Lesson 3.1   What Is a Rational Number?   Practice (pages 101–103)

Check

5. To identify equal numbers, group the rational numbers with the same numerator and denominator:

$$\frac{-3}{2}, \frac{3}{-2}, -\frac{3}{2} \text{ and } \frac{2}{3}, \frac{-2}{3}, -\frac{2}{3}, \frac{2}{-3}$$

Check: $\frac{-3}{2}, \frac{3}{-2}, -\frac{3}{2}$

The value of a fraction remains the same if its numerator and denominator are multiplied by the same non-zero number.

$\frac{-3}{2}$ can be written as $\frac{-3}{2} \times \frac{-1}{-1} = \frac{3}{-2}$.

Since $\frac{-3}{2} = \frac{3}{-2}$ and $\frac{-3}{2} = -\frac{3}{2}$, then $\frac{-3}{2} = \frac{3}{-2} = -\frac{3}{2}$

Check: $\frac{2}{3}, \frac{-2}{3}, -\frac{2}{3}, \frac{2}{-3}$

$\frac{2}{3}$ is positive; $\frac{-2}{3}, -\frac{2}{3}, \frac{2}{-3}$ are negative.

$\frac{-2}{3}$ can be written as $\frac{-2}{3} \times \frac{-1}{-1} = \frac{2}{-3}$

Since $\frac{-2}{3} = \frac{2}{-3}$ and $\frac{-2}{3} = -\frac{2}{3}$, then $\frac{-2}{3} = -\frac{2}{3} = \frac{2}{-3}$

6. a) $\frac{7}{-9}$ is the same as $-\frac{7}{9}$.

$\frac{7}{-9}$ can also be written as $\frac{7}{-9} \times \frac{-1}{-1} = \frac{-7}{9}$.

Since $\frac{7}{-9} = -\frac{7}{9}$ and $\frac{7}{-9} = \frac{-7}{9}$, then $\frac{7}{-9} = -\frac{7}{9} = \frac{-7}{9}$

b) $\frac{-5}{3}$ is the same as $-\frac{5}{3}$.

$\frac{-5}{3}$ can also be written as $\frac{-5}{3} \times \frac{-1}{-1} = \frac{5}{-3}$.

Since $\frac{-5}{3} = -\frac{5}{3}$ and $\frac{-5}{3} = \frac{5}{-3}$, then $\frac{-5}{3} = -\frac{5}{3} = \frac{5}{-3}$

c) $-\frac{6}{11}$ is the same as $\frac{-6}{11}$.

Then, $\frac{-6}{11}$ can be written as $\frac{-6}{11} \times \frac{-1}{-1} = \frac{6}{-11}$.

Since $-\frac{6}{11} = \frac{-6}{11}$ and $\frac{-6}{11} = \frac{6}{-11}$, then $-\frac{6}{11} = \frac{6}{-11} = \frac{-6}{11}$

7. Methods may vary.

   a) Since $5 \times 20 = 100$, multiply both the numerator and the denominator by 20.

$$\frac{6}{5} = \frac{6 \times 20}{5 \times 20}$$
$$= \frac{120}{100}$$
$$= 1.2$$

   b)
$$-\frac{6}{5} = -\frac{6 \times 20}{5 \times 20}$$
$$= -\frac{120}{100}$$
$$= -1.2$$

   c) Since $4 \times 25 = 100$, multiply both the numerator and the denominator by 25.

$$\frac{9}{4} = \frac{9 \times 25}{4 \times 25}$$
$$= \frac{225}{100}$$
$$= 2.25$$

   d) In this case, use a calculator to divide 11 by 6.

$$-\frac{11}{6} = -1.8\overline{3}$$

8. **a)** The number line is labelled from –8 to –7. It is divided in tenths.

   A is 9 tenths to the left of –7.

   Write 9 tenths as a decimal:

$$\frac{9}{10} = 0.9$$

   Then, A represents –7.9.

   B is 2 tenths to the left of –7.

   Write 2 tenths as a decimal:

$$\frac{2}{10} = 0.2$$

   So, B represents –7.2.

   b) The number line is labelled from –5 to –3. It is divided in fifths.

   C is 2 fifths to the left of –4.

   Write 2 fifths as a decimal:

$$\frac{2}{5} = \frac{4}{10}$$
$$= 0.4$$

   So, C represents –4.4.

   D is 1 fifth to the left of –3.

   Write 1 fifth as a decimal:

$$\frac{1}{5} = \frac{2}{10}$$
$$= 0.2$$

   D represents –3.2.

c) The number line is labelled from –1 to 0. It is divided in tenths.
J is 7 tenths to the left of 0. J represents –0.7.
K is 2 tenths to the left of 0. K represents –0.2.

d) The number line is labelled from –15.4 to –15.3.
Each tenth is divided in 10 equal parts, or hundredths.
G is 7 hundredths to the left of –15.3.
Write 7 hundredths as a decimal.
$$\frac{7}{100} = 0.07$$
G represents –15.37.
H is 2 hundredths to the left of –15.3.
Write 2 hundredths as a decimal.
$$\frac{2}{100} = 0.02$$
So, H represents –15.32.

9. The greater number is the number to the right on each number line.
a) –7.2; –7.2 > –7.9

b) –3.2; –3.2 > –4.4

c) –0.2; –0.2 > –0.7

d) –15.32; –15.32 > –15.37

10. a) The number line is labelled from –12 to –10. It is divided in fourths.
E is 1 fourth to the left of –11.
Then, E represents $-11\frac{1}{4}$, or $\frac{-45}{4}$.
F is 3 fourths to the left of –10.
Then, F represents $-10\frac{3}{4}$, or $\frac{-43}{4}$.

b) The number line is labelled from –6 to –5. It is divided in eighths.
L is 1 eighth to the left of –5.
Then, L represents $-5\frac{1}{8}$, or $\frac{-41}{8}$.
M is 6 eighths to the left of –5.
Then, M represents $-5\frac{6}{8} = -5\frac{3}{4}$, or $\frac{-23}{4}$.

c) The number line is labelled from –5 to –3. It is divided in sixths.
N is 1 sixth to the left of –4.
So, N represents $-4\frac{1}{6}$, or $\frac{-25}{6}$.
P is 4 sixths to the left of –3.
So, P represents $-3\frac{4}{6} = -3\frac{2}{3}$, or $\frac{-11}{3}$.

d) The number line is labelled in fourths from $-\dfrac{3}{4}$ to 0.

Each fourth is then divided in fourths, so the number line is divided in sixteenths.
Q is 9 sixteenths to the left of 0.

Then, Q represents $-\dfrac{9}{16}$.

R is 3 sixteenths to the left of 0.

Then, R represents $-\dfrac{3}{16}$.

11. The lesser number is the fraction to the left on each number line.

a) $\dfrac{-45}{4}$; $\dfrac{-45}{4} < \dfrac{-43}{4}$

b) $\dfrac{-23}{4}$; $\dfrac{-23}{4} = \dfrac{-46}{8} < \dfrac{-41}{8}$

c) $\dfrac{-25}{6}$; $\dfrac{-25}{6} < \dfrac{-22}{6}$

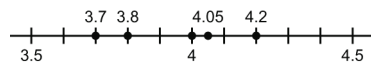**d)** $-\dfrac{9}{16}$; $-\dfrac{9}{16} < -\dfrac{3}{16}$

Apply

12. Answers will vary. For example: There are many rational numbers between each pair of numbers.

a) 3.7, 4.2

Sketch a number line divided in tenths and label it from 3.5 to 4.5.
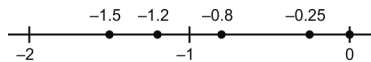Mark 3.7 and 4.2 on the number line.

From the number line, 3 possible rational numbers are:
3.8, 4, and 4.05

b) –1.5, 0

Sketch a number line labelled from –2 to 0.
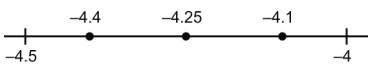Mark –1.5 on the number line.

From the number line, 3 possible rational numbers are:
–1.2, –0.8, –0.25

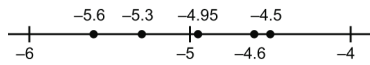**c)** –4.5, –4

Sketch a number line labelled from –4.5 to –4.

From the number line, 3 possible rational numbers are:
–4.4, –4.25, –4.1

d) –5.6, –4.5

Sketch a number line labelled from –6 to –4.
Mark –5.6 and –4.5 on the number line.

From the number line, 3 possible rational numbers are:
–5.3, –4.95, –4.6

e) –5.6, 5.7
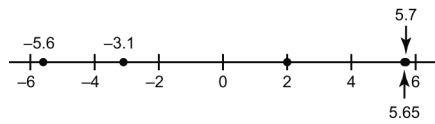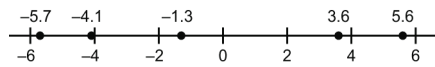
Sketch a number line labelled from –6 to 6.
Mark –5.6 and 5.7 on the number line.

From the number line, 3 possible rational numbers are:
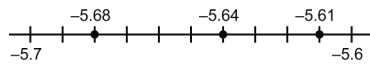–3.1, 2, 5.65

f) 5.6, –5.7

Sketch a number line labelled from –6 to 6.
Mark –5.7 and 5.6 on the number line.

From the number line, 3 possible rational numbers are:
–4.1, –1.3, 3.6

g) –5.6, –5.7
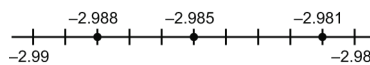
Sketch a number line divided in ten equal parts, labelled from –5.7 to –5.6.

From the number line, 3 possible rational numbers are:
–5.68, –5.64, –5.61

h) –2.98, –2.99

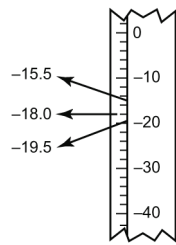Sketch a number line divided in ten equal parts, labelled from –2.99 to –2.98.

From the number line, 3 possible rational numbers are:
–2.988, –2.985, –2.981

13. a) Sketches may vary. For example:
Draw a vertical number line labelled with integers from –40 to 0.
Mark the 3 freezer temperatures on the number line: –15.5, –18, –19.5

b) From the number line, –15.5 is greater than –18.
The freezer's temperature may rise above –18°C.
The freezer should not be used to store meat.

14. Answers will vary. For example:
There are many rational numbers between each pair of numbers.

a) $\dfrac{5}{8}, \dfrac{13}{8}$

Sketch a number line labelled from 0 to 2.
Since both fractions have denominator 8, divide the number line in eighths.
Mark $\dfrac{5}{8}$ and $\dfrac{13}{8}$ on the number line.
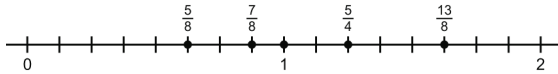
From the number line, 3 possible rational numbers are:
$\dfrac{7}{8}, 1, \dfrac{10}{8}$ or $\dfrac{5}{4}$

b) $\dfrac{17}{10}, -\dfrac{11}{5}$

Sketch a number line labelled from –2 to 2.
Mark $\dfrac{17}{10}$ and $-\dfrac{11}{5}$ on the number line.

From the number line, 3 possible rational numbers are:
$-\dfrac{8}{5}, -\dfrac{4}{5}, \dfrac{1}{5}$

c) $-\dfrac{15}{4}$, $-\dfrac{11}{3}$

Sketch a number line from $-\dfrac{15}{4}$ to $-\dfrac{11}{3}$.

Write both fractions as equivalent fractions with denominator 12.
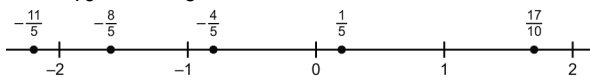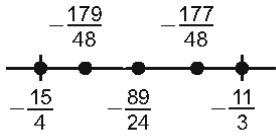
$-\dfrac{15}{4} = -\dfrac{45}{12}$ and $-\dfrac{11}{3} = -\dfrac{44}{12}$

Multiply the numerator and the denominator by 4 to be able to identify fractions between these two fractions.

$-\dfrac{45}{12} \times \dfrac{4}{4} = -\dfrac{180}{48}$; $-\dfrac{44}{12} \times \dfrac{4}{4} = -\dfrac{176}{48}$

From the number line, 3 possible rational numbers are:

$-\dfrac{179}{48}, -\dfrac{178}{48}$ or $-\dfrac{89}{24}, -\dfrac{177}{48}$

d) $-\dfrac{1}{2}$, $-\dfrac{1}{8}$

Sketch a number line from $-\dfrac{1}{2}$ to 0. Divide the line into eighths.

Write equivalent fractions for $-\dfrac{1}{2}$ and $-\dfrac{1}{8}$ with denominator 16 to identify fractions between the two numbers.

$-\dfrac{1}{2} = -\dfrac{8}{16}$ and $-\dfrac{1}{8} = -\dfrac{2}{16}$
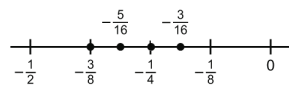
From the number line, 3 possible rational numbers between $-\dfrac{1}{2}$ and $-\dfrac{1}{8}$ are:

$-\dfrac{6}{16}$ or $-\dfrac{3}{8}$, $-\dfrac{4}{16}$ or $-\dfrac{1}{4}$, $-\dfrac{3}{16}$

e) $\dfrac{1}{6}$, 0.5

Sketch a number line from 0 to 1. Divide the line in sixths.

Write 0.5 as a fraction: $\dfrac{1}{2}$ or $\dfrac{3}{6}$

To write 3 rational numbers between $\dfrac{1}{6}$ and $\dfrac{3}{6}$, write the fractions with a common denominator of 12.

$\dfrac{1}{6} = \dfrac{2}{12}$ and $\dfrac{3}{6} = \dfrac{6}{12}$
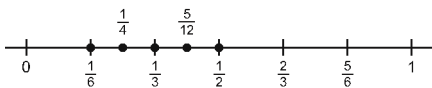
From the number line, 3 possible rational numbers are:

$\dfrac{3}{12}$ or $\dfrac{1}{4}$, $\dfrac{4}{12}$ or $\dfrac{1}{3}$, $\dfrac{5}{12}$

f) $-0.25,\ -\dfrac{1}{3}$

Write –0.25 as a fraction: $-\dfrac{1}{4}$

Sketch a number line from $-\dfrac{1}{3}$ to $-\dfrac{1}{4}$.

Write equivalent fractions for $-\dfrac{1}{3}$ and $-\dfrac{1}{4}$ with denominator 12 to identify fractions between the two numbers.

$-\dfrac{1}{3} = -\dfrac{4}{12}$ and $-\dfrac{1}{4} = -\dfrac{3}{12}$

We cannot easily identify fractions between these numbers, so write the fractions with denominator 48:

$-\dfrac{4}{12} = -\dfrac{16}{48}$ and $-\dfrac{3}{12} = -\dfrac{12}{48}$
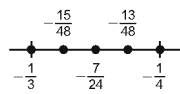
From the number line, 3 possible fractions between are:

$-\dfrac{15}{48},\ -\dfrac{14}{48}$ or $-\dfrac{7}{24},\ -\dfrac{13}{48}$

g) $-\dfrac{14}{5},\ -3$

Write $-\dfrac{14}{5}$ as a decimal: $-\dfrac{14}{5} = -2.8$

Sketch a number line from –3 to $-\dfrac{14}{5}$, or –2.8.
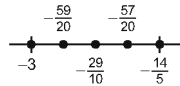
From the number line, 3 possible rational numbers are: –2.95, –2.9, –2.85

We could write each decimal as a fraction:

$-2.95 = -\dfrac{295}{100} = -\dfrac{59}{20}$

$-2.9 = -\dfrac{290}{100} = -\dfrac{29}{10}$

$-2.85 = -\dfrac{285}{100} = -\dfrac{57}{20}$

3 possible rational numbers between are: $-\dfrac{59}{20},\ -\dfrac{29}{10},\ -\dfrac{57}{20}$

h) $5\frac{3}{5}$, $5\frac{4}{5}$

Sketch a number line from $5\frac{3}{5}$ to $5\frac{4}{5}$.

To identify rational numbers between $5\frac{3}{5}$ and $5\frac{4}{5}$, write equivalent fractions with denominator 10:

$\frac{3}{5} = \frac{6}{10}$　　　　　　　$\frac{4}{5} = \frac{8}{10}$

Between $\frac{6}{10}$ and $\frac{8}{10}$ there is only one fraction with denominator 10: $\frac{7}{10}$

So, one possible rational number between $5\frac{3}{5}$ and $5\frac{4}{5}$ is $5\frac{7}{10}$.

Write equivalent fractions with denominator 20:

$\frac{3}{5} = \frac{6}{10} = \frac{12}{20}$　　　　　$\frac{4}{5} = \frac{8}{10} = \frac{16}{20}$

Write 2 more fractions between $\frac{12}{20}$ and $\frac{16}{20}$ with denominator 20: $\frac{13}{20}$ and $\frac{15}{20}$ or $\frac{3}{4}$.

So, two more possible rational numbers are: $5\frac{13}{20}$ and $5\frac{3}{4}$.
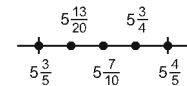

From the number line, 3 possible rational numbers are: $5\frac{7}{10}$, $5\frac{13}{20}$, $5\frac{3}{4}$

15. $\frac{3}{5}$ is close to 1 and $-\frac{19}{5}$ is close to –4.

Sketch a number line labelled from –4 to 1.

Write each fraction as a decimal before marking it on the number line. Use a calculator when necessary.

$\frac{3}{5} = \frac{6}{10} = 0.6$　　　$-\frac{5}{7} = 0.714\,285...$　　　$-\frac{8}{3} = -2.\overline{6}$　　　$-\frac{19}{5} = -3.8$
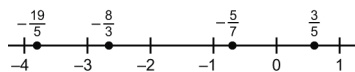
16. Strategies may vary. For example:

a) 2.34, 2.3

Both numbers are positive decimals, so compare the digits with the same place value.

Ones: Both decimals have the same digit, 2, in the ones place.

Tenths: Both decimals have the same digit, 3, in the tenths place.

Hundredths: Write 2.3 as 2.30. Since 4 is greater than 0, 2.34 is greater than 2.3.

b) –2.34, –2.3

Both numbers are negative. Use a number line from –2.4 to –2.3, divided in ten equal parts.
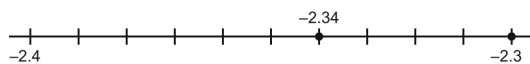
From the number line, –2.3 is to the right of –2.34. So, –2.3 is the greater number.

c) –1.4, 1.4

–1.4 and 1.4 are opposites. A positive number is always greater than a negative number.

So, 1.4 is greater than –1.4.

d) 3.96, –4.12

A positive number is always greater than a negative number.
So, 3.96 is greater than –4.12.

e) $-5.\overline{6}$, –5.6

Sketch a number line from –6 to –5. Divide the number line in ten equal parts.
Record both numbers on the number line.
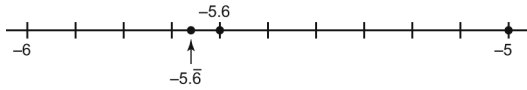
Since $-5.\overline{6}$ means –5.6666..., $-5.\overline{6}$ is to the left of –5.6 on the number line.
So, –5.6 is greater than $-5.\overline{6}$.

f) $2.8\overline{6}$, 2.866

Compare the digits beyond thousandths.
$2.8\overline{6}$ means 2.86666..., so $2.8\overline{6}$ is greater than 2.866.

17. Explanations may vary. For example:

a) $\dfrac{3}{4}$, $\dfrac{3}{5}$

I know that $\dfrac{1}{5}$ is less than $\dfrac{1}{4}$, so $\dfrac{3}{5}$ must be less than $\dfrac{3}{4}$.

I can also convert the fractions to decimals to compare:

$\dfrac{3}{4} = 0.75$ and $\dfrac{3}{5} = 0.6$

So, $\dfrac{3}{5}$ is less than $\dfrac{3}{4}$.

b) $2\dfrac{1}{2}$, $-1\dfrac{7}{8}$

A negative number is always less than a positive number. So, $-1\dfrac{7}{8}$ is less than $2\dfrac{1}{2}$.

c) $-\dfrac{13}{10}$, $-\dfrac{13}{5}$

Write the improper fractions as decimals, then compare.

$-\dfrac{13}{10} = -1.3$

$-\dfrac{13}{5} = -\dfrac{26}{10} = -2.6$

–2.6 is less than –1.3, so $-\dfrac{13}{5}$ is less than $-\dfrac{13}{10}$.

d) $-\dfrac{11}{3}$, $-\dfrac{5}{6}$

Since $-\dfrac{11}{3}$ is an improper fraction, it is less than –1, while $-\dfrac{5}{6}$ is greater than –1.

So, $-\dfrac{11}{3}$ is less than $-\dfrac{5}{6}$.

18. Strategies may vary. For example:

a) $\dfrac{3}{4}, \dfrac{6}{7}$

Use common denominators to compare.

$\dfrac{3}{4} = \dfrac{3 \times 7}{4 \times 7} = \dfrac{21}{28}, \dfrac{6}{7} = \dfrac{6 \times 4}{7 \times 4} = \dfrac{24}{28}$

Since $\dfrac{24}{28} > \dfrac{21}{28}, \dfrac{6}{7} > \dfrac{3}{4}$

So, $\dfrac{6}{7}$ is greater than $\dfrac{3}{4}$.

b) $-\dfrac{3}{4}, -\dfrac{6}{7}$

Use the equivalent fractions from part a.

Since $-\dfrac{21}{28} > -\dfrac{24}{28}, -\dfrac{3}{4} > -\dfrac{6}{7}$

So, $-\dfrac{3}{4}$ is greater than $-\dfrac{6}{7}$.

c) $-\dfrac{6}{7}, -\dfrac{7}{6}$

Since $-\dfrac{6}{7}$ is a proper fraction, it is greater than –1.

Since $-\dfrac{7}{6}$ is an improper fraction, it is less than –1.

So, $-\dfrac{6}{7}$ is greater than $-\dfrac{7}{6}$.

d) $-\dfrac{9}{5}, \dfrac{5}{9}$

A positive number is always greater than a negative number.

So, $\dfrac{5}{9}$ is greater than $-\dfrac{9}{5}$.

