\ 623$$

**d)** $\dfrac{(-12)^2 \times (-12)^4}{(-2)^4 - 12^0} = \dfrac{(-12)^{(2+4)}}{(-2)^4 - 12^0}$  Use the exponent law for products in the numerator. Evaluate

$$= \dfrac{(-12)^6}{16 - 1}$$  the exponents.

$$= \dfrac{2\ 985\ 984}{15}$$

$$= 199\ 065.6$$

**Practice Test**                                                                 **(page 90)**

1.  **a)** $(3 \times 4)^3 = 3^3 \times 4^3$

    **b)** $[(-5) \times 2]^4 = (-5)^4 \times 2^4$

    **c)** $\left(\dfrac{1}{4}\right)^4 = \dfrac{1^4}{4^4}$

    **d)** $-\left(\dfrac{9}{3}\right)^3 = -\dfrac{9^3}{3^3}$

2.  Use the power of a power law.
    **a)** $-(2^3)^3 = -(2^{3 \times 3})$
    $\qquad\qquad = -2^9$

    **b)** $(6^2)^0 = 6^{2 \times 0}$
    $\qquad\quad = 6^0$

    **c)** $[(-5)^2]^3 = (-5)^{2 \times 3}$
    $\qquad\qquad = (-5)^6$

    **d)** $-[(-3)^2]^4 = -(-3)^{2 \times 4}$
    $\qquad\qquad\; = -(-3)^8$

3.  **a)** $[(-3) \times (-2)]^4 = 6^4$
    $\qquad\qquad\qquad\;\; = 1296$

    **b)** $\left(\dfrac{1}{2}\right)^5 = \dfrac{1^5}{2^5}$

    $\qquad\qquad = \dfrac{1}{32}$, or 0.031 25

    **c)** $(6^0)^4 = 6^{0 \times 4}$
    $\qquad\quad = 6^0$
    $\qquad\quad = 1$

    **d)** $[(-3)^2]^3 = (-3)^{2 \times 3}$
    $\qquad\qquad = (-3)^6$
    $\qquad\qquad = 729$

4.  The value of a power with a negative base is negative if the exponent is odd because an odd number of negative factors produces a negative product; for example,
    $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

    The value of a power with a negative base is positive if the exponent is even because an even number of negative factors produces a positive product; for example,
    $(-3)^2 = (-3) \times (-3) = 9$

5.  $10^3 \text{ m}^2 = (10 \times 10 \times 10) \text{ m}^2 = 1000 \text{ m}^2$.
    The area of the baseball diamond is $27 \text{ m} \times 27 \text{ m} = 729 \text{ m}^2$.
    So, the area of the baseball diamond is less than $10^3 \text{ m}^2$.

**6.** $(-3^5 \times 10) - (9 \div 3)$

The brackets are not necessary because the order of operations matches the order in which the multiplication and division are written and should be carried out.

$-3^5 \times 10 - 9 \div 3$            Evaluate the exponent first.

$= -243 \times 10 - 9 \div 3$         Then multiply and divide.

$= -2430 - 3$                Then subtract.

$= -2433$

**7. a)** This answer is incorrect. The expression $(2^3 + 4)^2$ was calculated as $(2^3 + 4) \times 2$.

This could be the incorrect solution:

$$(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2 = (8 + 4)^2 \times (-1000) \div 10^2$$
$$= 12^2 \times (-1000) \div 100$$
$$= 24 \times (-1000) \div 100$$
$$= (-24\,000) \div 100$$
$$= -240$$

**b)** This answer is correct. The solution is:

$$(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2 = (8 + 4)^2 \times (-1000) \div 10^2$$
$$= 12^2 \times (-1000) \div 100$$
$$= 144 \times (-1000) \div 100$$
$$= (-144\,000) \div 100$$
$$= -1440$$

**c)** This answer is incorrect. The expression $(-10)^3$ was evaluated as 1000.

This could be the incorrect solution:

$$(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2 = (8 + 4)^2 \times 1000 \div 10^2$$
$$= 12^2 \times 1000 \div 100$$
$$= 144 \times 1000 \div 100$$
$$= 144\,000 \div 100$$
$$= 1440$$

**d)** This answer is incorrect. In this part of the expression, $(-10)^3 \div (5 + 5)^2$, the brackets were ignored and $(-10)^3$ was divided by 5 before $5^2$ was added. This could be the incorrect solution:

$$(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2 = (8 + 4)^2 \times (-1000) \div 5 + 5^2$$
$$= 12^2 \times (-1000) \div 5 + 25$$
$$= 144 \times (-1000) \div 5 + 25$$
$$= (-144\,000) \div 5 + 25$$
$$= -28\,800 + 25$$
$$= -28\,825$$

**8. a)** $(-5)^3 \times (-5)^2 \div (-5)^1$

The product in this expression is negative because the exponent of its base is an odd number, 5. The divisor is negative because its exponent is odd. The quotient of two negative numbers is a positive number, so the expression is positive.

$$(-5)^3 \times (-5)^2 \div (-5)^1 = (-5)^{3 + 2 - 1}$$
$$= (-5)^4$$
$$= 625$$

**b)** $[(-9)^6 - (-9)^3]^0$

This expression is equal to 1 because it has a zero exponent, so its value is positive.

$$[(-9)^6 - (-9)^3]^0 = 1$$

c) $\dfrac{(-1)^2 \times (-1)^4}{(-1)^3 \times (-1)^2} = \dfrac{(-1)^6}{(-1)^5}$

The numerator is positive because the exponent of its base is an even number, 6. The denominator is negative because the exponent of its base is an odd number, 5. The quotient of a positive number and a negative number is negative, so the expression is negative.

d) $(-4)^6 + (-4)^4 \times (-4)^0$

The product in this expression is positive because the exponent of the base is an even number, 4. The first term is positive because it has an even number exponent. The sum of two positive numbers is positive, so the expression is positive.

$$(-4)^6 + (-4)^4 \times (-4)^0 = (-4)^6 + (-4)^4 \times 1$$
$$= (-4)^6 + (-4)^4$$
$$= 4096 + 256$$
$$= 4352$$

**Unit Problem          How Thick Is a Pile of Paper?                    (page 91)**

| Number of Folds | Number of Layers | Number of Layers as Powers |
|---|---|---|
| 0 | 1 | $2^0$ |
| 1 | 2 | $2^1$ |
| 2 | 4 | $2^2$ |
| 3 | 8 | $2^3$ |
| 4 | 16 | $2^4$ |
| 5 | 32 | $2^5$ |
| 6 | 64 | $2^6$ |

The number of layers doubles after each fold. The number of layers can be expressed as powers of 2.

Using the patterns from the table, I predict that after 25 folds, the number of layers would be: $2^{25}$ = 33 554 432.

The thickness of 100 sheets in my math textbook is about 7 mm.

So, the thickness of 1 sheet of paper is: $\dfrac{7\,mm}{100}$ = 0.07 mm.

After 25 folds, there are 33 554 432 layers.
So, the height of this many layers would be: 0.07 mm × 33 554 432 = 2 348 810.24 mm

33 554 432 layers is about 2 348 810 mm tall.

2 348 810 mm = 234 881 cm
                = 2348.8 m
                = 2.348 km

It is about 2 and a half kilometres from my house to school.