19. Explanations may vary. For example:
For positive numbers, the statement is always true. The lesser number is closer to 0.
Example:
6.5 < 8.35; 6.5 is closer to 0 than 8.35 is.
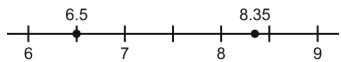
For negative numbers, the statement is never true. The lesser number is farther from 0.
Example:
–3.4 < –1.5; –3.4 is farther from 0 than –1.5 is.
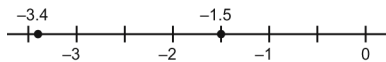
For numbers with opposite signs, the statement is sometimes true.
Example:

Consider 3 and $-\dfrac{1}{2}$.
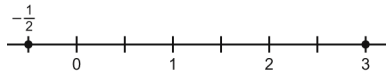
The statement is true. $-\dfrac{1}{2} < 3$ and $-\dfrac{1}{2}$ is closer to 0 than 3 is.

Consider –3 and $\dfrac{1}{2}$.
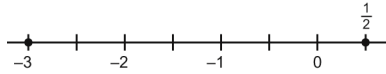
The statement is false. $-3 < \dfrac{1}{2}$; however, $\dfrac{1}{2}$ is closer to zero than –3 is.

20. a) $1.4, -\dfrac{11}{8}, -3.6, 4\dfrac{1}{3}, 0.8, -\dfrac{17}{3}$

To help place the numbers on the number line, write the fractions and mixed numbers as decimals.

$-\dfrac{11}{8} = -1.375$

$4\dfrac{1}{3} = 4.\overline{3}$

$-\dfrac{17}{3} = -5.\overline{6}$

Record each number on a number line labelled from –6 to 5.
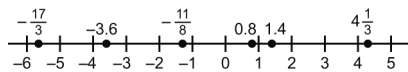
b) $-\dfrac{17}{3}, -3.6, -\dfrac{11}{8}$ are to the left of –1 on the number line.

So, $-\dfrac{17}{3}, -3.6, -\dfrac{11}{8}$ are less than –1.

c) $-\dfrac{11}{8}, 0.8, 1.4, 4\dfrac{1}{3}$ are to the right of –2 on the number line.

So, $-\dfrac{11}{8}, 0.8, 1.4, 4\dfrac{1}{3}$ are greater than –2.

d) Answers may vary. For example:

A possible rational number between $-\dfrac{17}{3} = -5.\overline{6}$ and -3.6 is -4.5.

A possible rational number between -3.6 and $-\dfrac{11}{8} = -1.375$ is $-2\dfrac{1}{6}$.

A possible rational number between $-\dfrac{11}{8} = -1.375$ and 0.8 is $\dfrac{1}{2}$.

A possible rational number between 0.8 and 1.4 is 1.25.

A possible rational number between 1.4 and $4\dfrac{1}{3}$ is $3.\overline{3}$.
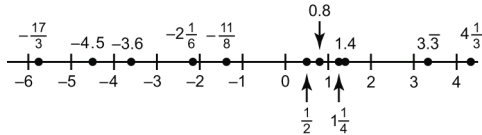
21. a) $-\dfrac{5}{7}$ ☐ $-\dfrac{4}{7}$

To order fractions with the same denominators, compare the numerators.

$-5 < -4$

So, $-\dfrac{5}{7} < -\dfrac{4}{7}$

b) $-\dfrac{5}{6}$ ☐ $-\dfrac{5}{7}$

Think: $\dfrac{1}{6}$ is greater than $\dfrac{1}{7}$.

So, $\dfrac{5}{6} > \dfrac{5}{7}$

And, $-\dfrac{5}{6} < -\dfrac{5}{7}$

c) -2.2 ☐ $-\dfrac{11}{5}$

Write $-\dfrac{11}{5}$ as a decimal.

$-\dfrac{11}{5} = -\dfrac{11 \times 2}{5 \times 2}$

So, $-2.2 = -\dfrac{11}{5}$

$= -\dfrac{22}{10}$

$= -2.2$

d) $-4.4\overline{6}$ ☐ -4.46

$-4.4\overline{6} = -4.4666...$

So, $-4.4\overline{6} < -4.46$

22. a) Distances above base camp are represented by positive numbers; distances below base camp are represented by negative numbers; the base camp is represented by 0.
Hiker A: 26.4 m above base camp, or 26.4
Hiker B: 37.2 m below base camp, or –37.2
Hiker C: 15.7 m below base camp, or –15.7

b) Sketch a vertical number line labelled from –40 to 30.
Mark each point on the number line.
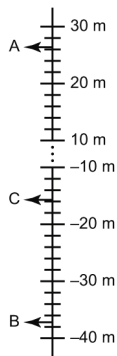
c) From the number line, –15.7 is closest to 0.
So, hiker C is closest to base camp.

d) The lowest altitude is the lowest point on the number line.
Hiker B has the lowest altitude.

23. Mark each number on a number line. For least to greatest, read the numbers from left to right.
a) 1.5, –3.5, 4, 0, –2.5, 7.5
Sketch a number line labelled from –4 to 8.
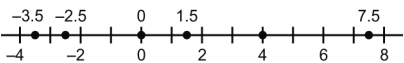
From least to greatest: –3.5, –2.5, 0, 1.5, 4, 7.5

b) –1.7, 5.9, –3.2, –0.8, 1, 4.3
Sketch a number line labelled from –4 to 6.
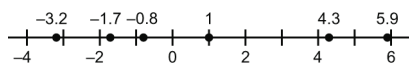
From least to greatest: –3.2, –1.7, –0.8, 1, 4.3, 5.9

c) 1.2, 2.1, –2.01, –1.2, $1.\overline{2}$, –1.22
Sketch a number line from –2 to 2, divided in fourths.
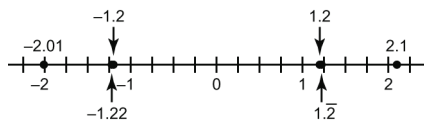
From least to greatest: –2.01, –1.22, –1.2, 1.2, $1.\overline{2}$, 2.1

d) 5.44, –5.4, –5.04, 5.$\overline{4}$, 5.04, –5.44

Sketch a number line from –5.5 to 5.5, divided in halves.
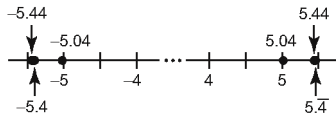
From least to greatest: –5.44, –5.4, –5.04, 5.04, 5.44, 5.$\overline{4}$

24. a) $\frac{3}{8}, -\frac{3}{4}, -\frac{1}{2}, -\frac{5}{8}, \frac{1}{4}, 0$

Sketch a number line from –1 to 1, divided in fourths.
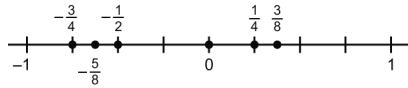
From greatest to least: $\frac{3}{8}, \frac{1}{4}, 0, -\frac{1}{2}, -\frac{5}{8}, -\frac{3}{4}$

b) $\frac{10}{9}, -\frac{5}{3}, \frac{7}{2}, -\frac{3}{2}, -\frac{7}{6}, \frac{17}{3}$

To help place each number on a number line, write each fraction as a decimal:

$\frac{10}{9} = 1.\overline{1}$ $-\frac{5}{3} = -1.\overline{6}$ $\frac{7}{2} = 3.5$ $-\frac{3}{2} = -1.5$ $-\frac{7}{6} = -1.1\overline{6}$ $\frac{17}{3} = 5.\overline{6}$

Sketch a number line labelled from –2 to 6.
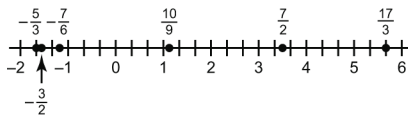
From greatest to least: $\frac{17}{3}, \frac{7}{2}, \frac{10}{9}, -\frac{7}{6}, -\frac{3}{2}, -\frac{5}{3}$

c) $-\frac{9}{5}, -\frac{17}{10}, -1\frac{1}{2}, \frac{16}{4}, -\frac{11}{4}, \frac{21}{5}$

To help place each number on a number line, write each fraction as a decimal:

$-\frac{9}{5} = -1.8$ $-\frac{17}{10} = -1.7$ $-1\frac{1}{2} = -1.5$ $\frac{16}{4} = 4$ $-\frac{11}{4} = -2.75$ $\frac{21}{5} = 4.2$

Sketch a number line labelled from –3 to 5.
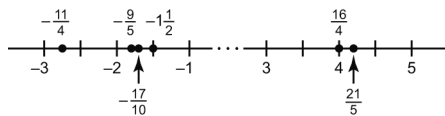
From greatest to least: $\frac{21}{5}, \frac{16}{4}, -1\frac{1}{2}, -\frac{17}{10}, -\frac{9}{5}, -\frac{11}{4}$

d) $-\dfrac{11}{2}, \dfrac{10}{3}, 2\dfrac{1}{4}, -\dfrac{8}{6}, \dfrac{7}{12}, -\dfrac{6}{4}$

Write each fraction as a decimal.

$-\dfrac{11}{2} = -5.5$ $\dfrac{10}{3} = 3.\overline{3}$ $2\dfrac{1}{4} = 2.25$ $-\dfrac{8}{6} = -1.\overline{3}$ $\dfrac{7}{12} = 0.58\overline{3}$ $-\dfrac{6}{4} = -1.5$

Sketch a number labelled from –6 to 4.
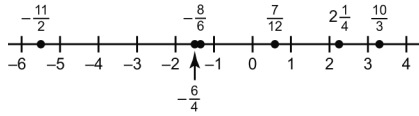
From greatest to least: $\dfrac{10}{3}, 2\dfrac{1}{4}, \dfrac{7}{12}, -\dfrac{8}{6}, -\dfrac{6}{4}, -\dfrac{11}{2}$

25. a) $3.8, \dfrac{3}{8}, -1.5, \dfrac{5}{3}, -2.3, -\dfrac{3}{2}$

Write each fractions as a decimal.

$\dfrac{3}{8} = 0.375$ $\dfrac{5}{3} = 1.\overline{6}$ $-\dfrac{3}{2} = -1.5$

Sketch a number line labelled from –3 to 4.
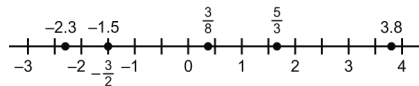
From least to greatest: $-2.3, -1.5\ (-\dfrac{3}{2}), \dfrac{3}{8}, \dfrac{5}{3}, 3.8$

b) $0.3, -0.\overline{3}, \dfrac{1}{3}, -0.3, 0.33, -3$

Write $\dfrac{1}{3}$ as a decimal: $\dfrac{1}{3} = 0.\overline{3}$

Sketch a number line labelled from –3 to 1.
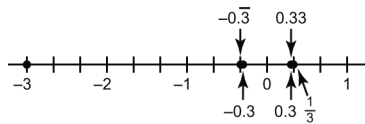
From least to greatest: $-3, -0.\overline{3}, -0.3, 0.3, 0.33, \dfrac{1}{3}$

Take It Further

26. A rational number is any number that can be written in the form $\dfrac{x}{y}$, where x and y are integers, $y \neq 0$.

Show that we can write each number in the form $\dfrac{x}{y}$.

a) $3 = \dfrac{3}{1}$

b) $-2 = \dfrac{-2}{1}$

c) $-0.5 = \dfrac{-1}{2}$

d) $-7.45 = -7\dfrac{45}{100}$

$\qquad = -\dfrac{745}{100}$

$\qquad = \dfrac{-745}{100}$

27. Any number that can be represented as a fraction is a rational number. Decimals that terminate or repeat can be expressed as fractions, so they are rational numbers.

   a) $4.\overline{21}$ is a repeating decimal, so it is a rational number.

   b) –3.121 121 112 111 12... does not appear to be a repeating decimal, so it is probably not a rational number.

   c) 2.78 is a terminating decimal, so it is a rational number.

   d) –2.122 222 22... appears to be a repeating decimal, so it is probably a rational number.

Lesson 3.2   Adding Rational Numbers   Practice (pages 111–113)

Check

3. To add a negative number on a number line, move to the left.
    To add a positive number, move to the right.
    a) 0.8 + 1.5 = 2.3

    b) 1.5 + (−0.8) = 0.7

    c) (−0.8) + (−1.5) = −2.3

    d) (−1.5) + 0.8 = −0.7

4. To add a negative number on a number line, move to the left.
    To add a positive number, move to the right.
    a) $\dfrac{1}{2}+\dfrac{5}{4}=\dfrac{7}{4}$

    b) $\left(-\dfrac{5}{4}\right)+\dfrac{1}{2}=-\dfrac{3}{4}$

    c) $\dfrac{5}{4}+\left(-\dfrac{1}{2}\right)=\dfrac{3}{4}$

    d) $\left(-\dfrac{1}{2}\right)+\left(-\dfrac{5}{4}\right)=-\dfrac{7}{4}$

5. **a) i)** 5   **ii)** 6.2

    b) i) −5   **ii)** −6.2

    c) i) −1   **ii)** −1.4

    d) i) 1   **ii)** 1.4

6. (−2.3) + (−1.9)
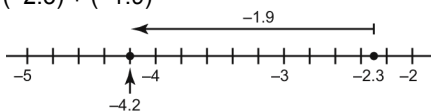
    From the number line, (−2.3) + (−1.9) = −4.2

    a) 2.3 + 1.9
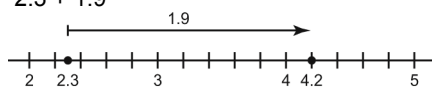
    From the number line, 2.3 + 1.9 = 4.2

    b) (−2.3) + 1.9
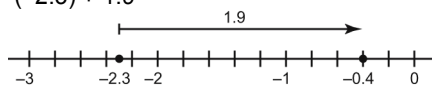
    From the number line, (−2.3) + 1.9 = −0.4

c) −1.9 + (−2.3)
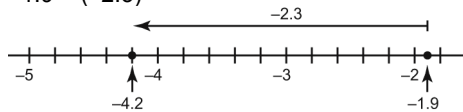
From the number line, −1.9 + (−2.3) = −4.2

d) −2.3 + (−1.9)
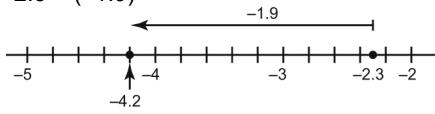
From the number line, −2.3 + (−1.9) = −4.2

The expressions in parts c and d have the same sum as −2.3 + (−1.9): −4.2

7. Strategies may vary. For example:

For parts a to d, use the result in each part i to determine the sum in part ii.

a) i) 9 + 3 = 12   **ii)** $\dfrac{9}{2} + \dfrac{3}{2} = \dfrac{12}{2} = 6$

b) i) −9 + (−3) = −12   **ii)** $-\dfrac{9}{2} + \left(-\dfrac{3}{2}\right) = -\dfrac{12}{2} = -6$

c) i) −9 + 3 = −6   **ii)** $-\dfrac{9}{2} + \dfrac{3}{2} = -\dfrac{6}{2} = -3$

d) i) 9 + (−3) = 6   **ii)** $\dfrac{9}{2} + \left(-\dfrac{3}{2}\right) = \dfrac{6}{2} = 3$

8. $-\dfrac{3}{4} + \dfrac{7}{8}$
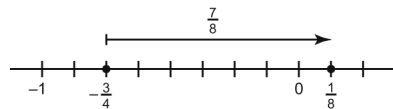
From the number line, $-\dfrac{3}{4} + \dfrac{7}{8} = \dfrac{1}{8}$

a) $-\dfrac{3}{4} + \left(-\dfrac{7}{8}\right)$
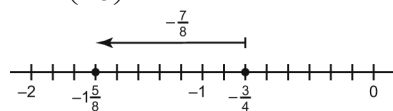
From the number line, $-\dfrac{3}{4} + \left(-\dfrac{7}{8}\right) = -1\dfrac{5}{8}$

b) $-\dfrac{7}{8} + \dfrac{3}{4}$
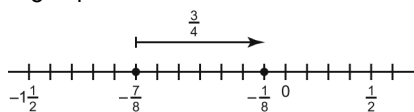
From the number line, $-\dfrac{7}{8} + \dfrac{3}{4} = -\dfrac{1}{8}$

c) $\dfrac{7}{8} + \left(-\dfrac{3}{4}\right)$
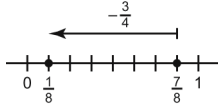
From the number line, $\dfrac{7}{8} + \left(-\dfrac{3}{4}\right) = \dfrac{1}{8}$

d) $\dfrac{7}{8} + \dfrac{3}{4}$
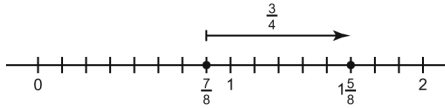
From the number line, $\dfrac{7}{8} + \dfrac{3}{4} = 1\dfrac{5}{8}$.

The expression in part c has the same sum as $-\dfrac{3}{4} + \dfrac{7}{8} : \dfrac{1}{8}$

Apply

9.   a) Estimate: –6 + 3 = –3
    –5.6 + 3.2 = –2.4

   b) Estimate: 8 + (–5) = 3
    7.95 + (–4.51) = 3.44

   c) Estimate: 0 + (–33) = –33
    –0.325 + (–32.5) = –32.825

   d) Estimate: –124 + 27 = –97
    –123.5 + 27.45 = –96.05

   e) Estimate: 82 + 100 = 182
    82.001 +100.28 = 182.281

   f) Estimate: –18 + 0 = –18
    –17.84 + (–0.098) = –17.938

10. Explanations may vary. For example:
Visualize a number line.

When we add a positive number, we move to the right. The sum of two positive rational numbers is always greater than the numbers added. For example: 2.45 + 3.27 = 5.72; 5.72 is greater than 2.45 and 3.27.

When we add a negative number, we move to the left. The sum of a positive rational number and a negative rational number is less than the positive number, but greater than the negative number. For example: 9.03 + (–24.56) = –15.53; –15.53 is less than 9.03, but greater than –24.56.

The sum of two negative rational numbers is always less than both numbers.
For example: (–1.5) + (–3.6) = –5.1. –5.1 is less than both –1.5 and –3.6.

11. Strategies may vary.

a) $-\dfrac{2}{3}+\dfrac{1}{2}=-\dfrac{4}{6}+\dfrac{3}{6}$

$=-\dfrac{1}{6}$

b) $\dfrac{4}{5}+\left(-\dfrac{1}{3}\right)=\dfrac{12}{15}+\left(-\dfrac{5}{15}\right)$

$=\dfrac{12-5}{15}$

$=\dfrac{7}{15}$

c) $-\dfrac{11}{4}+\left(-\dfrac{6}{5}\right)=-\dfrac{55}{20}+\left(-\dfrac{24}{20}\right)$

$=\dfrac{-55-24}{20}$

$=-\dfrac{79}{20},\text{ or }-3\dfrac{19}{20}$

d) $\dfrac{13}{5}+\dfrac{9}{2}=\dfrac{26}{10}+\dfrac{45}{10}$

$=\dfrac{26+45}{10}$

$=\dfrac{71}{10},\text{ or }7\dfrac{1}{10}$

e) $-2\dfrac{1}{3}+\left(-1\dfrac{3}{4}\right)$

Add the whole numbers and add the fractions.

$-2\dfrac{1}{3}+\left(-1\dfrac{3}{4}\right)=(-2)+\left(-\dfrac{1}{3}\right)+(-1)+\left(-\dfrac{3}{4}\right)$

$=\left[-2-1\right]+\left[-\dfrac{1}{3}+\left(-\dfrac{3}{4}\right)\right]$

$=(-3)+\left[-\dfrac{4}{12}+\left(-\dfrac{9}{12}\right)\right]$

$=(-3)+\left[\dfrac{-4-9}{12}\right]$

$=(-3)+\left[\dfrac{-13}{12}\right]$

$=(-3)+\left[-1\dfrac{1}{12}\right]$

$=-4\dfrac{1}{12}$

f) $\dfrac{9}{5}+\left(-\dfrac{17}{6}\right)$

Use equivalent fractions with denominator 30.

$$\dfrac{9}{5}+\left(-\dfrac{17}{6}\right)=\dfrac{54}{30}+\left(-\dfrac{85}{30}\right)$$

$$=\dfrac{54-85}{30}$$

$$=\dfrac{-31}{30},\text{ or }-1\dfrac{1}{30}$$

g) $-3\dfrac{3}{4}+4\dfrac{5}{8}$

Add the whole numbers and add the fractions.

$$-3\dfrac{3}{4}+4\dfrac{5}{8}=(-3)+\left(-\dfrac{3}{4}\right)+4+\dfrac{5}{8}$$

$$=[-3+4]+\left[-\dfrac{3}{4}+\dfrac{5}{8}\right]$$

$$=1+\left[-\dfrac{6}{8}+\dfrac{5}{8}\right]$$

$$=1+\left(-\dfrac{1}{8}\right)$$

$$=\dfrac{8}{8}-\dfrac{1}{8}$$

$$=\dfrac{8-1}{8}$$

$$=\dfrac{7}{8}$$

h) $1\dfrac{5}{6}+\left(-5\dfrac{2}{3}\right)$

Add the whole numbers and add the fractions.

$$1\dfrac{5}{6}+\left(-5\dfrac{2}{3}\right)=1+\dfrac{5}{6}+(-5)+\left(-\dfrac{2}{3}\right)$$

$$=[1-5]+\left[\dfrac{5}{6}+\left(-\dfrac{2}{3}\right)\right]$$

$$=(-4)+\left[\dfrac{5}{6}+\left(-\dfrac{4}{6}\right)\right]$$

$$=(-4)+\left[\dfrac{1}{6}\right]$$

$$=-\dfrac{24}{6}+\dfrac{1}{6}$$

$$=\dfrac{-24+1}{6}$$

$$=-\dfrac{23}{6},\text{ or }-3\dfrac{5}{6}$$

i) $-3\frac{1}{4}+\left(-2\frac{1}{6}\right)$

Add the whole numbers and add the fractions.

$$-3\frac{1}{4}+\left(-2\frac{1}{6}\right)=(-3)+\left(-\frac{1}{4}\right)+(-2)+\left(-\frac{1}{6}\right)$$

$$=(-5)+\left[\left(-\frac{1}{4}\right)+\left(-\frac{1}{6}\right)\right]$$

$$=(-5)+\left[\left(-\frac{3}{12}\right)+\left(-\frac{2}{12}\right)\right]$$

$$=(-5)+\left(-\frac{5}{12}\right)$$

$$=-5\frac{5}{12}$$

j) $2\frac{3}{5}+\left(-1\frac{7}{8}\right)$

Add the whole numbers and add the fractions.

$$2\frac{3}{5}+\left(-1\frac{7}{8}\right)=2+\frac{3}{5}+(-1)+\left(-\frac{7}{8}\right)$$

$$=1+\left[\frac{3}{5}+\left(-\frac{7}{8}\right)\right]$$

$$=1+\left[\frac{24}{40}+\left(-\frac{35}{40}\right)\right]$$

$$=1+\left(-\frac{11}{40}\right)$$

$$=\frac{40}{40}+\left(-\frac{11}{40}\right)$$

$$=\frac{29}{40}$$

12. Explanations may vary. For example:
Visualize a number line.

a) When we add a positive number, we move to the right.
The sum of two positive rational numbers is always greater than the numbers added.
For example: 1.5 + 3.6 = 5.1; 5.1 > 0. So, the sum of two positive rational numbers is positive.

b) When we add a negative number, we move to the left.
The sum of two negative rational numbers is always negative and less than both numbers.
For example: (–1.5) + (–3.6) = –5.1; –5.1 < 0. So, the sum of two negative rational numbers is negative.

c) For example: –1.5 + 3.6 = 2.1; 2.1 > 0
When the positive number is farther from 0 than the negative rational number is, the sum is positive.

For example: 1.5 + (–3.6) = –2.1; –2.1 <0
When the negative number is farther from 0 than the positive rational number is, the sum is negative.

When the rational numbers are opposites, their sum is 0.
For example: (–5.3) + 5.3 = 0

13. a) Owing or borrowing money can be represented with negative numbers.
Zoe owes $36.25; this can be represented as –36.25.
Then, she borrows another $25.35; this can be represented as –25.35.

b) i) In dollars, Zoe owes: (–36.25) + (–25.35)

ii) To determine the total amount owed, calculate: (–36.25) + (–25.35) = –61.60
Zoe owes $61.60.

c) i) In dollars, Zoe owes: $(-61.60) + 14.75$

ii) To determine the amount owed after Zoe paid back $14.75, calculate: $(-61.60) + 14.75 = -46.85$
Zoe now owes $46.85.

14. a) –0.61 + 0.23
Less than 0 since –0.61 is farther from 0 than 0.23 is; –0.61 + 0.23 = –0.38

b) 12.94 + (–12.56)
Greater than 0 since 12.94 is farther from 0 than –12.56 is;
12.94 + (–12.56) = 0.38

c) $-\dfrac{7}{3} + \dfrac{17}{5}$

Greater than 0 since $\dfrac{17}{5}$ is farther from 0 than $-\dfrac{7}{3}$ is;

$$-\frac{7}{3} + \frac{17}{5} = -\frac{35}{15} + \frac{51}{15}$$
$$= \frac{16}{15}, \text{ or } 1\frac{1}{15}$$

d) $\dfrac{7}{4} + \left(-\dfrac{6}{5}\right)$

Greater than 0 since $\dfrac{7}{4}$ is farther from 0 than $-\dfrac{6}{5}$ is;

$$\frac{7}{4} + \left(-\frac{6}{5}\right) = \frac{35}{20} + \left(-\frac{24}{20}\right)$$
$$= \frac{11}{20}$$

15. a) Use a rational number to represent each temperature: –13.4; 5.7
To determine the temperature at noon, add: –13.4 + 5.7 = –7.7, or –7.7°C

b) To determine the lowest temperature on Wednesday, add: –13.4 + (–3.7) = –17.1, or –17.1°C

c) Sketch a vertical number line labelled from 0 to –20. Mark both changes in temperature:
from –13.4 to –7.7 and from –13.4 to –17.1
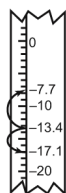
16. Explanations may vary. For example:
To determine which sum is greater without adding, compare the numbers in each expression.

  a) i) Think: Which number is farther to 0, –9.23 or 3.46?
     –9.23 is farther from 0.
     So, the sum in part i is negative.

    ii) Think: Which number is farther from 0, 9.23 or –3.46?
     9.23 is farther from 0.
     So, the sum in part ii is positive.
     The sum in part ii is greater.

    Check: **i)** –9.23 + 3.46 = –5.77
                ii) 9.23 + (–3.46) = 5.77

  b) The sum in part i is negative, because it is the sum of 2 negative numbers.
    The sum in part ii is a sum of a negative number and a positive number, where the positive number is
    farther from 0 than the negative number is. The sum in part ii is positive.
    So, the sum in part ii is greater.

    Check:

    i) $-\dfrac{2}{3}+\left(-\dfrac{3}{4}\right)=-\dfrac{8}{12}+\left(-\dfrac{9}{12}\right)$

                    $=-\dfrac{17}{12}$

    ii) $-\dfrac{2}{3}+\dfrac{3}{4}=-\dfrac{8}{12}+\dfrac{9}{12}$

                $=\dfrac{1}{12}$

17. a) Earnings can be represented with positive numbers, and expenses can be represented with negative
    numbers. Since Keith earned $45.50 and $22.25, these can be represented as 45.50 and 22.25. Since
    Keith spent $15.77 and $33.10, these can be represented as –15.77 and –33.10.

  b) At the end of January, Keith's balance in dollars is: 45.50 + 22.25 + (–15.77) + (–33.10)

  c) Keith's balance in dollars is: 45.50 + 22.25 + (–15.77) + (–33.10) = 18.88
    Keith's balance is $18.88.

18. Income can be represented with positive numbers, and expenses can be represented with negative
    numbers.
    $2115.70+2570.40+(-545.50)+(-978.44)+(-888.00)+(-2540.20)=-266.04$

    Since Lucille's balance is negative, she lost $266.04. Lucille's business did not make a profit.

19. Answers may vary. For example:
  a) 5.6 + ☐ ≤ 9.1

    Investigate: What do I add to 5.6 to get less than or equal to 9.1?
    Write and solve the corresponding addition equation:
    5.6 + ☐ = 9.1
    Then, ☐ = 9.1 – 5.6, or 3.5
    Any rational number less than or equal to 3.5 will make the sentence true.

b) $11.8 + (-\boxed{}) \le 23.4$

Investigate: What do I add to 11.8 to get less than or equal to 23.4?
Write and solve the corresponding addition equation:
$11.8 + (-\boxed{}) = 23.4$
Then, $\boxed{} = -(23.4 - 11.8)$, or -11.6
Any number greater than or equal to –11.6 will make the sentence true.

c) $-7.2 + \boxed{} \ge 7.2$

Investigate: What do I add to –7.2 to get greater than or equal to 7.2?
Write and solve the corresponding addition equation:
$-7.2 + \boxed{} = 7.2$
Then, $\boxed{} = 7.2 + 7.2$, or 14.4
Any number greater than or equal to 14.4 will make the sentence true.

d) $-7.2 + \boxed{} \le 7.2$

Investigate: What do I add to –7.2 to get less than or equal to 7.2?
From part c, we add 14.2 to –7.2 to get 7.2.
Any number less than or equal to 14.4 will make the sentence true.

Take It Further

20. **a)** Use a number line. Mark $-\dfrac{3}{4}$ and $\dfrac{7}{8}$ on the number line. To get from $-\dfrac{3}{4}$ to $\dfrac{7}{8}$, move 13 eighths to the

right. So, the number is $\dfrac{13}{8} = 1\dfrac{5}{8}$.
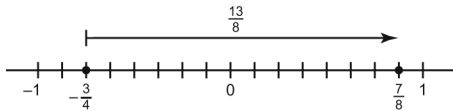
b) Investigate: If I have to add $\dfrac{4}{5}$ to a number to get $-\dfrac{2}{3}$, the number I'm looking for must be

$\dfrac{4}{5}$ less than $-\dfrac{2}{3}$. So, $-\dfrac{2}{3} + \left(-\dfrac{4}{5}\right) = -\dfrac{22}{15}$ and $-\dfrac{22}{15} + \dfrac{4}{5} = -\dfrac{2}{3}$

So, the number is $-\dfrac{22}{15}$, or $-1\dfrac{7}{15}$.

c) Use a number line. Mark $3\dfrac{1}{8}$ on the number line.

Investigate: Where do I start so that when I add $-\dfrac{5}{2}$ (move 5 halves to the left), I get $3\dfrac{1}{8}$?

I must start at a point that is $\dfrac{5}{2}$ to the right of $3\dfrac{1}{8}$; that is, $3\dfrac{1}{8} + \dfrac{5}{2}$

$$= 3\dfrac{1}{8} + 2\dfrac{1}{2}$$
$$= 5\dfrac{5}{8}$$

So, the number is $5\dfrac{5}{8}$.
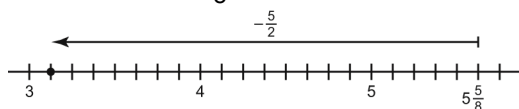
d) Use a number line. Mark $\dfrac{7}{3}$ and $-\dfrac{5}{4}$ on the number line.

Investigate: What do I add to $\dfrac{7}{3}$ to get $-\dfrac{5}{4}$?

To get from $\dfrac{7}{3}$ to $-\dfrac{5}{4}$, move 43 twelfths to the left.

So, the number is $-\dfrac{43}{12}$ $-\dfrac{43}{12}$, or $-3\dfrac{7}{12}$.
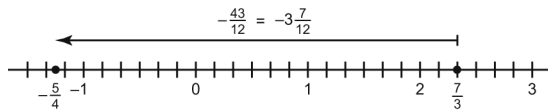
21. $7.9 + \boxed{} \leq 11.2$

Investigate: What do I add to 7.9 to get less than or equal to 11.2?

Write and solve the corresponding addition equation: $7.9 + \boxed{} = 11.2$

Since $7.9 + 3.3 = 11.2$, any number less than or equal to 3.3 will make the expression true.

22. Strategies may vary. For example:

Use what you know about adding rational numbers.

The possible negative fractions using –1, –2, –3, –4, 1, 2, 3, 4 are:

$\dfrac{-1}{1}, \dfrac{-1}{2}, \dfrac{-1}{3}, \dfrac{-1}{4}$ or $\dfrac{1}{-1}, \dfrac{1}{-2}, \dfrac{1}{-3}, \dfrac{1}{-4}$

$\dfrac{-2}{1}, \dfrac{-2}{2}, \dfrac{-2}{3}, \dfrac{-2}{4}$ or $\dfrac{2}{-1}, \dfrac{2}{-2}, \dfrac{2}{-3}, \dfrac{2}{-4}$

$\dfrac{-3}{1}, \dfrac{-3}{2}, \dfrac{-3}{3}, \dfrac{-3}{4}$ or $\dfrac{3}{-1}, \dfrac{3}{-2}, \dfrac{3}{-3}, \dfrac{3}{-4}$

$\dfrac{-4}{1}, \dfrac{-4}{2}, \dfrac{-4}{3}, \dfrac{-4}{4}$ or $\dfrac{4}{-1}, \dfrac{4}{-2}, \dfrac{4}{-3}, \dfrac{4}{-4}$

The possible positive fractions using –1, –2, –3, –4, 1, 2, 3, 4 are:

$\dfrac{-1}{-2}, \dfrac{-1}{-3}, \dfrac{-1}{-4}$ or $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}$

$\dfrac{-2}{-1}, \dfrac{-2}{-3}, \dfrac{-2}{-4}$ or $\dfrac{2}{1}, \dfrac{2}{3}, \dfrac{2}{4}$

$\dfrac{-3}{-1}, \dfrac{-3}{-2}, \dfrac{-3}{-4}$ or $\dfrac{3}{1}, \dfrac{3}{2}, \dfrac{3}{4}$

$\dfrac{-4}{-1}, \dfrac{-4}{-2}, \dfrac{-4}{-3}$ or $\dfrac{4}{1}, \dfrac{4}{2}, \dfrac{4}{3}, \dfrac{4}{4}$

Since the denominators are 1, 2, 3, or 4, the rational number closer to 0 will have a denominator of 12.

Then, the greatest possible sum less than 0 is $-\dfrac{1}{12}$. Use guess and test to determine the fractions.

Possible addition statements are: $\dfrac{2}{3} + \left(-\dfrac{3}{4}\right) = -\dfrac{1}{12}$ and $\left(-\dfrac{1}{3}\right) + \dfrac{1}{4} = -\dfrac{1}{12}$

Lesson 3.3   **Subtracting Rational Numbers**   **Practice (pages 119–120)**

Check

3. **a) i)** $5 - 3 = 2$

    ii) $5.1 - 3.3 = 1.8$

    b) i) $-5 - 3 = -8$

    ii) $-5.1 - 3.3 = -8.4$

    c) i) $-3 - (-5) = 2$

    ii) $-3 - (-5.1) = 1.8$

    d) i) $3 - 5 = -2$

    ii) $3.3 - 5.1 = -1.8$

4. I think of a number line.
    I start at –7.2 and move 1.8 to the left to reach –9; so –7.2 –1.8 = –9.

    a) $7.2 - 1.8$
    I start at 7.2 and move 1.8 to the left, so the answer is positive and cannot be the same as –7.2 – 1.8.

    b) $-7.2 - (-1.8)$
    I start at –7.2 and move 1.8 to the right, so the answer cannot be the same as –7.2 – 1.8.

    c) $1.8 - (-7.2)$
    I start at 1.8 and move 7.2 to the right, so the answer is positive and cannot be the same as –7.2 – 1.8.

    d) $-1.8 - 7.2$
    I start at –1.8 and move 7.2 to the left; this is the same as starting at –7.2 and moving 1.8 to the left.
    So, –1.8 – 7.2 = –7.2 – 1.8 = –9

5. Strategies may vary. For example: The fractions in part ii have the same denominator. To subtract the fractions, write the denominator and subtract the numerators only.
    Use the answers in part i to help check your work in part ii.

    a) i) $11 - 2 = 9$

    ii) $\dfrac{11}{5} - \dfrac{2}{5} = \dfrac{11-2}{5}$

    $$= \dfrac{9}{5}, \text{ or } 1\dfrac{4}{5}$$

    b) i) $-11 - 2 = -13$

    ii) $-\dfrac{11}{5} - \dfrac{2}{5} = \dfrac{-11-2}{5}$

    $$= -\dfrac{13}{5}, \text{ or } -2\dfrac{3}{5}$$

c) i) 11 − (−2) = 11 + 2, or 13

ii) $\dfrac{11}{5} - \left(-\dfrac{2}{5}\right) = \dfrac{11-(-2)}{5}$

$= \dfrac{11+2}{5}$

$= \dfrac{13}{5}$, or $2\dfrac{3}{5}$

d) i) 2 − (−11) = 2 + 11, or 13

ii) $\dfrac{2}{5} - \left(-\dfrac{11}{5}\right) = \dfrac{2-(-11)}{5}$

$= \dfrac{2+11}{5}$

$= \dfrac{13}{5}$, or $2\dfrac{3}{5}$

Apply

6. Use equivalent fractions.

$-\dfrac{3}{10} - \dfrac{9}{5} = -\dfrac{3}{10} - \dfrac{18}{10}$

$= -\dfrac{21}{10}$

a) $-\dfrac{3}{10} - \left(-\dfrac{9}{5}\right)$

Subtracting a negative number is the same as adding the opposite (positive) number.

So, $-\dfrac{3}{10} - \left(-\dfrac{9}{5}\right)$ does not have the same answer as $-\dfrac{3}{10} - \dfrac{9}{5}$.

$-\dfrac{3}{10} - \left(-\dfrac{9}{5}\right) = -\dfrac{3}{10} - \left(-\dfrac{18}{10}\right)$

$= -\dfrac{3}{10} + \dfrac{18}{10}$

$= \dfrac{15}{10}$

b) $\dfrac{3}{10} - \dfrac{9}{5}$

The expression is the difference of two positive rational numbers.

So, $\dfrac{3}{10} - \dfrac{9}{5}$ does not have the same answer as $-\dfrac{3}{10} - \dfrac{9}{5}$.

$\dfrac{3}{10} - \dfrac{9}{5} = \dfrac{3}{10} - \dfrac{18}{10}$

$= -\dfrac{15}{10}$

c) $-\dfrac{9}{5} - \dfrac{3}{10}$

Since $-\dfrac{9}{5} - \dfrac{3}{10}$ is equivalent to $-\dfrac{3}{10} - \dfrac{9}{5}$, $-\dfrac{9}{5} - \dfrac{3}{10}$ has the same answer as $-\dfrac{3}{10} - \dfrac{9}{5}$.

$-\dfrac{9}{5} - \dfrac{3}{10} = -\dfrac{18}{10} - \dfrac{3}{10}$

$\qquad\qquad = -\dfrac{21}{10}$

d) $\dfrac{9}{5} - \dfrac{3}{10}$

The expression is the difference of two positive rational numbers.

So, $\dfrac{9}{5} - \dfrac{3}{10}$ does not have the same answer as $-\dfrac{3}{10} - \dfrac{9}{5}$.

$\dfrac{9}{5} - \dfrac{3}{10} = \dfrac{18}{10} - \dfrac{3}{10}$

$\qquad\qquad = \dfrac{15}{10}$

7. a) $10.8 - 3.5$
   Estimate: $11 - 4 = 7$
   Difference: $10.8 - 3.5 = 7.3$

   b) $-37.23 - 48.54$
   Estimate: $-37 - 49 = -86$
   Difference: $-37.23 - 48.54 = -85.77$

   c) $50.06 - (-14.67)$
   Estimate: $50 - (-15) = 65$
   Difference: $50.06 - (-14.67) = 64.73$

   d) $64.19 - 95.76$
   Estimate: $64 - 96 = -32$
   Difference: $64.19 - 95.76 = -31.57$

   e) $-28.31 - 9.72$
   Estimate: $-28 - 10 = -38$
   Difference: $-28.31 - 9.72 = -38.03$

   f) $70.59 - (-81.25)$
   Estimate: $71 - (-81) = 152$
   Difference: $70.59 - (-81.25) = 151.84$

8. a) In degrees Celsius, the difference could be:
   $-28.5 - (-33.1) = 4.6$
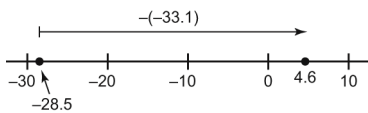
or, in degrees Celsius, the difference could be:

$-(-33.1) - (-28.5) = -4.6$
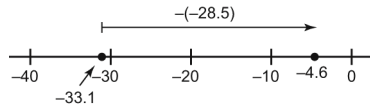
b) There are two possible answers depending on the order in which we subtract the temperatures.

9. Strategies may vary.
For example: Write equivalent fractions with a common denominator.

a) $\dfrac{17}{3} - \dfrac{19}{2} = \dfrac{34}{6} - \dfrac{57}{6}$

$= \dfrac{34 - 57}{6}$

$= -\dfrac{23}{6}$

$= -3\dfrac{5}{6}$

b) Write equivalent fractions with a common denominator.

$-\dfrac{13}{5} - \dfrac{7}{3} = -\dfrac{39}{15} - \dfrac{35}{15}$

$= \dfrac{-39 - 35}{15}$

$= -\dfrac{74}{15}$

$= -4\dfrac{14}{15}$

c) Write each mixed number as an improper fraction.

$1\dfrac{5}{6} - 6\dfrac{3}{4} = \dfrac{11}{6} - \dfrac{27}{4}$

$= \dfrac{22}{12} - \dfrac{81}{12}$

$= \dfrac{22 - 81}{12}$

$= \dfrac{-59}{12}$

$= -4\dfrac{11}{12}$

d) Write equivalent fractions with a common denominator.

$-\dfrac{19}{6} - \dfrac{7}{8} = -\dfrac{76}{24} - \dfrac{21}{24}$

$= \dfrac{-76 - 21}{24}$

$= -\dfrac{97}{24}$

$= -4\dfrac{1}{24}$

e) Write equivalent fractions with a common denominator.

$$\frac{15}{4} - \frac{5}{12} = \frac{45}{12} - \frac{5}{12}$$
$$= \frac{45 - 5}{12}$$
$$= \frac{40}{12}$$
$$= \frac{10}{3}$$
$$= 3\frac{1}{3}$$

f) Write each mixed number as an improper fraction.

$$-2\frac{1}{8} - \left(-4\frac{1}{3}\right) = -\frac{17}{8} - \left(-\frac{13}{3}\right)$$
$$= -\frac{51}{24} - \left(-\frac{104}{24}\right)$$
$$= \frac{-51 + 104}{24}$$
$$= \frac{53}{24}, \text{ or } 2\frac{5}{24}$$

10. Explanations may vary. For example:

Choose 4 rational numbers: 1.5, 3.5, –1.5, and –3.5; and their fraction forms: $\frac{3}{2}$, $\frac{7}{2}$, $-\frac{3}{2}$, and $-\frac{7}{2}$.

Use 2 of these rational numbers to write as many different subtraction expressions as possible:

Expression	Difference
3.5 – 1.5 = 2.0 $\frac{7}{2} - \frac{3}{2} = 2$	The difference is greater than one of the numbers.
3.5 – (–1.5) = 5.0 $\frac{7}{2} - \left(-\frac{3}{2}\right) = 5$	Since subtracting a negative number is the same as adding a positive number, the difference will be greater than both the numbers you started with.
–3.5 – 1.5 = –5.0 $-\frac{7}{2} - \frac{3}{2} = -5$	The difference of a negative number and a positive number is less than both numbers subtracted.
–3.5 – (–1.5) = –2.0 $-\frac{7}{2} - \left(-\frac{3}{2}\right) = -2$	The difference is greater than one of the numbers.
1.5 – (–3.5) = 5.0 $\frac{3}{2} - \left(-\frac{7}{2}\right) = 5$	Since subtracting a negative number is the same as adding a positive number, the difference is greater than both the numbers you started with.
1.5 – 3.5 = –2.0 $\frac{3}{2} - \frac{7}{2} = -2$	The difference is less than both positive rational numbers.
–1.5 – 3.5 = –5.0 $-\frac{3}{2} - \frac{7}{2} = -5$	The difference of a positive number and a negative number is less than both numbers you started with.
–1.5 – (–3.5) = 2.0 $-\frac{3}{2} - \left(-\frac{7}{2}\right) = 2$	The difference is greater than both negative rational numbers.

So, I got a difference greater than both the numbers I subtracted when I subtracted a negative number from a positive number, or when I subtracted a negative number from a negative number, if the number I subtracted from was closer to 0 than the number I subtracted was.

11. a) A distance measured above the sea level is positive.
A distance measured below the sea level is negative.
Sea level is represented as 0. Then, the distance in metres below sea level of the shore of the Dead Sea is –417.5. The altitude in metres of the peak of Mt. Everest is 8844.43.

b) The distance between the highest point and the lowest point represents the difference in heights:
8844.43 – (–417.5) = 9261.93
The distance is 9261.93 m.

12. Strategies may vary. For example:
a) –3.86 – 41.12
Subtracting a positive number from a negative number will result in a negative number. The difference is negative.
–3.86 – 41.12 = –44.98

b) 1.32 – (–5.79)
Subtracting a negative number is the same as adding a positive number. Since we start with a positive number and add a positive number to it, the result will be positive. The difference is positive.
1.32 – (–5.79) = 7.11

c) $-\dfrac{5}{4} - \left(-\dfrac{7}{2}\right)$

Subtracting $-\dfrac{7}{2}$ from $-\dfrac{5}{4}$ gives the same result as adding $\dfrac{7}{2}$ to $-\dfrac{5}{4}$.

Since $\dfrac{7}{2}$ is farther from 0 than $-\dfrac{5}{4}$, the result will be positive. The difference is positive.

$$-\dfrac{5}{4} - \left(-\dfrac{7}{2}\right) = -\dfrac{5}{4} - \left(-\dfrac{14}{4}\right)$$
$$= \dfrac{-5+14}{4}$$
$$= \dfrac{9}{4}, \text{or } 2\dfrac{1}{4}$$

d) $-\dfrac{23}{5} - \dfrac{5}{3}$

Subtracting a positive number from a negative number results in a negative number. The difference is negative.

$$-\dfrac{23}{5} - \dfrac{5}{3} = -\dfrac{69}{15} - \dfrac{25}{15}$$
$$= \dfrac{-69-25}{15}$$
$$= -\dfrac{94}{15}, \text{or } -6\dfrac{4}{15}$$

13. Strategies may vary. For example:

a) $\dfrac{3}{5} - \left(-\dfrac{1}{2}\right) + \dfrac{2}{3}$ Write equivalent fractions with the common denominator 30.

$\dfrac{3}{5} = \dfrac{18}{30}$ $-\dfrac{1}{2} = -\dfrac{15}{30}$ $\dfrac{2}{3} = \dfrac{20}{30}$

Then,

$$\dfrac{3}{5} - \left(-\dfrac{1}{2}\right) + \dfrac{2}{3} = \dfrac{18}{30} - \left(-\dfrac{15}{30}\right) + \dfrac{20}{30}$$
$$= \dfrac{18 - (-15) + 20}{30}$$
$$= \dfrac{18 + 15 + 20}{30}$$
$$= \dfrac{53}{30}, \text{ or } 1\dfrac{23}{30}$$

b) $-2.34 + 8.6 + (-5.71)$

Add the first 2 terms: $-2.34 + 8.6 = 6.26$

Then add (-5.71) to the calculated sum: $6.26 + (-5.71) = 0.55$

c) $-\dfrac{16}{5} - \left(-\dfrac{14}{3}\right) + \dfrac{13}{4}$ Write equivalent fractions with the common denominator 60.

$\dfrac{16}{5} = \dfrac{192}{60}$ $-\dfrac{14}{3} = -\dfrac{280}{60}$ $\dfrac{13}{4} = \dfrac{195}{60}$

$$-\dfrac{16}{5} - \left(-\dfrac{14}{3}\right) + \dfrac{13}{4} = -\dfrac{192}{60} - \left(-\dfrac{280}{60}\right) + \dfrac{195}{60}$$
$$= \dfrac{-192 - (-280) + 195}{60}$$
$$= \dfrac{-192 + 280 + 195}{60}$$
$$= \dfrac{283}{60}, \text{ or } 4\dfrac{43}{60}$$

d) $23.5 + (-12.61) - 3.2$

Add the first 2 terms: $23.5 + (-12.61) = 10.89$

Then subtract 3.2 from the calculated sum: $10.89 - 3.2 = 7.69$

14. a) $-1.2 - \boxed{} \le 3.7$

Investigate: What do I subtract from -1.2 to get a rational number less than or equal to 3.7?

Write and solve the corresponding subtraction equation:

$-1.2 - \boxed{} = 3.7$

Use a number line. To move from -1.2 to 3.7, we move 4.9 units to the right.
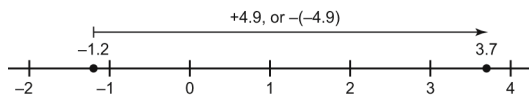
Then, $\boxed{} = -4.9$

Any number greater than or equal to -4.9 will make the statement true.

Use a calculator to check the answer. Substitute: $\boxed{} = -4.8$

$-1.2 - (-4.8) = 3.6$, which is less than 3.7

b) $4.3 - \boxed{} \geq 8.9$

Investigate: What do I subtract from 4.3 to get at least 8.9?
Write and solve the corresponding subtraction equation:
$4.3 - \boxed{} = 8.9$

Use a number line. To move from 4.3 to 8.9, we move 4.6 units to the right.
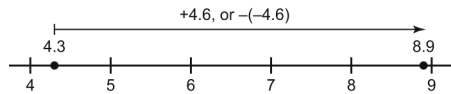
Then, $\boxed{} = -4.6$
Any number less than or equal to –4.6 will make the statement true.
Use a calculator to check the answer. Substitute: $\boxed{} = -4.7$
4.3 – (–4.7) = 9, which is greater than 8.9

c) $\boxed{} - 2.9 \geq 5.3$

Investigate: What do I subtract 2.9 from to get at least 5.3?
Write and solve the corresponding subtraction equation:
$\boxed{} - 2.9 = 5.3$
Then, $\boxed{} = 5.3 + 2.9$, or 8.2
Any number greater than or equal to 8.2 will make the statement true.
Use a calculator to check the answer. Substitute: $\boxed{} = 8.3$
8.3 – 2.9 = 5.4, which is greater than 5.3

d) $\boxed{} - 7.2 \leq -10.9$

Investigate: What do I subtract 7.2 from to get a rational number less than or equal to –10.9?
Write and solve the corresponding subtraction equation:
$\boxed{} - 7.2 = -10.9$
Then, $\boxed{} = -10.9 + 7.2$, or –3.7
Any number less than or equal to –3.7 will make the statement true.
Use a calculator to check the answer. Substitute: $\boxed{} = -3.8$
–3.8 – 7.2 = –11.0, which is less than –10.9

Take It Further

15. To determine the missing numbers in parts a and c, rewrite the statements as addition statements.
To determine the missing numbers in parts b, d, e, and f, rewrite the questions as related subtraction statements.

a) To determine the missing number, add 37.3 + 28.4. The missing number is 65.7.

b) To determine the missing number, subtract $\dfrac{9}{10} - \dfrac{3}{5}$. The missing number is $\dfrac{3}{10}$.

c) To determine the missing number, add –2.08 + 0.05. The missing number is –2.03.

d) To determine the missing number, subtract $\dfrac{11}{6} - \left(-\dfrac{7}{3}\right)$. The missing number is $\dfrac{25}{6} = 4\dfrac{1}{6}$.

e) To determine the missing number, subtract –1.25 – 3.75. The missing number is –5.

f) To determine the missing number, subtract $-3\frac{1}{2} - 5\frac{1}{4}$. The missing number is $-8\frac{3}{4}$.

16. a) $-7.4 + \square - \square = -10.9$

$\square - \square = -10.9 + 7.4$

$\quad\quad = -3.5$

Any 2 numbers with a difference of $-10.9 + 7.4 = -3.5$ will satisfy the equation.
For example:-4 and -0.5, -1.2 and 2.3

b) $\square - (-12.8) + \square = -1.1$

$\square + \square = -1.1 + (-12.8)$

$\quad\quad = -13.9$

Any 2 numbers with a sum of -13.9 will satisfy the equation.
For example:-13 and -0.9, -5.8 and -8.1

c) $-21.6 - \square - \square = -15.4$

$-21.6 - (\square + \square) = -15.4$

$\square + \square = 15.4 - 21.6$

$\quad\quad = -6.2$

Any 2 numbers with a sum of -6.2 will satisfy the equation.
For example:-4 and -2.2, -8 and 1.8

17. a) $-11.8 - \square \leq 5.7$

Investigate: What number would I subtract from -11.8 to get less than or equal to 5.7?
Consider the equation: $-11.8 - \square = 5.7$
To determine the missing number, write the related subtraction equation:

$\square = -11.8 - 5.7$

$\quad = -17.5$

So, any number greater than or equal to -17.5 will satisfy the expression $-11.8 - \square \leq 5.7$.
Check: Substitute: $\square = -10$.
$-11.8 - (-10) = -1.8$, which is less than 5.7

b) $6.3 - \square \geq 9.4$

Consider the equation $6.3 - \square = 9.4$. To determine the missing number, write the related subtraction
equation:

$\square = 6.3 - 9.4$

$\quad = -3.1$

So, any number less than or equal to -3.1 will satisfy the expression $6.3 - \square \geq 9.4$.
Check: Substitute: $\square = -4$
$6.3 - (-4) = 10.3$, which is greater than 9.4

Mid-Unit Review   **(page 121)**

Lesson 3.1

1. **a)** -1.3, $2\frac{3}{4}$, 1.51, $-\frac{8}{5}$, $-\frac{9}{3}$

    Sketch a number line labelled from -3 to 3, divided in halves.
    Write each fraction and mixed number as a decimal.

    $2\frac{3}{4} = 2.75$

    $-\frac{8}{5} = -1.6$

    $-\frac{9}{3} = -3.0$

    Mark each number on the number line.
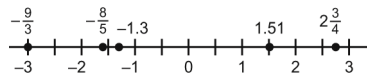
    b) Record the numbers to the left of -1.5 on the number line: $-\frac{9}{3}$, $-\frac{8}{5}$

2. $-\frac{6}{5}$, 1.2, -1.1, $-\frac{1}{4}$, 0.2, $-1\frac{3}{8}$

    Write each fraction as a decimal.

    $-\frac{6}{5} = -1.2$

    $-\frac{1}{4} = -0.25$

    $-1\frac{3}{8} = -1.375$

    Use the decimals to order the numbers.

    From least to greatest: $-1\frac{3}{8}$, $-\frac{6}{5}$, -1.1, $-\frac{1}{4}$, 0.2, 1.2
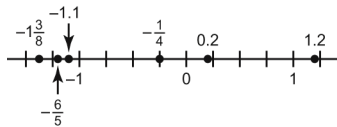
3. Strategies may vary. For example:

    a) $-\frac{2}{3}$ $\square$ $-\frac{3}{4}$

    Compare the equivalent fractions with common denominator 12.

    $-\frac{8}{12} > -\frac{9}{12}$

    So, $-\frac{2}{3} > -\frac{3}{4}$

    I use a number line to check my answer.
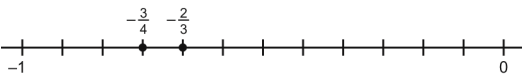
    From the number line, since $-\frac{2}{3}$ is closer to 0 than $-\frac{3}{4}$ is, then $-\frac{2}{3} > -\frac{3}{4}$ is correct.

b) $-\dfrac{8}{3}$ ☐ $-\dfrac{9}{4}$

Compare the equivalent fractions with common denominator 12.

$-\dfrac{32}{12} < -\dfrac{27}{12}$

So, $-\dfrac{8}{3} < -\dfrac{9}{4}$

I write each fraction as a decimal to check my answer.

$-\dfrac{8}{3} = -2.\overline{6}$

$-\dfrac{9}{4} = -2.25$

Since $-2.\overline{6} < -2.25$, then $-\dfrac{8}{3} < -\dfrac{9}{4}$ is correct.

c) −2.5 ☐ 0.5

Any positive number is greater than a negative number.
So, −2.5 < 0.5

d) $-\dfrac{4}{5}$ ☐ −0.9

Write the fraction as a decimal.

$-\dfrac{4}{5} = -0.8$

Since −0.8 > −0.9, then $-\dfrac{4}{5} > -0.9$

I write equivalent fractions with denominator 10 to check my answer.

$-\dfrac{4}{5} = -\dfrac{8}{10}$

$-0.9 = -\dfrac{9}{10}$

Since $-\dfrac{8}{10} > -\dfrac{9}{10}$, then $-\dfrac{4}{5} > -0.9$ is correct.

4. Answers will vary. For example:

a) 1.3 is between 1.2 and 1.4.
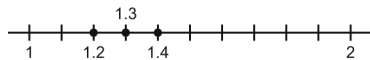
b) $-\dfrac{1}{8}$ is between $-\dfrac{3}{4}$ and $-\dfrac{5}{8}$.
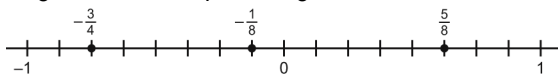
c) 0.35 is between $\dfrac{1}{3}$ and 0.4. I think of $\dfrac{1}{3}$ as $0.\overline{3}$.
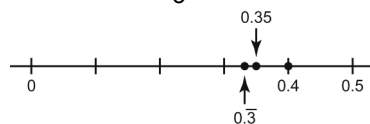
d) −0.95 is between −1.05 and $-\dfrac{9}{10}$.
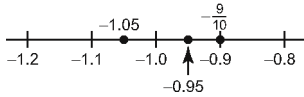
Lesson 3.2

5. a) The sum of two positive numbers is positive.

The sum of two negative numbers is negative.

The sum of a negative number and a positive number has the same sign as the number farther away from 0 on a number line.

b) i) Positive; 5.82   **ii)** Negative; −6.03

    iii) Negative; $-1\dfrac{19}{24}$ **iv)** Positive; 1.31

    v) Negative; $-2\dfrac{43}{45}$ **vi)** Negative; −0.04

6. a) 8.37 + 0.58 = 8.95

b) −21.25 + (−36.57) = −57.82

c) −157.4 + 32.7 = −124.7

d) $\dfrac{5}{8}+\left(-\dfrac{1}{9}\right)$

Write equivalent fractions with denominator 72.

$$=\dfrac{45}{72}+\left(-\dfrac{8}{72}\right)$$

$$=\dfrac{45-8}{72}$$

$$=\dfrac{37}{72}$$

e) $-8\dfrac{1}{4}+5\dfrac{1}{5}$

Add the whole numbers and add the fractions.

$$=(-8)+\left(-\dfrac{1}{4}\right)+5+\dfrac{1}{5}$$

$$=[(-8)+5]+\left[\left(-\dfrac{1}{4}\right)+\dfrac{1}{5}\right]$$

$$=-3+\left(-\dfrac{5}{20}+\dfrac{4}{20}\right)$$

$$=-3-\dfrac{1}{20}$$

$$=-3\dfrac{1}{20}$$

f) $-\dfrac{5}{3}+\left(-\dfrac{23}{7}\right)$

Write equivalent fractions with common denominator 21.

$=-\dfrac{35}{21}+\left(-\dfrac{69}{21}\right)$

$=\dfrac{-35-69}{21}$

$=-\dfrac{104}{21}$, or $-4\dfrac{20}{21}$

Lesson 3.3

7. **a) i)** Write the temperatures in degrees Celsius as rational numbers: –16.1 and –14.7
To determine the change in temperature, subtract:
–16.1 – (–14.7) = 1.4, or 1.4°C

   ii) This is an increase in temperature because –14.7°C is warmer than –16.1°C.

   b) To determine the increase in degrees Celsius from –14.7 to –3.8, subtract:
–3.8 – (–14.7) = 10.9, or 10.9°C

8. **a)** 40.25 – 63.10 = –22.85

   b) –112.2 – (–14.8) = –112.2 + 14.8
                = –97.4

   c) Write equivalent fractions with the common denominator 10.

   $\dfrac{2}{5}-\dfrac{9}{10}=\dfrac{4}{10}-\dfrac{9}{10}$

   $=\dfrac{4-9}{10}$

   $=-\dfrac{5}{10}$, or $-\dfrac{1}{2}$

   d) Write each mixed number as an improper fraction. Then, use equivalent fractions.

   $-4\dfrac{4}{9}-3\dfrac{5}{6}=-\dfrac{40}{9}-\dfrac{23}{6}$

   $=-\dfrac{80}{18}-\dfrac{69}{18}$

   $=\dfrac{-80-69}{18}$

   $=\dfrac{-149}{18}$, or $-8\dfrac{5}{18}$

   e) –1.8 – 4.3 = –6.1

   f) $\dfrac{23}{8}-\left(-\dfrac{7}{2}\right)=\dfrac{23}{8}+\dfrac{7}{2}$

   $=\dfrac{23}{8}+\dfrac{28}{8}$

   $=\dfrac{51}{8}$, or $6\dfrac{3}{8}$

9. Distances below sea level are written as negative rational numbers.
Write 86 m below sea level as –86.
Distances above sea level are written as positive rational numbers.
Write 6193.7 m above sea level as 6193.7.
Sea level is 0.
To represent the distance between these two points, subtract:
$6193.7 - (-86) = 6279.7$, or 6279.7 m

10. a) If both numbers are positive, and the lesser number is subtracted from the greater number, the difference is positive, as in part b, i.
If the first number is negative and a positive number is subtracted, then the difference is negative, as in part b, ii.

If both numbers are positive, and the greater number is subtracted from the lesser number, then the difference is negative, as in part b, iii.
If the first number is positive and a negative number is subtracted, then the difference is positive, as in part b, iv.

b) i) Positive; 8.7   **ii)** Negative; –2.52

iii) Negative; $-\dfrac{49}{60}$ **iv)** Positive; $13\dfrac{1}{6}$

Check

3. Use the properties for determining the sign of the product of 2 rational numbers.
   If two numbers have the same sign, their product is positive, or greater than 0.
   If two numbers have opposite signs, their product is negative, or less than 0.

   a) $3 \times (-5.2)$
   The product is less than 0.
   Use mental math to multiply: $3 \times 52 = 156$
   Estimate to place the decimal point: Since 5.2 is close to 5, 3×5.2 is close to $3 \times 5 = 15$.
   So, place the decimal point after 5 in 156: 15.6
   Then, $3 \times (-5.2) = -15.6$

   b) $2.6 \times (-4)$
   The product is less than 0.
   Use mental math to multiply: $26 \times 4 = 104$
   Estimate to place the decimal point: Since 2.6 is close to 3, 2.6×4 is close to $3 \times 4 = 12$.
   So, place the decimal point after 0 in 104: 10.4
   Then, $2.6 \times (-4) = -10.4$

   c) $(-1.3) \times 5$
   The product is less than 0.
   Use mental math to multiply: $13 \times 5 = 65$
   Estimate to place the decimal point: Since 1.3 is close to 1, 1.3×5 is close to $1 \times 5 = 5$.
   So, place the decimal point after 6 in 65: 6.5
   Then, $(-1.3) \times 5 = -6.5$

   d) $(-0.9) \times (-7.1)$
   The product is greater than 0.
   Use mental math to multiply: $9 \times 71 = 639$
   Estimate to place the decimal point: Since 0.9 is close to 1 and 7.1 is close to 7, 0.9×7.1 is close to
   $1 \times 7 = 7$.
   So, place the decimal point after 6 in 639: 6.39
   Then, $(-0.9) \times (-7.1) = 6.39$

4. Use the properties for determining the sign of the product of 2 rational numbers. If two numbers have the
   same sign, their product is positive, or greater than 0. If two numbers have opposite signs, their product is
   negative, or less than 0.

   a) $(-3) \times \dfrac{2}{3}$

   The product is less than 0.
   Use fraction multiplication to determine the numerical value of the product.
   Simplify the fractions before multiplying.
   $$3 \times \frac{2}{3} = \frac{{}^1\cancel{3}}{1} \times \frac{2}{\cancel{3}^1}$$
   $$= 2$$
   So, $(-3) \times \dfrac{2}{3} = -2$

b) $\left(-\dfrac{1}{4}\right) \times (-5)$

The product is greater than 0.
Use fraction multiplication to determine the numerical value of the product.

$\dfrac{1}{4} \times 5 = \dfrac{1}{4} \times \dfrac{5}{1}$

$\qquad = \dfrac{5}{4}, \text{ or } 1\dfrac{1}{4}$

So, $\left(-\dfrac{1}{4}\right) \times (-5) = 1\dfrac{1}{4}$

c) $\left(\dfrac{4}{5}\right) \times (-2)$

The product is less than 0. Use fraction multiplication to determine the numerical value of the product.

$\dfrac{4}{5} \times 2 = \dfrac{4}{5} \times \dfrac{2}{1}$

$\qquad = \dfrac{8}{5}, \text{ or } 1\dfrac{3}{5}$

So, $\left(\dfrac{4}{5}\right) \times (-2) = -\dfrac{8}{5}, \text{ or } -1\dfrac{3}{5}$

d) $\left(-\dfrac{1}{2}\right) \times \dfrac{7}{8}$

The product is less than 0. Use fraction multiplication to determine the numerical value of the product.

$\dfrac{1}{2} \times \dfrac{7}{8} = \dfrac{7}{16}$

So, $\left(-\dfrac{1}{2}\right) \times \dfrac{7}{8} = -\dfrac{7}{16}$

5. When 2 rational numbers have the same sign, their product is positive.
When 2 rational numbers have opposite signs, their product is negative.
I multiply the rational numbers as I would integers, then estimate the product to place the decimal point.

a) $(-0.64)(0.2)$
$(-64)(20) = -1280$
Estimate: $(-0.64)(0.2)$ is close to $(-1)(0.2) = -0.2$
So, $(-0.64)(0.2) = -0.128$

b) $(-0.5)(-5.71)$
$(-5)(-571) = 2855$
Estimate: $(-0.5)(-5.71)$ is close to $(-0.5)(-6) = 3$
So, $(-0.5)(-5.71) = 2.855$

c) $(-4.13)(-0.8)$
$(-413)(-8) = 3304$
Estimate: $(-4.13)(-0.8)$ is close to $(-4)(-1) = 4$
So, $(-4.13)(-0.8) = 3.304$

d) $(0.7)(8.5)$
$(7)(85) = 595$
Estimate: $(0.7)(8.5)$ is close to $(1)(8.5) = 8.5$
So, $(0.7)(8.5) = 5.95$

6. To determine the expressions with the same product as $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$, predict the sign of each product.

When 2 rational numbers have the same sign, their product is positive.
When 2 rational numbers have opposite signs, their product is negative.
Then, compare the numerators and denominators of the fractions in each product with the numerators
and denominators in $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$, which has a negative product.

a) $\left(\dfrac{5}{2}\right)\left(-\dfrac{3}{4}\right)$

The product is negative.
Both expressions have the same numerators and the same denominators.
So, the expression above has the same product as $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

b) $\left(\dfrac{3}{4}\right)\left(-\dfrac{5}{2}\right)$

The product is negative.
Both expressions have the same numerators and the same denominators.
So, the expression above has the same product as $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

c) $\left(-\dfrac{3}{2}\right)\left(\dfrac{5}{4}\right)$

The product is negative.
Both expressions have the same numerators and the same denominators.
So, the expression above has the same product as $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

d) $\left(\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$

The product is positive.
So, the expression above has a different product than $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

e) $\left(\dfrac{3}{2}\right)\left(-\dfrac{5}{4}\right)$

The product is negative.
Both expressions have the same numerators and the same denominators.
So, the expression in part e has the same product as $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

f) $\left(-\dfrac{3}{4}\right)\left(-\dfrac{5}{2}\right)$

The product is positive.
So, the expression above has a different product than $\left(-\dfrac{3}{4}\right)\left(\dfrac{5}{2}\right)$.

7. Predict the sign of each product.
When 2 rational numbers have the same sign, their product is positive.
When 2 rational numbers have opposite signs, their product is negative.
Then, use fraction multiplication to determine the numerical value of the product.

a) $\left(-\dfrac{1}{3}\right)\left(\dfrac{2}{5}\right)$

The product is negative.

$$\left(\dfrac{1}{3}\right)\left(\dfrac{2}{5}\right) = \dfrac{1 \times 2}{3 \times 5}$$

$$= \dfrac{2}{15}$$

So, $\left(-\dfrac{1}{3}\right)\left(\dfrac{2}{5}\right) = -\dfrac{2}{15}$

b) $\left(\dfrac{1}{4}\right)\left(-\dfrac{3}{5}\right)$

The product is negative.

$$\left(\dfrac{1}{4}\right)\left(\dfrac{3}{5}\right) = \dfrac{1 \times 3}{4 \times 5}$$

$$= \dfrac{3}{20}$$

So, $\left(\dfrac{1}{4}\right)\left(-\dfrac{3}{5}\right) = -\dfrac{3}{20}$

c) $\left(\dfrac{4}{5}\right)\left(\dfrac{1}{2}\right)$

The product is positive.
Simplify before multiplying.

$$\left(\dfrac{4}{5}\right)\left(\dfrac{1}{2}\right) = \dfrac{{}^{2}\cancel{4} \times 1}{5 \times \cancel{2}^{1}}$$

$$= \dfrac{2}{5}$$

So, $\left(\dfrac{4}{5}\right)\left(\dfrac{1}{2}\right) = \dfrac{2}{5}$

d) $\left(-\dfrac{5}{6}\right)\left(-\dfrac{2}{3}\right)$

The product is positive.
Simplify before multiplying.

$$\left(\dfrac{5}{6}\right)\left(\dfrac{2}{3}\right) = \dfrac{5 \times \cancel{2}^{1}}{{}^{3}\cancel{6} \times 3}$$

$$= \dfrac{5}{9}$$

So, $\left(-\dfrac{5}{6}\right)\left(-\dfrac{2}{3}\right) = \dfrac{5}{9}$

Apply

8. Strategies may vary. For example:

Both numbers multiplied must be negative to produce a positive product. So, parts b and f can be excluded, as they cannot represent correct answers. This leaves parts a, c, d, and e as possible answers. Since $(-4)(-2.5) = 10$, then any factor greater than -4 will *not* produce a product greater than 10. This excludes part d.

a) $(-5.1)(-2.5) = 12.75$

c) $(-4.4)(-2.5) = 11$

e) $(-5)(-2.5) = 12.5$

9. Write each change in value as the product: price per share $\times$ number of shares
Use a calculator. A positive product represents an increase in value. A negative product represents a decrease in value.
a) Change in value: $120 \times (-0.8) = -96$
The value of shares dropped by $96.

b) Change in value: $50 \times (-2.10) = -105$
The value of shares dropped by $105.

c) Change in value: $65 \times 0.23 = 14.95$
The value of shares increased by $14.95.

10. Write the change in depth as the product: average speed per minute $\times$ number of minutes
Since the diver is descending, her average speed is negative.
Change in depth after 3.6 min: $(-10.4) \times 3.6 = -37.44$
After 3.6 min, the diver descended 37.44 m.

11. Use a calculator.
a) $(-1.23)(2.8) = -3.444$

b) $(-23.7)(-1.2) = 28.44$

c) $(15.2)(15.2) = 231.04$

d) $(-20.1)(-5.2) = 104.52$

12. Predict the sign of each product. When 2 rational numbers have the same sign, their product is positive. When 2 rational numbers have opposite signs, their product is negative.
Then, use fraction multiplication to determine the numerical value of each product. Simplify before multiplying when needed.

a) $\left(\dfrac{5}{4}\right)\left(-\dfrac{16}{5}\right)$

The product is negative since the rational numbers have opposite signs.

$$\frac{\overset{1}{\cancel{5}}}{4} \times \frac{\overset{4}{\cancel{16}}}{\cancel{5}^{1}} = \frac{1}{1} \times \frac{4}{1}$$
$$= 4$$

So, $\left(\dfrac{5}{4}\right)\left(-\dfrac{16}{5}\right) = -4$

b) $\left(-\dfrac{2}{3}\right)\left(-\dfrac{5}{6}\right)$

The product is positive since the rational numbers have the same sign.

$$\dfrac{\overset{1}{\cancel{2}}}{3} \times \dfrac{5}{\cancel{6}^{\,3}} = \dfrac{1}{3} \times \dfrac{5}{3}$$

$$= \dfrac{5}{9}$$

So, $\left(-\dfrac{2}{3}\right)\left(-\dfrac{5}{6}\right) = \dfrac{5}{9}$

c) $\left(-2\dfrac{8}{9}\right)\left(5\dfrac{1}{8}\right)$

The product is negative since the rational numbers have opposite signs.
Write each mixed number as an improper fraction.

$$2\dfrac{8}{9} \times 5\dfrac{1}{8} = \dfrac{\overset{13}{\cancel{26}}}{9} \times \dfrac{41}{\cancel{8}^{\,4}}$$

$$= \dfrac{13}{9} \times \dfrac{41}{4}$$

$$= \dfrac{533}{36}$$

$$= 14\dfrac{29}{36}$$

So, $\left(-2\dfrac{8}{9}\right)\left(5\dfrac{1}{8}\right) = -14\dfrac{29}{36}$

d) $\left(-4\dfrac{2}{5}\right)\left(-\dfrac{5}{3}\right)$

The product is positive since the rational numbers have the same sign.

Write $4\dfrac{2}{5}$ as an improper fraction.

$$4\dfrac{2}{5} \times \dfrac{5}{3} = \dfrac{22}{\cancel{5}^{\,1}} \times \dfrac{\cancel{5}^{\,1}}{3}$$

$$= \dfrac{22}{1} \times \dfrac{1}{3}$$

$$= \dfrac{22}{3}$$

$$= 7\dfrac{1}{3}$$

So, $\left(-4\dfrac{2}{5}\right)\left(-\dfrac{5}{3}\right) = 7\dfrac{1}{3}$

13. a) $(-26)(-4)$

Since the 2 rational numbers have the same sign, their product is positive.
Use mental math: $26 \times 4 = 104$
So, $(-26)(-4) = 104$

b) i) $(-2.6)(-0.4) = 1.04$

ii) $(-0.26)(0.4) = -0.104$

iii) $(260)(-0.04) = -10.4$

iv) $(-0.026)(-4) = 0.104$

c) I used the same pair of whole-number factors to determine the numerical value of each product:
$26 \times 4 = 104$. So, all products have the same digits.
I used estimation to place the decimal point in the digits 104 each time.
I predicted the sign of each product.
When 2 rational numbers have the same sign, their product is positive.
When 2 rational numbers have opposite signs, their product is negative.

d) Products may vary. For example:

$(-0.26)(-4) = 1.04$

$(-26)(0.4) = -10.4$

$(2.6)(-4) = -10.4$

14. a) Write the cost in dollars of painting the trucks as a product: average cost per truck × number of trucks
The average cost per truck is negative because it is money that the company pays.
Then, $(-3457.25) \times 25 = -86\ 431.25$
It costs $86 431.25 to repaint all the trucks.

b) $45\ 567.87 - 86\ 431.25 = -40\ 863.38$

The company's bank account is overdrawn by $40 863.38 after the bill for painting has been paid.

15. Methods may vary. For example:

a) The first two factors are negative, so their product is positive.
The third factor is positive, so the product of 3 factors is positive.
$(-2.0)(-0.5)(3.1) = (1)(3.1)$
$= 3.1$

b) Only one factor is negative, so the product of the 3 factors is negative.

$$\left(\frac{5}{6}\right)\left(-\frac{4}{7}\right)\left(\frac{3}{2}\right) = \left(\frac{5}{{}^{1}\!\!\not{2}\,\not{6}}\right)\left(-\frac{\not{4}^{2}}{7}\right)\left(\frac{\not{3}^{1}}{\not{2}^{1}}\right)$$

$$= -\frac{5}{7}$$

Take It Further

16. To determine the numerical value of each missing number, use the fact that multiplication and division are inverse operations.

a) The sign of the missing number is negative because the product of 2 negative numbers is positive.
Use division to determine the numerical value.

$\square = 15.275 \div 3.25$

$= 4.7$

The missing number is –4.7.

b) The sign of the missing number is positive because the product of a negative number and a positive number is negative.

Use division to determine the numerical value.

$$\square = \frac{35}{8} \div \frac{5}{4}$$

$$= \frac{35}{8} \div \frac{10}{8}$$

$$= \frac{35}{10}$$

$$= \frac{7}{2}$$

The missing number is $\frac{7}{2}$.

c) The sign of the missing number is negative because the product of a negative number and a positive number is negative.

Use division to determine the numerical value.

$$\square = 0.018 \div 0.045$$

$$= 0.4$$

The missing number is –0.4.

d) The sign of the missing number is positive because the product of 2 positive numbers is positive.

Use division to determine the numerical value.

$$\square = 5\frac{1}{4} \div 3\frac{3}{4}$$

$$= \frac{21}{4} \div \frac{15}{4}$$

$$= \frac{21}{15}$$

$$= \frac{7}{5}$$

$$= 1\frac{2}{5}$$

The missing number is $1\frac{2}{5}$.

17. Choose a pair of rational numbers. For the product to be closer to 0 than both of these numbers, the rational numbers must be between –1 and 1.

For example,

(–0.4)(0.5) = –0.2

(0.4)(–0.5) = –0.2

For each product statement, the product is closer to 0 than either factor is.

Choose a pair of rational numbers where one number is greater than 1. For example:

(1.4)(–0.5) = –0.7

The product is closer to 0 than 1.4 is.

Choose a pair of rational numbers where one number is less than –1. For example,

(–1.4)(0.5) = –0.7

The product is closer to 0 than –1.4 is.

Choose pairs of rational numbers where both numbers are greater than or less than 1. For example,

(–1.4)(–1.5) = 2.1 and (1.4)(1.5) = 2.1

Neither product is closer to 0 than either of the two factors is.

18. a) Since there are no common factors in the numerators and denominators, the fractions cannot be simplified before multiplying.

The first fraction in decimal form is 0.978 021 978…; this appears to be a repeating decimal.

The second fraction in decimal form is 0.360 465 116…; this appears not to terminate, but it may repeat.

So, the product of the fractions cannot be a terminating decimal.

b)   The exact answer is:

$$-\frac{89}{91} \times \frac{31}{86} = \frac{-89 \times 31}{91 \times 86} = -\frac{2759}{7826}$$

Check

3. Predict the sign of the quotient. When two rational numbers have the same sign, their quotient is positive. When two rational numbers have different signs, their quotient is negative.
Then, use the procedures for dividing decimals to determine the numerical value of the quotient.
Use mental math when the dividend and the divisor are compatible numbers

a) $(-1.5) \div 3$
The quotient is negative. Use mental math to determine the quotient.
$1.5 \div 3 = 0.5$
So, $(-1.5) \div 3 = -0.5$

b) $2.8 \div (-2)$
The quotient is negative. Use mental math to determine the quotient.
$2.8 \div (-2) = -1.4$

c) $(-8.4) \div (-4)$
The quotient is positive. Use mental math to determine the quotient.
$(-8.4) \div (-4) = 2.1$

d) $1.6 \div (-8)$
The quotient is negative. Use mental math to determine the quotient.
$(1.6) \div (-8) = -0.2$

e) $(-14.4) \div (-6)$
The quotient is positive. Use mental math to determine the quotient.
$(-14.4) \div (-6) = 2.4$

f) $(-6.3) \div 7$
The quotient is negative. Use mental math to determine the quotient.
$(-6.3) \div 7 = -0.9$

4. Predict the sign of the quotient. When two rational numbers have the same sign, their quotient is positive. When two rational numbers have different signs, their quotient is negative.
Then, use the procedures for dividing fractions to determine the numerical value of the quotient.

a) $\dfrac{1}{2} \div \left(-\dfrac{3}{4}\right)$

The quotient is negative.

Determine the numerical value of the quotient: $\dfrac{1}{2} \div \dfrac{3}{4}$

Use the strategy of multiplying the dividend by the reciprocal of the divisor.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{{}^1\cancel{2}} \times \frac{\cancel{4}^2}{3}$$

$$= \frac{2}{3}$$

So, $\dfrac{1}{2} \div \left(-\dfrac{3}{4}\right) = -\dfrac{2}{3}$

b) $\left(-\dfrac{2}{5}\right) \div \dfrac{3}{10}$

The quotient is negative.

Determine the numerical value of the quotient: $\dfrac{2}{5} \div \dfrac{3}{10}$

Use the strategy of dividing fractions with a common denominator.
Write equivalent fractions with denominator 10.

$$\dfrac{2}{5} \div \dfrac{3}{10} = \dfrac{4}{10} \div \dfrac{3}{10}$$

$$= \dfrac{4}{3}$$

So, $\left(-\dfrac{2}{5}\right) \div \dfrac{3}{10} = -\dfrac{4}{3}$, or $-1\dfrac{1}{3}$

c) $\left(-\dfrac{7}{6}\right) \div \left(-\dfrac{8}{3}\right)$

The quotient is positive.

Determine the numerical value of the quotient: $\dfrac{7}{6} \div \dfrac{8}{3}$

Use the strategy of multiplying the dividend by the reciprocal of the divisor.

$$\dfrac{7}{6} \div \dfrac{8}{3} = \dfrac{7}{{}^{2}\cancel{6}} \times \dfrac{\cancel{3}^{\,1}}{8}$$

$$= \dfrac{7}{16}$$

So, $\left(-\dfrac{7}{6}\right) \div \left(-\dfrac{8}{3}\right) = \dfrac{7}{16}$

d) $\dfrac{1}{4} \div \dfrac{11}{3}$

The quotient is positive.

Determine the quotient: $\dfrac{1}{4} \div \dfrac{11}{3}$

Use the strategy of dividing fractions with a common denominator.
Write equivalent fractions with denominator 12.

$$\dfrac{1}{4} \div \dfrac{11}{3} = \dfrac{3}{12} \div \dfrac{44}{12}$$

$$= \dfrac{3}{44}$$

So, $\dfrac{1}{4} \div \dfrac{11}{3} = \dfrac{3}{44}$

e) $\dfrac{5}{2} \div \left(-\dfrac{2}{3}\right)$

The quotient is negative.

Determine the numerical value of the quotient: $\dfrac{5}{2} \div \dfrac{2}{3}$

Use the strategy of multiplying the dividend by the reciprocal of the divisor.

$$\dfrac{5}{2} \div \dfrac{2}{3} = \dfrac{5}{2} \times \dfrac{3}{2}$$

$$= \dfrac{15}{4}$$

So, $\dfrac{5}{2} \div \left(-\dfrac{2}{3}\right) = -\dfrac{15}{4}$, or $-3\dfrac{3}{4}$

f) $\left(-\dfrac{9}{5}\right) \div \left(-\dfrac{11}{4}\right)$

The quotient is positive.

Determine the numerical value of the quotient: $\dfrac{9}{5} \div \dfrac{11}{4}$

Use the strategy of dividing fractions with a common denominator.
Write equivalent fractions with denominator 20.

$$\dfrac{9}{5} \div \dfrac{11}{4} = \dfrac{36}{20} \div \dfrac{55}{20}$$

$$= \dfrac{36}{55}$$

So, $\left(-\dfrac{9}{5}\right) \div \left(-\dfrac{11}{4}\right) = \dfrac{36}{55}$

5. $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$ Since both rational numbers are negative, the quotient is positive.

Write the equivalent multiplication statement: $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{4}{3}\right)$

Predict the sign of each product or quotient in parts a to f.
Compare the numerators and denominators of each quotient (parts b and e) with the numerators and
denominators of $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

Compare the numerators and denominators of each product (parts a, c, d, f) with the numerators and
denominators of the equivalent multiplication statement $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{4}{3}\right)$.

a) $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{3}{4}\right)$

The rational numbers are the same as in $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$, but the expression represents multiplication,

not division. So, the expression has a different answer than $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

b) $\left(-\dfrac{3}{4}\right) \div \left(-\dfrac{1}{3}\right)$

The quotient is positive. The position of the rational numbers in the expression is reversed, so the expression has a different answer than $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

c) $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{4}{3}\right)$

The quotient is positive. The numerators and denominators are the same as in $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{4}{3}\right)$. So, the

answer is the same as $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

d) $\left(-\dfrac{4}{3}\right) \times \left(-\dfrac{1}{3}\right)$

Multiplication is commutative. The expression is equivalent to the expression in part c. So, the answer is the same as $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

e) $\dfrac{1}{3} \div \dfrac{3}{4}$

The quotient is positive. The numerators and denominators are the same as in $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

So, the expression has the same answer as $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

f) $\dfrac{4}{3} \times \dfrac{1}{3}$

The product is positive. The numerators and denominators are the same as in $\left(-\dfrac{1}{3}\right) \times \left(-\dfrac{4}{3}\right)$. So, the

expression has the same answer as $\left(-\dfrac{1}{3}\right) \div \left(-\dfrac{3}{4}\right)$.

Apply

6. To determine the mean hourly change in level, divide: $(-5.6) \div 3.5$
   Since the dividend and divisor have different signs, the quotient is negative.
   Determine the numerical value of the quotient: $5.6 \div 3.5 = 1.6$
   So, $(-5.6) \div 3.5 = -1.6$
   The mean change in water level per hour was -1.6 m.

7. **a)** $0.32 \div 0.4$ Since the dividend and divisor have the same sign, the quotient is positive.
   Divide with whole numbers: $32 \div 4 = 8$
   Estimate: 0.32 is close to 1, and 0.4 is close to 1. Then, $0.32 \div 0.4$ is close to $1 \div 1 = 1$.
   Place the decimal point before 8: 0.8
   So, $0.32 \div 0.4 = 0.8$

   b) $(-1.17) \div 0.8$ Since the dividend and divisor have different signs, the quotient is negative.
   Divide with whole numbers: $-117 \div 8 = -14.625$
   Estimate: -1.17 is close to -1, and 0.8 is close to 1. Then, $(-1.17) \div 0.8$ is close to $(-1) \div 1 = -1$.
   Place the decimal point after 1 in -14.625: -1.4625
   So, $(-1.17) \div 0.8 = -1.4625$

c) 0.25 ÷ (–0.6) Since the dividend and divisor have different signs, the quotient is negative.
Divide with whole numbers: 25 ÷ (–6) = – 4.1$\overline{6}$
Estimate: –0.6 is close to –0.5. Then, 0.25 ÷ (–0.6) is close to 0.25 ÷ (–0.5) = –0.5.
Place the decimal point before 4 in – 4.1$\overline{6}$: – 0.41$\overline{6}$
So, 0.25 ÷ (–0.6) = – 0.41$\overline{6}$

d) (–1.02) ÷ (–0.2) Since the dividend and divisor have the same sign, the quotient is positive.
Divide with whole numbers: (–102) ÷ (–2) = 51
Estimate: –1.02 is close to –1. Then, (–1.02) ÷ (–0.2) is close to (–1) ÷ (–0.2) = 5.
Place the decimal point after 5 in 51: 5.1
So, (–1.02) ÷ (–0.2) = 5.1

e) 3.76 ÷ (–0.3) Since the dividend and divisor have different signs, the quotient is negative.
Divide with whole numbers: 376 ÷ (–3) = – 125.$\overline{3}$
Estimate: 3.76 is close to 4, and –0.3 is close to –0.5. Then, 3.76 ÷ (–0.3) is close to 4 ÷ (–0.5) = –8.
Place the decimal point after 2 in – 125.$\overline{3}$: – 12.5$\overline{3}$
So, 3.76 ÷ (–0.3) = – 12.5$\overline{3}$

f) 3.15 ÷ 0.9 Since the dividend and divisor have the same sign, the quotient is positive.
Divide with whole numbers: 315 ÷ 9 = 35
Estimate: 3.15 is close to 3, and 0.9 is close to 1. Then, 3.15 ÷ 0.9 is close to 3 ÷ 1 = 3.
Place the decimal point after 3 in 35: 3.5
So, 3.15 ÷ 0.9 = 3.5

8. To determine how long it took for the temperature to reach –12.5°C, divide: (–12.5) ÷ (–2.5) = 5
It took 5 h for the temperature to reach –12.5°C.

9. a) 20.736 ÷ (–1.8) = –11.52

b) (–27.94) ÷ 1.2 = –23.28$\overline{3}$

c) (–84.41) ÷ (–2.3) = 36.7

d) 23.04 ÷ 4.8 = 4.8

e) 76.63 ÷ (–7.5) = –10.217$\overline{3}$

f) (–0.1081) ÷ 0.45 = –0.2402$\overline{2}$

10. a) Any negative number divided by –0.5 will produce a positive quotient, so the quotient will be greater than –10. –20.5 ÷ (–0.5) = 41

b) 5 ÷ (–0.5) = –10; so any number greater than 5 that is divided by –0.5 will have a quotient less than –10. So, 18.8 ÷ (–0.5) will have a quotient less than –10.

c) I used the same strategy as in part b. 10.7 ÷ (–0.5) will have a quotient less than –10.

d) From part b, any number less than 5 that is divided by –0.5 will have a quotient greater than –10. Since 0.6 is less than 5, the quotient will be greater than –10.
0.6 ÷ (–0.5) = –1.2

11. a) $(-1450.50) \div (-30.75) \doteq 47.17$; it will take 48 weeks for Paige to pay her parents back.

  b) Debt is represented by a negative number. So, the amount in dollars Paige borrowed is represented by -1450.50. When Paige pays back her parents, she is spending money, so the amount in dollars she pays back is also represented by a negative number: -30.75

12. a) $\dfrac{5}{4} \div \left(-\dfrac{7}{6}\right)$

Use the strategy of multiplying by the reciprocal of the divisor.

$$\dfrac{5}{4} \div \left(-\dfrac{7}{6}\right) = \dfrac{5}{{}^2\cancel{4}} \times \left(-\dfrac{\cancel{6}^{\,3}}{7}\right)$$
$$= \dfrac{5 \times (-3)}{2 \times 7}$$
$$= -\dfrac{15}{14}, \text{ or } -1\dfrac{1}{14}$$

  b) $\dfrac{3}{10} \div \dfrac{12}{5}$

Use the strategy of multiplying by the reciprocal of the divisor.

$$\dfrac{3}{10} \div \dfrac{12}{5} = \dfrac{{}^1\cancel{3}}{{}^2\cancel{10}} \times \dfrac{\cancel{5}^{\,1}}{\cancel{12}^{\,4}}$$
$$= \dfrac{1}{8}$$

  c) $\left(-\dfrac{3}{4}\right) \div \left(-1\dfrac{1}{8}\right)$

Write the mixed number as an improper fraction.

$$\left(-\dfrac{3}{4}\right) \div \left(-1\dfrac{1}{8}\right) = \left(-\dfrac{3}{4}\right) \div \left(-\dfrac{9}{8}\right)$$

Use the strategy of dividing fractions with a common denominator.

$$\left(-\dfrac{3}{4}\right) \div \left(-\dfrac{9}{8}\right) = \left(-\dfrac{6}{8}\right) \div \left(-\dfrac{9}{8}\right)$$
$$= \dfrac{6}{9}, \text{ or } \dfrac{2}{3}$$

  d) $\left(-4\dfrac{3}{5}\right) \div \dfrac{3}{4}$

Write the mixed number as an improper fraction.

$$\left(-4\dfrac{3}{5}\right) \div \dfrac{3}{4} = \left(-\dfrac{23}{5}\right) \div \dfrac{3}{4}$$

Use the strategy of dividing fractions with a common denominator.

$$\left(-\dfrac{23}{5}\right) \div \dfrac{3}{4} = \left(-\dfrac{92}{20}\right) \div \dfrac{15}{20}$$
$$= -\dfrac{92}{15}, \text{ or } -6\dfrac{2}{15}$$

e) $3\dfrac{2}{3} \div \left(-2\dfrac{1}{4}\right)$

Write the mixed numbers as improper fractions.

$3\dfrac{2}{3} \div \left(-2\dfrac{1}{4}\right) = \dfrac{11}{3} \div \left(-\dfrac{9}{4}\right)$

Use the strategy of multiplying by the reciprocal of the divisor.

$\dfrac{11}{3} \div \left(-\dfrac{9}{4}\right) = \dfrac{11}{3} \times \left(-\dfrac{4}{9}\right)$

$= -\dfrac{44}{27}, \text{ or } -1\dfrac{17}{27}$

f) $3\dfrac{4}{9} \div 6\dfrac{1}{3}$

Write the mixed numbers as improper fractions.

$3\dfrac{4}{9} \div 6\dfrac{1}{3} = \dfrac{31}{9} \div \dfrac{19}{3}$

Use the strategy of multiplying by the reciprocal of the divisor.

$\dfrac{31}{9} \div \dfrac{19}{3} = \dfrac{31}{{}^{3}\cancel{9}} \times \dfrac{\cancel{3}}{19}$

$= \dfrac{31}{57}$

13. The difference between 5°C and –5.5°C is: 5°C – (–5.5°C) = 10.5°C.
Each time the freezer door is open, the temperature increases 0.3°C.
To determine how many times the door fridge can be opened for a 10.5°C increase in the temperature, divide: 10.5 ÷ 0.3
Divide whole numbers: 105 ÷ 3 = 35
Then: 10.5 ÷ 0.3 = 35
So, the door can be opened 35 times before the temperature increases to 5°C.

14. To determine the mean change in temperature per hour, divide: (–15.4) ÷ 5.5 = –2.8
The mean change in temperature was –2.8°C per hour.

15. To determine the change in value of one share, I divide the change in value of all shares by the number of shares.
(–17.28) ÷ 54 Since the dividend and the divisor have different signs, the quotient is negative.
(–17.28) ÷ 54 = –0.32
So, the change in value of one share was –$0.32.

16. Explanations may vary. For example:

a) $\left(-\dfrac{2}{3}\right) \div \left(-\dfrac{2}{3}\right)$

Any number (except zero) divided by itself is 1. So, the quotient is not less than $-\dfrac{1}{2}$.

b) $\dfrac{1}{3} \div \left(-\dfrac{2}{3}\right)$

Since the dividend and the divisor have different signs, the quotient is negative.
$\dfrac{1}{3}$ is one-half of $\dfrac{2}{3}$. So, the quotient is $-\dfrac{1}{2}$.

c) $\dfrac{5}{6} \div \left(-\dfrac{2}{3}\right)$

To predict, I first ignore the signs. Since $\dfrac{5}{6}$ is greater than $\dfrac{2}{3}$, or $\dfrac{4}{6}$, the quotient $\dfrac{5}{6} \div \dfrac{2}{3}$ is greater than

1. The quotient of a positive number and a negative number is negative. So, $\dfrac{5}{6} \div \left(-\dfrac{2}{3}\right)$ is less than –1,

and, therefore, less than $-\dfrac{1}{2}$.

d) $\dfrac{1}{4} \div \left(-\dfrac{2}{3}\right)$

To predict, I first ignore the signs, then I use common denominators to compare the fractions.

$\dfrac{1}{4} = \dfrac{3}{12}$ and $\dfrac{2}{3} = \dfrac{8}{12}$

The quotient $\dfrac{1}{4} \div \dfrac{2}{3} = \dfrac{3}{12} \div \dfrac{8}{12} = \dfrac{3}{8}$, which is less than $\dfrac{1}{2}$. So, the quotient $\dfrac{1}{4} \div \left(-\dfrac{2}{3}\right)$, which is

negative, is greater than $-\dfrac{1}{2}$.

17. Write a related division or multiplication statement to determine each missing number.

a) $\square \div 1.25 = -3.6$

$\square = (-3.6) \times 1.25$

$\quad = -4.5$

So, the missing number is –4.5.

b) $\square \div \left(-\dfrac{3}{4}\right) = \dfrac{7}{8}$

$\square = \dfrac{7}{8} \times \left(-\dfrac{3}{4}\right)$

$\quad = -\dfrac{21}{32}$

So, the missing number is $-\dfrac{21}{32}$.

c) $(-0.5875) \div \square = -0.25$

$\square = (-0.5875) \div (-0.25)$

Since the divisor and the dividend have the same sign, the quotient is positive.

$\square = 2.35$

So, the missing number is 2.35.

d) $\dfrac{68}{15} \div \square = -\dfrac{4}{5}$

$\square = \dfrac{68}{15} \div \left(-\dfrac{4}{5}\right)$

$= \dfrac{^{17}\cancel{68}}{^{3}\cancel{15}} \times \left(-\dfrac{\cancel{5}^{1}}{\cancel{4}^{1}}\right)$

$= -\dfrac{17}{3}$

So, the missing number is $-\dfrac{17}{3}$.

18. Use a related division or multiplication statement to determine each missing number.

a) $(-0.3) \times \square = 0.78$

$\square = 0.78 \div (-0.3)$

Since the divisor and the dividend have the same signs, the quotient is negative.

Divide whole numbers.

$78 \div 3 = 26$

Estimate to place the decimal point. Since 0.78 is close to 1 and 0.3 is close to 0.5, 0.78 ÷ 0.3 is close to

$1 \div 0.5 = 2$

Place the decimal point after 2 in 26: 2.6

So, $\square$ = –2.6

The missing number is –2.6.

b) $0.8 \times \square = -5.52$

$\square = (-5.52) \div 0.8$

Since the divisor and dividend have different signs, the quotient is negative.

Divide whole numbers.

$552 \div 8 = 69$

Estimate to place the decimal point. Since 5.52 is close to 6 and 0.8 is close to 1, 5.52 ÷ 0.8 is close to

$6 \div 1 = 6$

Place the decimal point after 6 in 69: 6.9

So, $\square$ = –6.9

The missing number is –6.9.

c) $(-1.26) \div \square = 0.2$

$\square = (-1.26) \div 0.2$

Since the divisor and the dividend have different signs, the quotient is negative.

Divide whole numbers.

$126 \div 2 = 65$

Estimate to place the decimal point. Since 1.26 is close to 1, 1.26 ÷ 0.2 is close to 1 ÷ 0.2 = 5.

Place the decimal point after 6 in 65: 6.5

So, $\square$ = –6.5

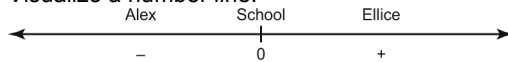The missing number is –6.5.

d) $\square \div (-1.1) = 3.26$

$\square = 3.26 \times (-1.1)$

$\square = -3.586$

The missing number is –3.586.

Take It Further

19. a) Visualize a number line.

```
       Alex        School        Ellice
  ←——————|——————————|——————————|——————————→
          −          0          +
```

Since Ellice and Alex are running in opposite directions from school, we can represent these directions
as positive and negative.

Ellice's average speed (in kilometres per minute): 1.3 ÷ 7.8 = $0.1\overline{6}$

An average speed of $0.1\overline{6}$ km/min indicates Ellice is running in the positive direction.

Convert 630 m to 0.630 km.

Alex's average speed (in kilometres per minute): $(-0.630) \div 4.2 = -0.15$

An average speed of –0.15 km/min indicates Alex is running in the negative direction.

b) Since the signs of the average speeds tell us in which direction each student is running, we ignore the
signs when comparing speeds.

$0.1\overline{6} > 0.15$

So, Ellice has the greater average speed.

20. Answers will vary. For example:

For fractions:

I know that $\dfrac{3}{5}$ is $2\left(\dfrac{3}{10}\right)$, so $\dfrac{3}{10} \div \dfrac{3}{5} = \dfrac{1}{2}$ and $\left(-\dfrac{3}{10}\right) \div \dfrac{3}{5} = -\dfrac{1}{2}$

I know that $\dfrac{4}{5}$ is $3\left(\dfrac{4}{15}\right)$, so $\dfrac{4}{15} \div \dfrac{4}{5} = \dfrac{1}{3}$ and $\dfrac{4}{15} \div \left(-\dfrac{4}{5}\right) = -\dfrac{1}{3}$

I know that $\dfrac{5}{8} \times \dfrac{3}{5} = \dfrac{3}{8}$, so $\dfrac{3}{8} \div \dfrac{3}{5} = \dfrac{5}{8}$ and $\left(-\dfrac{3}{8}\right) \div \dfrac{3}{5} = -\dfrac{5}{8}$

For decimals:

I choose a quotient between –0.75 and –0.25 and use that as one factor in a multiplication statement.

For –0.27, I write: (–0.27) × 9.1 = –2.457, then I write a related division statement:
$(-2.457) \div 9.1 = -0.27$

For –0.55, I write: (–0.55) × 1.3 = –0.715, then I write a related division statement:
$(-0.715) \div 1.3 = -0.55$

For –0.6, I write: (–0.6) × (–2.5) = 1.5, then a related division statement is:
$1.5 \div (-2.5) = -0.6$

21. In part a, since both terms are negative, $-\dfrac{1}{2} + \left(-\dfrac{2}{3}\right)$ is negative.

In part b, write the subtraction statement $-\dfrac{1}{2} - \left(-\dfrac{2}{3}\right)$ as $-\dfrac{1}{2} + \dfrac{2}{3}$.

Since $\dfrac{2}{3}$ is greater than $\dfrac{1}{2}$, the sum is positive.

In part c, since the factors have the same sign, the product $\left(-\dfrac{1}{2}\right) \times \left(-\dfrac{2}{3}\right)$ is positive.

In part d, since the divisor and the dividend have the same sign, the quotient is positive.

To compare the answers in parts c and d:

Rewrite part d as a multiplication statement:

$$\left(-\frac{1}{2}\right) \div \left(-\frac{2}{3}\right) = \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right);$$

Since $\frac{3}{2} > \frac{2}{3}$, $\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) > \left(-\frac{1}{2}\right) \times \left(-\frac{2}{3}\right)$; so the answer in part d is greater than the answer in part c.

Finally, calculate the answers in part b and part d to see which is greater:

b) $-\frac{1}{2} - \left(-\frac{2}{3}\right) = -\frac{1}{2} + \frac{2}{3}.$

$$= \frac{1}{6}$$

d)

$$\left(-\frac{1}{2}\right) \div \left(-\frac{2}{3}\right) = \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)$$

$$= \frac{3}{4}$$

Since $\frac{3}{4} > \frac{1}{6}$, part d has the greatest value.

Lesson 3.6   Order of Operations with Rational Numbers   Practice (pages 140–142)

Check

3. Use the order of operations.

   a) $2.3 - [(-1.6) \times (0.8)] = 2.3 - (-1.28)$
$$= 2.3 + 1.28$$
$$= 3.58$$

   b) $[(-14.8) \times 0.9] - 3.1 = -13.32 - 3.1$
$$= -16.42$$

   c) $[(-12.8) \div (-0.2)] + [4.5 \div 0.5] = 64 + (4.5 \div 0.5)$
$$= 64 + 9$$
$$= 73$$

   d) $(-4.8) \times (-0.4 + 0.6)2 = (-4.8) \times (0.2)2$
$$= (-4.8) \times (0.04)$$
$$= -0.192$$

4. **a)**
$$\frac{1}{2} + \left(-\frac{{}^1\cancel{3}}{4}\right) \times \frac{1}{\cancel{3}^1} = \frac{1}{2} + \left(-\frac{1}{4}\right)$$
$$= \frac{2}{4} + \left(-\frac{1}{4}\right)$$
$$= \frac{1}{4}$$

   b)
$$\left(-\frac{5}{4}\right) \div \left(-\frac{1}{4} + \frac{3}{2}\right)\left(-\frac{1}{4} + \frac{3}{2}\right) = \left(-\frac{5}{4}\right) \div \left(-\frac{1}{4} + \frac{6}{4}\right)\left(-\frac{1}{4} + \frac{6}{4}\right)$$
$$= \left(-\frac{5}{4}\right) \div \left(-\frac{5}{4}\right)\left(-\frac{5}{4}\right)$$
$$= \left[\left(-\frac{5}{4}\right) \div \left(-\frac{5}{4}\right)\right]\left(-\frac{5}{4}\right)$$
$$= 1 \times \left(-\frac{5}{4}\right)$$
$$= -\frac{5}{4}$$

   c)
$$\left(-\frac{7}{10}\right) \div \left(-\frac{2}{5}\right) - \left(-\frac{1}{4}\right) \times \frac{1}{2} = \left(-\frac{7}{{}^2\cancel{10}}\right) \times \left(-\frac{\cancel{5}^1}{2}\right) - \left(-\frac{1}{4}\right) \times \frac{1}{2}$$
$$= \frac{7}{4} - \left(-\frac{1}{8}\right)$$
$$= \frac{14}{8} - \left(-\frac{1}{8}\right)$$
$$= \frac{14}{8} + \frac{1}{8}$$
$$= \frac{15}{8}, \text{ or } 1\frac{7}{8}$$

d)
$$\frac{6}{5} \times \left(-\frac{2}{3} + \frac{8}{3}\right)^2 - \frac{5}{12} = \frac{6}{5} \times \left(\frac{-2+8}{3}\right)^2 - \frac{5}{12}$$
$$= \frac{6}{5} \times \left(\frac{6}{3}\right)^2 - \frac{5}{12}$$
$$= \frac{6}{5} \times (2)^2 - \frac{5}{12}$$
$$= \frac{24}{5} - \frac{5}{12}$$
$$= \frac{288}{60} - \frac{25}{60}$$
$$= \frac{263}{60}, \text{ or } 4\frac{23}{60}$$

Apply

5. Answers may vary. For example:

a) $-2.8 - 1.4 \times 4.5 = -9.1$

b) Follow the order of operations:
$-2.8 - 1.4 \times 4.5 = -2.8 - 6.3$
$= -9.1$
Perform the operations from left to right:
$-2.8 - 1.4 \times 4.5$ would be:
$(-2.8 - 1.4) \times 4.5 = (-4.2) \times 4.5$
$= -18.9$
Since my calculator displayed -9.1, it followed the order of operations.

6. Estimate first.

a) $9.1 - 3.5 \times (4.2)^2$ is close to
$9 - 4 \times 4^2 = 9 - 64$
$= -55$

b) $(9.1 - 3.5) \times (4.2)^2$ is close to
$(9 - 4) \times 4^2 = 5 \times 4^2$
$= 5 \times 16$
$= 80$

c) $9.1 - (3.5 \times 4.2)^2$ is close to
$9 - (4 \times 4)^2 = 9 - (16)^2$
$= 9 - 256$
$= -247$

d) $9.1[(-3.5) \times (4.2)^2]$ is close to
$9[(-4) \times 4^2] = 9[(-4) \times 16]$
$= 9(-64)$
$= -576$

From the estimates, part b appears to have the greatest value.

Evaluate using a calculator.
a) $9.1 - 3.5 \times (4.2)^2 = 9.1 - 3.5 \times 17.64$
$= 9.1 - 61.74$
$= -52.64$

b) $(9.1 - 3.5) \times (4.2)^2 = 5.6 \times (4.2)^2$
$$= 5.6 \times 17.64$$
$$= 98.784$$

c) $9.1 - (3.5 \times 4.2)^2 = 9.1 - [14.7]^2$
$$= 9.1 - 216.09$$
$$= -206.99$$

d) $9.1[(-3.5) \times (4.2)^2] = 9.1[(-3.5) \times 17.64]$
$$= 9.1(-61.74)]$$
$$= -561.834$$

The actual values verify my prediction.

7. a)

$$\left(-\frac{2}{3}\right) \div \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \left(-\frac{2}{3}\right) \times \frac{4}{1} + \frac{1}{2 \times 2 \times 3}$$
$$= -\frac{8}{3} + \frac{1}{12}$$
$$= -\frac{32}{12} + \frac{1}{12}$$
$$= -\frac{31}{12}, \text{ or } -2\frac{7}{12}$$

b)

$$\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right)\right] \times \frac{1}{3} = \left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{2}{4}\right)\right] \times \frac{1}{3}$$
$$= \left(-\frac{2}{3}\right) \div \left(-\frac{1}{4}\right) \times \frac{1}{3}$$
$$= \left(-\frac{2}{3}\right) \times \left(-\frac{4}{1}\right) \times \frac{1}{3}$$
$$= \frac{8}{3} \times \frac{1}{3}$$
$$= \frac{8}{9}$$

c)

$$\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} - \left(-\frac{1}{2}\right)\right] \times \frac{1}{3} = \left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \frac{2}{4}\right] \times \frac{1}{3}$$
$$= \left(-\frac{2}{3}\right) \div \frac{3}{4} \times \frac{1}{3}$$
$$= \left(-\frac{2}{3}\right) \times \frac{4}{3} \times \frac{1}{3}$$
$$= \left(-\frac{8}{9}\right) \times \frac{1}{3}$$
$$= -\frac{8}{27}$$

d)

$$\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right) \times \frac{1}{3}\right] = \left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{6}\right)\right]$$

$$= \left(-\frac{2}{3}\right) \div \left[\frac{3}{12} + \left(-\frac{2}{12}\right)\right]$$

$$= \left(-\frac{2}{3}\right) \div \frac{1}{12}$$

$$= \left(-\frac{2}{{}^1\!\cancel{3}}\right) \times \cancel{12}^4$$

$$= -8$$

8. a) In the second line, the student evaluated (–2.8 + 1.5) as 1.3, when it should have been –1.3. And, in the third line, the student subtracted before dividing.

Correct solution:
(–3.7) × (–2.8 + 1.5) – 4.8 ÷ (–1.2)
= (–3.7) × (–1.3) – 4.8 ÷ (–1.2)
= 4.81 – 4.8 ÷ (–1.2)
= 4.81 – (–4)
= 8.81

b) In the second line, the student subtracted before multiplying and dividing.
Correct solution:

$$-\frac{3}{8} - \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\cancel{10}^5} \div \left(-\frac{4}{5}\right)$$

$$= -\frac{3}{8} - \frac{6}{25} \div \left(-\frac{4}{5}\right)$$

$$= -\frac{3}{8} - \frac{\overset{3}{\cancel{6}}}{\cancel{25}^5} \times \left(-\frac{\cancel{5}^1}{\cancel{4}^2}\right)$$

$$= -\frac{3}{8} - \left(-\frac{3}{10}\right)$$

$$= -\frac{3}{8} + \frac{3}{10}$$

$$= -\frac{15}{40} + \frac{12}{40}$$

$$= -\frac{3}{40}$$

9. Substitute $d = 4$ and $k = 2400$ in the formula $C = 1.15 \, [21.95d + 0.035 \, (k - 120)]$
$C = 1.15 \, [21.95(4) + 0.035(2400 - 120)]$
$ = 1.15 \, [21.95(4) + 0.035(2280)]$
$ = 1.15 \, [87.8 + 0.035(2280)]$
$ = 1.15 \, [87.8 + 79.8]$
$ = 1.15 \, [167.6]$
$ = 192.74$
The cost to rent the truck is $192.74.

10. a) Substitute r = 3.5 and height = 11.5 in the formula: Surface area = $2\pi(r)^2 + 2\pi r \times height$.

$$\text{Surface area} = 2\pi(3.5)^2 + 2\pi(3.5)(11.5)$$
$$= 2\pi(12.25) + \pi(80.5)$$
$$= \pi(24.5) + \pi(80.5)$$
$$\doteq 76.9690 + 252.8982$$
$$\doteq 329.8672$$

The area of tin is approximately 330 cm^2.

b) First I evaluated the exponent. Then, I multiplied from left to right. Finally, I added.

11. a) Substitute each value in the formula $C = \dfrac{F - 32}{1.8}$.

i) For F = 0,
$$C = \frac{-32}{1.8}$$
$$= 17.\overline{7}$$
So, 0°F is about −18°C

ii) For F = −40,
$$C = \frac{-40 - 32}{1.8}$$
$$= \frac{-72}{1.8}$$
$$= 40$$
So, −40°F is −40°C.

iii) For F = −53,
$$C = \frac{-53 - 32}{1.8}$$
$$= \frac{-85}{1.8}$$
$$= 47.\overline{2}$$
So, −53°F is about −47°C.

b) Substitute each value in the formula $C = \dfrac{5}{9}(F - 32)$.

i) For F = 50,
$$C = \frac{5}{9}(50 - 32)$$
$$= \frac{5}{9} \times \cancel{18}^2$$
$$= 10$$
So, 50°F is 10°C.

ii) For F = −13,
$$C = \frac{5}{9}(-13 - 32)$$
$$= \frac{5}{^1\cancel{9}} \times (\cancel{-45}^5)$$
$$= -25$$
So, −13°F is −25°C.

iii) For $F = 32$,

$$C = \frac{5}{9}(32 - 32)$$
$$= 0$$

So, 32°F is 0°C.

c) Answers may vary. For example: It seemed easier to use the formula in part a when I used a calculator. I could use mental math for most of the questions in part b.

12. a) Multiply first, then add.

$$\left(-4\frac{1}{2}\right)+\left(-\frac{2}{3}\right)\times2\frac{3}{4}=\left(-\frac{9}{2}\right)+\left(-\frac{{}^{1}\cancel{2}}{3}\right)\times\frac{11}{\cancel{4}^{2}}$$

$$=\left(-\frac{9}{2}\right)+\left(-\frac{11}{6}\right)$$

$$=\left(-\frac{27}{6}\right)+\left(-\frac{11}{6}\right)$$

$$=-\frac{38}{6}, \text{ or } -6\frac{1}{3}$$

b) Multiply first, then add.

$$\left(-3\frac{2}{5}\right)\times\left(-1\frac{5}{6}\right)+\frac{3}{10}=\left(-\frac{17}{5}\right)\left(-\frac{11}{6}\right)+\frac{3}{10}$$

$$=\frac{187}{30}+\frac{3}{10}$$

$$=\frac{187}{30}+\frac{9}{30}$$

$$=\frac{196}{30}$$

$$=\frac{98}{15}, \text{ or } 6\frac{8}{15}$$

c) Divide, multiply, then add.

$$(-3)\div\left(-\frac{4}{5}\right)+\left(-\frac{5}{12}\right)\times1\frac{1}{2}=(-3)\times\left(-\frac{5}{4}\right)+\left(-\frac{5}{12}\right)\times\frac{3}{2}$$

$$=\frac{15}{4}+\left(-\frac{5}{{}^{4}\cancel{12}}\right)\times\frac{\cancel{3}}{2}$$

$$=\frac{15}{4}+\left(-\frac{5}{8}\right)$$

$$=\frac{30}{8}+\left(-\frac{5}{8}\right)$$

$$=\frac{25}{8}, \text{ or } 3\frac{1}{8}$$

d) Add in brackets, multiply, then subtract.

$$\left(1\frac{5}{8}\right)-\left(-2\frac{3}{4}+2\right)\left(-2\frac{3}{4}+2\right)=\left(\frac{13}{8}\right)-\left(-\frac{11}{4}+2\right)\left(-\frac{11}{4}+2\right)$$

$$=\left(\frac{13}{8}\right)-\left(-\frac{11}{4}+\frac{8}{4}\right)\left(-\frac{11}{4}+\frac{8}{4}\right)$$

$$=\left(\frac{13}{8}\right)-\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)$$

$$=\frac{13}{8}-\frac{9}{16}$$

$$=\frac{13}{8}-\frac{9}{16}$$

$$=\frac{26}{16}-\frac{9}{16}$$

$$=\frac{17}{16},\text{ or }1\frac{1}{16}$$

13. a) $2.3 + (-11.2) \div (-0.2) - 3.7 = 54.6$

   b) $(-3.4) \times 0.7 - (-1.8)(-1.8) = -5.62$

   c) $\dfrac{0.67 - 4.2 \div (-0.2)}{(-7.3 + 8.6)^2} \doteq 12.82$

   d) $\dfrac{8.9 \times (-3.1 + 22.7)^2 + 4.7}{(-9.6) \div 0.04 - 0.4} \doteq -14.24$

14. a) To calculate the mean temperature, add the minimum value and the maximum value, then divide the
sum by 2.
[(–8.1) + (–16.7)] ÷ 2
= (–24.8) ÷ 2
= –12.4
The mean temperature was –12.4°C.

   b) I added first, then I divided.

15. Answers may vary. For example:

   a) $\dfrac{-3}{4}, \dfrac{10}{-8}, \dfrac{2}{-5}, \dfrac{6}{-12}$

$$\left(\frac{2}{-5}\right)\div\left(\frac{10}{-8}\right)\times\left[\left(\frac{-3}{4}\right)-\left(\frac{6}{-12}\right)\right]=\left(\frac{2}{-5}\right)\div\left(\frac{10}{-8}\right)\times\left[\left(\frac{-9}{12}\right)-\left(\frac{6}{-12}\right)\right]$$

$$=\left(\frac{2}{-5}\right)\div\left(\frac{10}{-8}\right)\times\left[\left(\frac{-9}{12}\right)+\left(\frac{6}{12}\right)\right]$$

$$=\left(\frac{2}{-5}\right)\div\left(\frac{10}{-8}\right)\times\left[\left(\frac{-3}{12}\right)\right]$$

$$=\left(\frac{2}{-5}\right)\times\left(\frac{-8}{10^{5}}\right)\times\left[\left(\frac{-1}{4}\right)\right]$$

$$=\left(\frac{{}^{2}\cancel{8}}{25}\right)\times\left(\frac{-1}{\cancel{4}^{1}}\right)$$

$$=-\frac{2}{25}$$

b) $\dfrac{2}{4}, \dfrac{-8}{10}, \dfrac{-3}{-5}, \dfrac{6}{-12}$

$$\left[\dfrac{-8}{10} + \dfrac{-3}{-5}\right] \div \dfrac{2}{4} - \left(\dfrac{6}{-12}\right) = \left[\dfrac{-8}{10} + \dfrac{6}{10}\right] \div \dfrac{2}{4} - \left(\dfrac{6}{-12}\right)$$

$$= \left(\dfrac{-2}{10}\right) \div \dfrac{2}{4} - \left(\dfrac{6}{-12}\right)$$

$$= \left(\dfrac{-\cancel{2}}{^{5}\cancel{10}}\right) \times \dfrac{\cancel{4}^{2}}{\cancel{2}^{1}} - \left(\dfrac{6}{-12}\right)$$

$$= \dfrac{-2}{5} - \left(-\dfrac{1}{2}\right)$$

$$= \dfrac{-4}{10} + \dfrac{5}{10}$$

$$= \dfrac{1}{10}$$

16. a) Since most of the maximum temperatures are below zero, I predict the mean maximum temperature is below zero.

b) $\dfrac{(-3.1) + (-4.5) + (-6.2) + (-1.2) + 1.5 + 2.3 + 4.1}{7} = \dfrac{-7.1}{7} \doteq -1.0$

The mean maximum temperature for the week was about –1.0° C.

17. In the third line, the product of 2.9 × (–5.7) is negative, not positive. In the fourth line, the student subtracted before dividing.
Correct solution:
$(-8.2)^{2} \div (-0.3) - 2.9 \times (-5.7)$
$= 67.24 \div (-0.3) - 2.9 \times (-5.7)$
$= -224.1\overline{3} - 2.9 \times (-5.7)$
$= -224.1\overline{3} - (-16.53)$
$= -224.1\overline{3} + 16.53$
$= -207.60\overline{3}$
So, the correct solution, to the nearest hundredth, is –207.60.

18. a)

$$\dfrac{23.7 - (-5.6) \div 0.7 + 6.8}{(-3) \times (-6.7) + 3.5}$$

$$= \dfrac{23.7 - (-8) + 6.8}{20.1 + 3.5}$$

$$= \dfrac{31.7 + 6.8}{23.6}$$

$$= \dfrac{38.5}{23.6}$$

$$\doteq 1.631$$

The correct answer, to the nearest hundredth, is 1.63.

b) The student likely calculated:
23.7 – (–5.6) ÷ 0.7 + 6.8 ÷ (–3) × (–6.7) + 3.5
$\doteq$ 23.7 + 5.6 ÷ 0.7 + 15.186 + 3.5
= 23.7 + 8 + 18.686
= 50.386
instead of calculating the numerator and the denominator, then finding the result of the division:
[23.7 – (–5.6) ÷ 0.7 + 6.8] ÷ [(–3) × (–6.7) + 3.5]

Take It Further

19. Multiplying by $\dfrac{5}{9}$ is equivalent to dividing by $\dfrac{9}{5}$, or 1.8.

To get from $C = \dfrac{5}{9}(F - 32)$ to $C = \dfrac{F - 32}{1.8}$:

$C = \dfrac{5}{9}(F - 32)$

$C = \dfrac{5}{9} \times (F - 32)$

$C = (F - 32) \times \dfrac{5}{9}$

$C = (F - 32) \div \dfrac{9}{5}$

$C = (F - 32) \div 1.8$

$C = \dfrac{F - 32}{1.8}$

To get from $C = \dfrac{F - 32}{1.8}$ to $C = \dfrac{5}{9}(F - 32)$:

$C = \dfrac{F - 32}{1.8}$

$C = (F - 32) \div 1.8$

$C = (F - 32) \div \dfrac{18}{10}$

$C = (F - 32) \times \dfrac{\cancel{10}^{5}}{^{9}\cancel{18}}$

$C = \dfrac{5}{9}(F - 32)$

20. Let $\square$ represent the minimum temperature.

$\dfrac{\square + (-11.5)}{2} = -12.8$

$\square + (-11.5) = -25.6$

$\square = (-25.6) - (-11.5)$

$\square = -14.1$

The minimum temperature was $-14.1°$C.

21. I used guess and test.

$-3.8 + 9.1 \times (-2.5 - 0.5) = -31.1$

To get a positive answer, I inserted the brackets so that I ended up having a product of two factors with the same sign.

$-[3.8 + 9.1 \times (-2.5) - 0.5]$

$= (-1)[3.8 - 22.75 - 0.5]$

$= (-1) \times (-19.45)$

$= 19.45$

Lesson 3.1

1. Convert all fractions to decimals.

$$-\frac{11}{3} = -3.\overline{6} \qquad -\frac{9}{4} = -2.25 \qquad -\frac{19}{6} = -3.1\overline{6}$$

To determine which rational numbers are between $-\frac{11}{3}$ and –2.5, place the rational numbers on a number line.
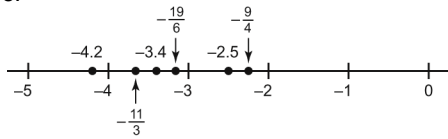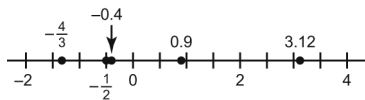

From the number line, -3.4 and $-\frac{19}{6}$ are between $-\frac{11}{3}$ and –2.5.

2. $3.12, -\frac{4}{3}, 0.9, -\frac{1}{2}, -0.4$

Write all fractions as decimals.

$$-\frac{4}{3} = -1.\overline{3} \qquad -\frac{1}{2} = -0.5$$

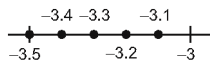Mark each number on a number line.

For least to greatest, read the numbers from left to right:

$$-\frac{4}{3}, -\frac{1}{2}, -0.4, 0.9, 3.12$$

3. Answers may vary. For example:

   a) Sketch and label a number line from –3.5 to –3.

   Mark –3.1 on the number line.

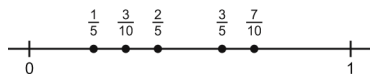   From the number line, 3 possible rational numbers are: –3.4, –3.3, –3.2

   b) Sketch and label a number line from 0 to 1.
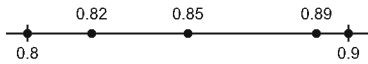
   $\frac{1}{5}$ is $\frac{2}{10}$.

   Mark $\frac{1}{5}$ and $\frac{7}{10}$ on the number line.

   From the number line, 3 possible rational numbers are: $\frac{3}{10}$, $\frac{2}{5}$, and $\frac{3}{5}$

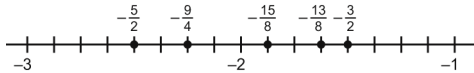**c)** Sketch and label a number line from 0.8 to 0.9.

From the number line, 3 possible rational numbers are: 0.82, 0.85, 0.89

d) Sketch and label a number line from –3 to –1. Divide the number line in eighths.
Use equivalent fractions.

$$-\frac{5}{2} = -\frac{20}{8} = -2\frac{4}{8} \qquad\qquad -\frac{3}{2} = -\frac{12}{8} = -1\frac{4}{8}$$
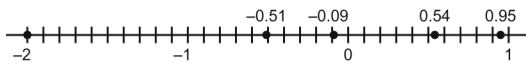
Mark $-\frac{5}{2}$ and $-\frac{3}{2}$ on the number line.

From the number line, 3 possible rational numbers are: $-\frac{9}{4}$, $-\frac{15}{8}$, and $-\frac{13}{8}$

4. –0.09, –0.51, +0.95, +0.54, –2.00
Use a number line to order the rational numbers.

The greatest loss represents the least amount.
The greatest gain represents the greatest amount.
For least to greatest, read the numbers from left to right: $-2.00, -0.51, -0.09, +0.54, +0.95$

Lesson 3.2

5. **a)** –1.2 + (–0.3) = –1.5

   b) 134.89 + (–56.45) = 78.44

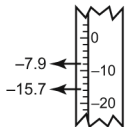   **c)** –23.6 –4.57 = –28.17

   d) 48.05 + 0.003 = 48.053

6. **a)** The temperature dropped 7.8°C from the day before, so its value the day before must have been 7.8°C
greater than –15.7°C.
Write the addition statement to represent the temperature the day before:
$-15.7 + 7.8 = -7.9$
The freezer's temperature was –7.9°C.

   b)

7. a) $\dfrac{3}{4} + \dfrac{7}{8}$

Use equivalent fractions.

$$\dfrac{3}{4} + \dfrac{7}{8} = \dfrac{6}{8} + \dfrac{7}{8}$$
$$= \dfrac{13}{8}$$

b) $-1\dfrac{1}{2} + 3\dfrac{1}{3}$

Write each mixed number as an improper fraction.

$$-1\dfrac{1}{2} + 3\dfrac{1}{3} = -\dfrac{3}{2} + \dfrac{10}{3}$$
$$= -\dfrac{9}{6} + \dfrac{20}{6}$$
$$= \dfrac{11}{6}, \text{ or } 1\dfrac{5}{6}$$

c) $-4\dfrac{5}{6} + \left(-1\dfrac{5}{12}\right)$

Write each mixed number as an improper fraction.

$$-4\dfrac{5}{6} + \left(-1\dfrac{5}{12}\right) = -\dfrac{29}{6} + \left(-\dfrac{17}{12}\right)$$
$$= -\dfrac{58}{12} + \left(-\dfrac{17}{12}\right)$$
$$= \dfrac{-58 - 17}{12}$$
$$= -\dfrac{75}{12}, \text{ or } -6\dfrac{1}{4}$$

d)

$$\dfrac{11}{9} + \left(-\dfrac{17}{6}\right) = \dfrac{22}{18} + \left(-\dfrac{51}{18}\right)$$
$$= \dfrac{22 - 51}{18}$$
$$= -\dfrac{29}{18}, \text{ or } -1\dfrac{11}{18}$$

Lesson 3.3

8. a) $(-3.4) - (-4.8) = 1.4$

b) $-71.91 - 11.23 = -83.14$

c) $90.74 - 100.38 = -9.64$

d) $63.2 - 80.02 = -16.82$

9. The stock price changed by −$0.75, so its value at the start of the day must have been $0.75 greater than $21.60. Write the subtraction statement that represents the price of stock at the beginning of the day.

$$21.60 - (-0.75) = 21.60 + 0.75$$
$$= 22.35$$

The stock price at the beginning of the day was $22.35.

10. a) $\dfrac{4}{3} - \dfrac{11}{6}$

Write equivalent fractions with common denominator 6.

$$\dfrac{4}{3} - \dfrac{11}{6} = \dfrac{8}{6} - \dfrac{11}{6}$$

$$= -\dfrac{3}{6}, \text{ or } -\dfrac{1}{2}$$

b) $-\dfrac{5}{8} - \left(-\dfrac{7}{5}\right)$

Write equivalent fractions with common denominator 40.

$$-\dfrac{5}{8} - \left(-\dfrac{7}{5}\right) = -\dfrac{25}{40} - \left(-\dfrac{56}{40}\right)$$

$$= -\dfrac{25}{40} + \dfrac{56}{40}$$

$$= \dfrac{31}{40}$$

c) $3\dfrac{5}{7} - \left(-6\dfrac{9}{10}\right)$

Write each mixed number as an improper fraction.
Then, use equivalent fractions with common denominator 70 to determine the difference.

$$3\dfrac{5}{7} - \left(-6\dfrac{9}{10}\right) = \dfrac{26}{7} - \left(-\dfrac{69}{10}\right)$$

$$= \dfrac{26}{7} + \dfrac{69}{10}$$

$$= \dfrac{260}{70} + \dfrac{483}{70}$$

$$= \dfrac{743}{70}, \text{ or } 10\dfrac{43}{70}$$

d) $-\dfrac{23}{4} - \dfrac{23}{3}$

Write equivalent fractions with common denominator 12.

$$-\dfrac{23}{4} - \dfrac{23}{3} = -\dfrac{69}{12} - \dfrac{92}{12}$$

$$= -\dfrac{161}{12}, \text{ or } -13\dfrac{5}{12}$$

Lesson 3.4

11. a) $(-1.4) \times (-0.8)$

Since both factors have the same sign, the product is positive.

1.4 can be written as $\dfrac{14}{10}$; 0.8 can be written as $\dfrac{8}{10}$.

Then, 1.4×0.8 can be written as $\dfrac{14}{10} \times \dfrac{8}{10}$, or $\dfrac{14 \times 8}{10 \times 10}$

I know that 14×8 is greater that 10×10.
So, the product $(-1.4) \times (-0.8)$ is greater than 1.

To multiply $(-1.4) \times (-0.8)$:
Since both factors have the same sign, the product is positive.
Multiply whole numbers: $14 \times 8 = 112$
Estimate to place the decimal point.
Since 1.4 is close to 1 and 0.8 is close to 1, then 1.4×0.8 is close to $1 \times 1 = 1$.
So, place the decimal point to follow the hundreds position in 112: 1.12
$(-1.4) \times (-0.8) = 1.12$

b) $25.6 \times (-0.05)$ Since the factors have different signs, the product is negative.

25.6 can be written as $\dfrac{256}{10}$; 0.05 can be written as $\dfrac{5}{100}$.

Then, 25.6×0.05 can be written as $\dfrac{256}{10} \times \dfrac{5}{100}$, or $\dfrac{256 \times 5}{10 \times 100}$

I know that 256×5 is greater that 10×100.
So, the product $25.6 \times (-0.05)$ is less than -1.

To multiply $25.6 \times (-0.05)$:
Since the factors have different signs, the product is negative.
Multiply whole numbers: $256 \times 5 = 1280$
Estimate to place the decimal point. Since 25.6 is close to 26 and 0.05 is close to 0.1, then
25.6×0.05 is close to $26 \times 0.1 = 2.6$.
So, place the decimal point after 1 in 1280: 1.28
$25.6 \times (-0.05) = -1.28$

c) $\left(-\dfrac{3}{5}\right)\left(\dfrac{4}{3}\right)$

Since the factors have opposite signs, the product is negative.

Think of the product: $\left(\dfrac{3}{5}\right)\left(\dfrac{4}{3}\right)$

The numerator is less than the denominator: $3 \times 4 < 5 \times 3$, or $\dfrac{3 \times 4}{5 \times 3} < 1$

Then, $\left(-\dfrac{3}{5}\right)\left(\dfrac{4}{3}\right)$ is negative and greater than -1, or between -1 and 0.

$$\left(-\dfrac{3}{5}\right)\left(\dfrac{4}{3}\right) = \left(-\dfrac{{}^{1}\cancel{3}}{5}\right)\left(\dfrac{4}{\cancel{3}^{1}}\right)$$

$$= -\dfrac{4}{5}$$

d) $\left(-\dfrac{5}{6}\right)\left(-\dfrac{2}{3}\right)$

Since both factors have the same sign, the product is positive.

The numerator of the product is less than the denominator: $5 \times 2 < 6 \times 3$, or $\dfrac{5 \times 2}{6 \times 3} < 1$

Then, the product $\left(-\dfrac{5}{6}\right)\left(-\dfrac{2}{3}\right)$ is positive and less than 1.

$$\left(-\dfrac{5}{6}\right)\left(-\dfrac{2}{3}\right) = \left(-\dfrac{5}{{}^{3}\cancel{6}}\right)\left(-\dfrac{\cancel{2}^{1}}{3}\right)$$

$$= \dfrac{5}{9}$$

The products in parts c and d are between -1 and 1.

12. The temperature drops 1.3°C per hour from 4 P.M. to 11 P.M., or for 7 h.
$7 \times (-1.3) = -9.1$; so the temperature drops by –9.1°C.

The temperature at 11:00 P.M. is 9.1°C less than the temperature at 4 P.M.
Write the subtraction sentence to represent the temperature at 11:00 P.M.:
2°C – 9.1°C = –7.1°C

13. Multiplication is commutative. So, the order of the factors does not matter.

I can write equivalent multiplication statements with the same numerators and denominators as $\left(-\dfrac{4}{9}\right)\left(\dfrac{7}{5}\right)$,

only in different positions. For example: $\left(-\dfrac{7}{9}\right)\left(\dfrac{4}{5}\right), \left(-\dfrac{4}{5}\right)\left(\dfrac{7}{9}\right)$;

I can also change the position of the negative sign. For example: $\left(\dfrac{4}{9}\right)\left(-\dfrac{7}{5}\right), \left(\dfrac{7}{9}\right)\left(-\dfrac{4}{5}\right)$

I can check by finding each product:

Each product is $-\dfrac{28}{45}$.

14. a) $3.5 \times (-0.3)$
Since the factors have different signs, the product is negative.
Multiply whole numbers.
35 × 3 = 105
Estimate to place the decimal point.
Since 3.5 is close to 4, then 3.5 × 0.3 is close to 4 × 0.3, or 1.2.
Place the decimal point after 1 in 105: 1.05
So, $3.5 \times (-0.3) = -1.05$

b) (–4.1)(2.3)
Since the factors have different signs, the product is negative.
Multiply whole numbers.
41 × 23 = 943
Estimate to place the decimal point.
Since 4.1 is close to 4 and 2.3 is close to 2, (4.1)(2.3) is close to 4 × 2, or 8.
Place the decimal point after 9 in 943: 9.43
So, (–4.1)(2.3) = –9.43

c)

$$\left(-\frac{4}{7}\right)\left(-\frac{2}{3}\right) = \frac{(-4)(-2)}{7 \times 3}$$
$$= \frac{8}{21}$$

d) $1\dfrac{3}{5} \times \left(-2\dfrac{1}{2}\right)$

Write each mixed number as an improper fraction, then simplify.

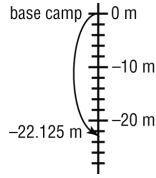$$1\frac{3}{5} \times \left(-2\frac{1}{2}\right) = \frac{\overset{4}{\cancel{8}}}{\cancel{5}} \times \left(-\frac{\overset{1}{\cancel{5}}}{\cancel{2}^{1}}\right)$$
$$= -4$$

15. Since the climber is descending, his average speed is negative.
Write a multiplication statement to represent the distance from base camp after 3.75 h at an average speed of –5.9 m/h:
$-5.9 \times 3.75 = -22.125$
The climber will be 22.125 m below base camp.

Lesson 3.5

16. a) $(-2.2) \div 0.4$
$(-2) \div 0.5$ is –4.
So, is less than –4.
Determine the numerical value of the quotient: $2.2 \div 0.4$
Estimate first; use compatible numbers.
From above, $2.2 \div 0.4$ is close to $2 \div 0.5$, or 4.
Divide whole numbers.
$22 \div 4 = 5.5$
The decimal place is correctly placed: 5.5
So, $(-2.2) \div 0.4 = -5.5$

b) $10.6 \div (-9.2)$
Since the dividend and the divisor have opposite signs, the quotient is negative.
Since 10.6 > 9.2, the dividend is farther from 0 than the divisor. So, the quotient is less than –1.
Determine the numerical value of the quotient.
$10.6 \div (-9.2) \doteq -1.15$

c) $\dfrac{9}{10} \div \left(-\dfrac{3}{2}\right)$

Since the dividend and the divisor have different signs, the quotient is negative.

Write $\dfrac{9}{10} \div \left(-\dfrac{3}{2}\right)$ as $\left(-\dfrac{9}{10}\right) \div \left(\dfrac{3}{2}\right)$.

Since $-\dfrac{9}{10} = -0.9$ is greater than –1 and $\dfrac{3}{2}$ is greater than 1, then the quotient $\dfrac{9}{10} \div \left(-\dfrac{3}{2}\right)$ is negative

and greater than –1.
Multiply by the reciprocal of the divisor and simplify:

$$\frac{9}{10} \div \left(-\frac{3}{2}\right) = \frac{^3\cancel{9}}{^5\cancel{10}} \times \left(-\frac{\cancel{2}^1}{\cancel{3}^1}\right)$$

$$= -\frac{3}{5}$$

d) $\left(-\dfrac{5}{12}\right) \div \left(-\dfrac{5}{4}\right)$

Since the dividend and the divisor have the same sign, the quotient is positive.

So, I can ignore the signs and write $\left(-\dfrac{5}{12}\right) \div \left(-\dfrac{5}{4}\right)$ as $\dfrac{5}{12} \div \dfrac{5}{4}$.

Since $\dfrac{5}{12} < 1$, $\dfrac{5}{4} > 1$, then $\dfrac{5}{12} \div \dfrac{5}{4}$ is between 0 and 1.

$$\left(-\dfrac{5}{12}\right) \div \left(-\dfrac{5}{4}\right) = \dfrac{5}{12} \div \dfrac{5}{4}$$

$$= \dfrac{{}^1\cancel{5}}{{}^3\cancel{12}} \times \dfrac{\cancel{4}^{}}{\cancel{5}^1}$$

$$= \dfrac{1}{3}$$

The quotients in parts c and d are between −1 and 1.

17. $\dfrac{3}{8} \div \left(-\dfrac{5}{11}\right)$

Answers may vary. For example:

Use the strategy of multiplying by the reciprocal of the divisor.

$$\dfrac{3}{8} \div \left(-\dfrac{5}{11}\right) = \dfrac{3}{8} \times \left(-\dfrac{11}{5}\right)$$

Multiplication is commutative.

To write equivalent products, interchange the position of the numerators, the position of the denominators, or change the position of the negative sign. Then, write the equivalent division sentences.

For example:

$$\dfrac{11}{8} \times \left(-\dfrac{3}{5}\right); \quad \left(\dfrac{11}{8}\right) \div \left(-\dfrac{5}{3}\right) = -\dfrac{33}{40}$$

$$\dfrac{3}{5} \times \left(-\dfrac{11}{8}\right); \quad \left(\dfrac{3}{5}\right) \div \left(-\dfrac{8}{11}\right) = -\dfrac{33}{40}$$

$$\dfrac{11}{5} \times \left(-\dfrac{3}{8}\right); \quad \left(\dfrac{11}{5}\right) \div \left(-\dfrac{8}{3}\right) = -\dfrac{33}{40}$$

18. a) Rewrite the equation as a division statement.

$$(-0.2) \times \square = 0.75$$
$$\square = 0.75 \div (-0.2)$$
$$\square = -3.75$$

b) Rewrite the equation as a division statement.

$$0.9 \times \square = -7.47$$
$$\square = -7.47 \div 0.9$$
$$\square = -8.3$$

c) Write a related division statement to solve for the missing number.

$$(-0.624) \div \square = -0.4$$
$$\square = (-0.624) \div (-0.4)$$
$$\square = 1.56$$

19. a) $8.4 \div (-1.2) = -7$

b) $(-20.6) \div (-0.9) = 22.\overline{8}$

c)

$$\left(-\frac{9}{11}\right) \div \frac{7}{5} = \left(-\frac{9}{11}\right) \times \frac{5}{7}$$
$$= -\frac{45}{77}$$

d)

$$\left(-1\frac{2}{3}\right) \div 3\frac{1}{2} = \left(-\frac{5}{3}\right) \div \frac{6}{2}$$
$$= \left(-\frac{5}{3}\right) \times \frac{1}{3}$$
$$= -\frac{5}{9}$$

Lesson 3.6

20. a) i) $-3.5 + 6.2 \times (-0.2)$
$$= -3.5 + (-1.24)$$
$$= -4.74$$

ii) $(-3.5 + 6.2) \times (-0.2)$
$$= 2.7 \times (-0.2)$$
$$= -0.54$$

b) The answer in part i is different from the answer in part ii.
In part i, I multiplied before I added. In part ii, I added before I multiplied.

21. a) $-\frac{3}{5} + \left[\frac{1}{3} \times \left(-\frac{3}{4}\right)\right]$

Start with the brackets.
The product of a negative number and a positive number is negative.
Then, the sum of 2 negative numbers is negative. I predict the value is negative.
Evaluate:

$$-\frac{3}{5} + \left[\frac{1}{3} \times \left(-\frac{3}{4}\right)\right] = -\frac{3}{5} + \left[\frac{1}{{}^1\cancel{3}} \times \left(-\frac{\cancel{3}^{\,1}}{4}\right)\right]$$
$$= -\frac{3}{5} + \left[-\frac{1}{4}\right]$$
$$= -\frac{12}{20} + \left[-\frac{5}{20}\right]$$
$$= \frac{-12-5}{20}$$
$$= -\frac{17}{20}$$

b) $\left(-\dfrac{3}{5}+\dfrac{1}{3}\right)\times\left(-\dfrac{3}{4}\right)$

The sum in brackets is negative, since $-\dfrac{3}{5}$ is farther from 0 than $\dfrac{1}{3}$ is.

Then, the product of 2 negative numbers is positive. I predict the value is positive.

Evaluate:

$$\left(-\dfrac{3}{5}+\dfrac{1}{3}\right)\times\left(-\dfrac{3}{4}\right)=\left(-\dfrac{9}{15}+\dfrac{5}{15}\right)\times\left(-\dfrac{3}{4}\right)$$

$$=-\dfrac{{}^{1}\cancel{4}}{{}_{5}\cancel{15}}\times\left(-\dfrac{\cancel{3}}{\cancel{4}^{1}}\right)$$

$$=\dfrac{1}{5}$$

c) $-\left(-\dfrac{3}{5}+\dfrac{1}{3}\right)\times\left(-\dfrac{3}{4}\right)$

The sum in brackets is negative, but the negative sign in front of the brackets changes the sign to positive.

Then, the product of a positive number and a negative number is negative. I predict the value is negative.

Evaluate:

$$-\left(-\dfrac{3}{5}+\dfrac{1}{3}\right)\times\left(-\dfrac{3}{4}\right)=-\left(-\dfrac{3}{5}+\dfrac{1}{3}\right)\times\left(-\dfrac{3}{4}\right)$$

$$=-\left(-\dfrac{9}{15}+\dfrac{5}{15}\right)\times\left(-\dfrac{3}{4}\right)$$

$$=-\left(-\dfrac{{}^{1}\cancel{4}}{{}_{5}\cancel{15}}\right)\times\left(-\dfrac{\cancel{3}}{\cancel{4}^{1}}\right)$$

$$=-\left(-\dfrac{1}{5}\right)(-1)$$

$$=-\dfrac{1}{5}$$

22. a) Substitute length = 25.3, width = 15.2, and height = 9.7 in the surface area formula for a right rectangular prism.

$2[25.3\times15.2+25.3\times9.7+15.2\times9.7]$

= 2[384.56 + 245.41 + 147.44]

= 2 × 777.41

= 1554.82

The surface area is 1554.82 cm^2

b) I multiplied the products in the brackets first. Then, I added the products. Finally, I multiplied by 2.

23. a) $-1.2\div(0.6)-\left[6.3+(-3.4)\right]$

$=-1.2\div(0.6)-(2.9)$

$=-2-2.9$

$=-4.9$

b) $-\dfrac{5}{12}+\left(\dfrac{4}{3}\right)\left(\dfrac{4}{3}\right)$

$=-\dfrac{5}{12}+\dfrac{16}{9}$

$=-\dfrac{15}{36}+\dfrac{64}{36}$

$=\dfrac{49}{36}$

$=1\dfrac{13}{36}$

c) $-\dfrac{4}{5}\div\left[\dfrac{1}{2}+\left(-\dfrac{1}{6}\right)\left(-\dfrac{1}{6}\right)\times\dfrac{1}{4}\right]$

$=-\dfrac{4}{5}\div\left[\dfrac{1}{2}+\dfrac{1}{36}\times\dfrac{1}{4}\right]$

$=-\dfrac{4}{5}\div\left[\dfrac{1}{2}+\dfrac{1}{144}\right]$

$=-\dfrac{4}{5}\div\left[\dfrac{72}{144}+\dfrac{1}{144}\right]$

$=-\dfrac{4}{5}\div\dfrac{73}{144}$

$=-\dfrac{4}{5}\times\dfrac{144}{73}$

$=-\dfrac{576}{365}$

$=-1\dfrac{211}{365}$

d) $\left(-\dfrac{2}{3}\right)\left(-\dfrac{2}{3}\right)\div\dfrac{2}{9}-\left(-\dfrac{4}{5}\right)$

$=\dfrac{4}{9}\div\dfrac{2}{9}-\left(-\dfrac{4}{5}\right)$

$=\dfrac{4}{2}-\left(-\dfrac{4}{5}\right)$

$=2+\dfrac{4}{5}$

$=2\dfrac{4}{5}$

e) $-1\dfrac{3}{7}\times\dfrac{1}{2}+\left(-3\dfrac{1}{7}\right)$

$=-\dfrac{\overset{5}{\cancel{10}}}{7}\times\dfrac{1}{\cancel{2}^{1}}+\left(-\dfrac{22}{7}\right)$

$=-\dfrac{5}{7}+\left(-\dfrac{22}{7}\right)$

$=-\dfrac{27}{7}$

$=-3\dfrac{6}{7}$

f) $0.2 - (-1.2) \times 0.5 \div (-0.1)$

$= 0.2 - (-0.6) \div (-0.1)$

$= 0.2 - 6$

$= -5.8$

g) $(-0.2 + 0.9)^2 + 9.8 \div (-0.7)$

$= (0.7)^2 + 9.8 \div (-0.7)$

$= 0.49 + 9.8 \div (-0.7)$

$= 0.49 + (-14)$

$= -13.51$

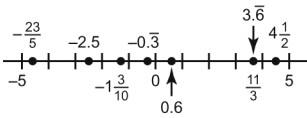**Practice Test**   **(page 146)**

1. Answers may vary. For example:
    a) –0.55 is between –0.5 and –0.6.

    b) –0.55 is a rational number because I can write it as a fraction with an integer numerator and non-zero
    integer denominator: $\dfrac{-55}{100}$

2. **a)** $0.6, -0.\overline{3}, -2.5, 3.\overline{6},\ 4\dfrac{1}{2}, -1\dfrac{3}{10}, -\dfrac{23}{5}, \dfrac{11}{3}$

    Mark each number on the number line.

    b) For greatest to least, read the numbers on the number line from right to left:

    $4\dfrac{1}{2}, \dfrac{11}{3} = 3.\overline{6},\ 0.6,\ -0.\overline{3},\ -1\dfrac{3}{10},\ -2.5,\ -\dfrac{23}{5}$

3. **a)** $-7.4 - (-6.1)$
        $= -7.4 + 6.1$
        $= -1.3$

    b)

    $$\dfrac{4}{5} + \left(-\dfrac{3}{10}\right) = \dfrac{8}{10} + \left(-\dfrac{3}{10}\right)$$
    $$= \dfrac{5}{10}$$
    $$= \dfrac{1}{2}$$

    c) $(-3.2)(-0.5)$
    Since both factors have the same sign, the product is positive.
    Multiply whole numbers.
    $32 \times 5 = 160$
    Estimate to place the decimal point.
    Since 3.2 is close to 3, 3.2×0.5 is close to 3×0.5, or 1.5.
    Place the decimal point after 1 in 160: 1.60
    So, $(-3.2)(-0.5) = 1.6$

    d)

    $$\left(-\dfrac{3}{4}\right) \div \dfrac{1}{3} = \left(-\dfrac{3}{4}\right) \times \dfrac{3}{1}$$
    $$= -\dfrac{9}{4}$$

4. **a)** Sarah has a negative balance. This means she owes the bank $2.34.

b) Write the addition statement to represent Sarah's new balance after three more withdrawals of $20.50 each. The $1.20 charge is applied at each withdrawal.

–2.34 + 3(–20.50) + 3(–1.20)

= –2.34 + (–61.50) + (–3.6)

= –67.44

Sarah has a balance of –$67.44.

c) $-500 - (-67.44) = -432.56$; Sarah can increase her debt by $432.56.

$-432.56 \div (-20.50 - 1.20)$

$= -432.56 \div (-21.7)$

$\doteq 19.93$

Sarah can make another 19 withdrawals.

5. a) (–56.8)(–14.5)

Since both factors are negative, the product is positive.

(–56.8)(–14.5) = 823.6

b) $\left(-3\dfrac{1}{3}\right)\left(-2\dfrac{3}{10}\right)$

Write each mixed number as an improper fraction.

$\left(-3\dfrac{1}{3}\right)\left(-2\dfrac{3}{10}\right) = \left(-\dfrac{{}^1\cancel{10}}{3}\right)\left(-\dfrac{23}{\cancel{10}^1}\right)$

$= \dfrac{23}{3}$, or $7\dfrac{2}{3}$

c) $\left(-4\dfrac{2}{5}\right) \div \left(-1\dfrac{5}{7}\right)$

Write each mixed number as an improper fraction.

$\left(-4\dfrac{2}{5}\right) \div \left(-1\dfrac{5}{7}\right) = \left(-\dfrac{22}{5}\right) \div \left(-\dfrac{12}{7}\right)$

$= \left(-\dfrac{{}^{11}\cancel{22}}{5}\right) \times \left(-\dfrac{7}{\cancel{12}^6}\right)$

$= \dfrac{77}{30}$, or $2\dfrac{17}{30}$

d) 45.8 ÷ (–12.2)

Since the dividend and the divisor have different signs, the quotient is negative.

45.8 ÷ (–12.2) $\doteq$ –3.75

I could check my answers by using a calculator. I could also check the division by multiplying.

6. a) $\dfrac{1}{2} + \left(-\dfrac{3}{4}\right) \div \left(-\dfrac{1}{4}\right)$

$= \dfrac{1}{2} + \left(-\dfrac{3}{{}^1\cancel{4}}\right) \times \left(-\dfrac{\cancel{4}^1}{1}\right)$

$= \dfrac{1}{2} + 3$

$= 3\dfrac{1}{2}$

The correct answer is $3\dfrac{1}{2}$.

b) The student added before dividing:

$$\frac{1}{2} + \left(-\frac{3}{4}\right) \div \left(-\frac{1}{4}\right)$$

$$= \left[\frac{1}{2} + \left(-\frac{3}{4}\right)\right] \div \left(-\frac{1}{4}\right)$$

$$= \left(\frac{2}{4} - \frac{3}{4}\right) \div \left(-\frac{1}{4}\right)$$

$$= \left(-\frac{1}{4}\right) \div \left(-\frac{1}{4}\right)$$

$$= 1$$

7.   **a)** $-3.1 + 4.5 \times (-2.9) - 7.2 \div (-3)$

$= -3.1 + (-13.05) - (-2.4)$

$= -13.75$

b) $(-9.7) \times (-1.2) + 5.4^2 \div (-3.6)$

$= 11.64 + 29.16 \div (-3.6)$

$= 11.64 + (-8.1)$

$= 3.54$

Unit Problem   **Investigating Temperature Data**   **(page 147)**

1. To determine the mean monthly lowest temperature, add the monthly temperatures in the table and divide the sum by the number of months in a year.
    Mean monthly lowest temperature =
    $$\frac{(-14.7)+(-12.8)+(-7.2)+0.5+5.9+10.1+12.3+10.6+6.1+(-0.8)+(-8.3)+(-13.4)}{12}$$
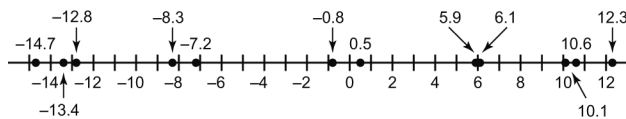
    $$=\frac{-11.7}{12}$$
    $$=-0.975$$

    a) I used the order of operations when I added first, then divided. Usually, I'd divide first but because the sum was in the numerator of a fraction, it was the same as being in brackets.
    The mean monthly lowest temperature in Edmonton for that year was about –1.0°C.

    b) Sketch a number line. Mark each temperature on the number line.

    For the temperatures below the mean, read the temperatures to the left of –0.9 on the number line.
    From least to greatest, the temperatures below the mean are: –14.7, –13.4, –12.8, –8.3, and –7.2

2. To determine the mean monthly highest temperature, add the monthly temperatures in the table and divide the sum by the number of months in a year.
    Mean monthly highest temperature =
    $$\frac{(-5.8)+(-2.9)+2.1+10.6+17.2+20.7+23.4+22.2+16.7+9.8+1.1+(-4.3)}{12}$$

    $$=\frac{110.8}{12}$$
    $$=9.2\overline{3}$$
    The mean monthly highest temperature in Edmonton for that year was about 9.2°C.

3. Answers may vary. For example:
    I choose June, July, and August.

    a) June:
    the mean monthly highest temperature: 9.2°C
    the monthly highest temperature: 20.7°C
    Difference: 9.2°C – 20.7°C = –11.5°C

    July:
    the mean monthly highest temperature: 9.2°C
    the monthly highest temperature: 23.4°C
    9.2°C – 23.4°C = –14.2°C

    August:
    the mean monthly highest temperature: 9.2°C
    the monthly highest temperature: 22.2°C
    Difference: 9.2°C – 22.2°C = –13°C

---

b) June:
the mean monthly lowest temperature: –0.9°C
the monthly lowest temperature: 10.1°C
Difference: –1.0°C – 10.1°C = –11.1°C

July:
the mean monthly lowest temperature: –0.9°C
the monthly lowest temperature: 12.3°C
Difference: –1.0°C – 12.3°C = –13.3°C

August:
the mean monthly lowest temperature: –0.9°C
the monthly lowest temperature: 10.6°C
Difference: –1.0°C – 10.6°C = –11.6°C

4. **a)** Determine the new mean monthly highest temperature for an increase of 1.4°C:
9.2°C + 1.4°C = 10.6°C
Determine the new mean monthly highest temperature for an increase of 11°C:
9.2°C + 11°C = 20.2°C
So, the mean monthly highest temperature for an increase between 1.4°C and 11°C will range from 10.6°C to 20.2°C.
Range: 20.2°C – 10.6°C = 9.6°C

b) Determine the new mean monthly lowest temperature for an increase of 1.4°C:
–1.0°C + 1.4°C = 0.4°C
Determine the new mean monthly lowest temperature for an increase of 11°C:
–1.0°C + 11°C = 10°C
So, the mean monthly lowest temperature for an increase between 1.4°C and 11°C will range from 0.4°C to 10°C.
Range: 10°C – 0.4°C = 9.6 °C