Lesson 5.1 Modelling Polynomials Practice (pages 214–216)

Check

4. Expressions a, c, d, and f are polynomials because they contain terms whose variables have whole-number exponents only. The terms in the polynomial are of degree 1, 2, or a constant (degree 0).

   Expression b is not a polynomial because it contains the square root of a variable.
   Expression e is not a polynomial because it contains a variable in the denominator.

5. Count the number of terms of different degrees in each polynomial.
   a) Trinomial; it has three terms of different degrees.

   b) Binomial; it has two terms of different degrees.

   c) Monomial: it has only one term of degree 1.

   d) Monomial: it has only one term of degree 0.

6. **a)** Coefficient –7, variable *x,* degree 1

   b) Coefficient 14, variable *a,* degree 2

   c) Coefficient 1, variable *m,* degree 1

   d) No coefficient, no variable, degree 0

7. Identify the variable with the greatest exponent in each polynomial.
   a) Degree 2; the variable with the greatest exponent is j^2

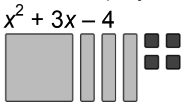
   b) Degree 1; the variable with the greatest exponent is x

   c) Degree 2; the variable with the greatest exponent is p^2

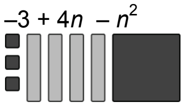
   d) Degree 0; this polynomial does not have a variable, so there is no exponent.

Apply

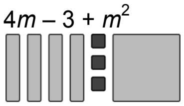
8. Model each polynomial and compare.
   a) $x^2 + 3x - 4$

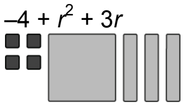
   b) $-3 + 4n - n^2$

   c) $4m - 3 + m^2$

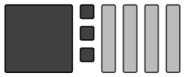
   d) $-4 + r^2 + 3r$

   This matches polynomial a.

e) $-3m^2 + 4m - 3$

f) $-h^2 - 3 + 4h$



This matches polynomial b.

The matching polynomials are a and d; and b and f.

9. **a)** Coefficients 5, –6, variable x, degree 2, constant term 2

b) Coefficient 7, variable b, degree 1, constant term –8

c) Coefficient 12, variable c, degree 2, constant term 2

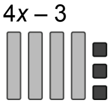
d) Coefficient 12, variable m, degree 1, no constant term

e) No coefficients, no variable, degree 0, constant term 18

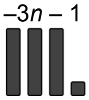
f) Coefficients 5, –8, variable x, degree 2, constant term 3

10. Both students are correct. $4a$ is a polynomial because its term has a variable with a whole-number exponent. It is also a monomial because it is a polynomial with only one term.

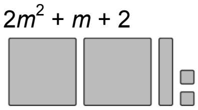
11. **a)** $4x - 3$



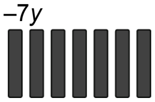
b) $-3n - 1$



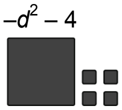
c) $2m^2 + m + 2$



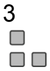
d) $-7y$



e) $-d^2 - 4$



f) 3



12. a) $r^2 - r + 3$ is represented with one r^2-tile, one $-r$-tile, and three 1-tiles. This matches Model B.

b) $-t^2 - 3$ is represented with one t^2-tile and three -3-tiles. This matches Model D.

c) $-2v$ is represented with two $-v$-tiles. This matches Model E.

d) $2w + 2$ is represented with two w-tiles and two 1-tiles. This matches Model A.

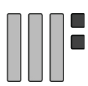
e) $2s^2 - 2s + 1$ is represented with two s^2-tiles, two $-s$-tiles, and one 1-tile. This matches Model C.

13. Use a table.

Part	Description of Tiles	Polynomial	Description of Polynomial
a	sixteen -1-tiles	-16	monomial, since there is only one term
b	one x-tile and eight -1-tiles	$x - 8$	binomial, since there are two terms of different degrees
c	four x-tiles	$4x$	monomial, since there is only one term
d	two x^2-tiles, eight $-x$-tiles, and three 1-tiles	$2x^2 - 8x + 3$	trinomial, since there are three terms of different degrees
e	five 1-tiles and five $-x$-tiles	$5 - 5x$	binomial, since there are two terms of different degrees
f	five x^2-tiles	$5x^2$	monomial, since there is only one term
g	two $-x^2$-tiles, two x-tiles, and three -1-tiles	$-2x^2 + 2x - 3$	trinomial, since there are three terms of different degrees
h	three $-x^2$-tiles and eight 1-tiles	$-3x^2 + 8$	binomial, since there are two terms of different degrees

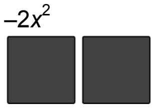
14. Answers may vary; for example:

a) $3x - 2$

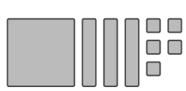


b) 5

c) $-2x^2$



d) $x^2 + 3x + 5$



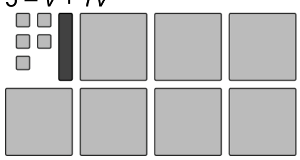
15. Use a table.

Model	Description of Tiles	Polynomial
a	two x^2-tiles, three x-tiles, four 1-tiles	$2x^2 + 3x - 4$
b	two $-x^2$-tiles, two x-tiles, four 1-tiles	$-2x^2 + 2x + 4$
c	three x^2-tiles, two $-x$-tiles, four 1-tiles	$3x^2 - 2x + 4$
d	two $-x^2$-tiles, two x-tiles, four 1-tiles	$-2x^2 + 2x + 4$
e	three x^2-tiles, two $-x$-tiles, four 1-tiles	$3x^2 - 2x + 4$
f	two x^2-tiles, three x-tiles, four 1-tiles	$2x^2 + 3x - 4$
g	two $-x^2$-tiles, five x-tiles, two -1-tiles	$-2x^2 + 5x - 2$
h	two $-x^2$-tiles, two x-tiles, four 1-tiles	$-2x^2 + 2x + 4$
i	two $-x^2$-tiles, five x-tiles, two -1-tiles	$-2x^2 + 5x - 2$

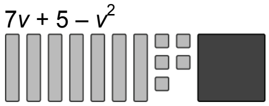
Look for matching results.
Models a and f are equivalent. They represent the same polynomial, $2x^2 + 3x - 4$.
Models b, d, and h are equivalent. They represent the same polynomial, $-2x^2 + 2x + 4$.
Models c and e are equivalent. They represent the same polynomial, $3x^2 - 2x + 4$.
Models g and i are equivalent. They represent the same polynomial, $-2x^2 + 5x - 2$.

16. Model each polynomial with algebra tiles.

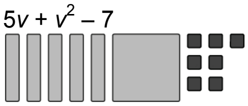
a) $5 - v + 7v^2$



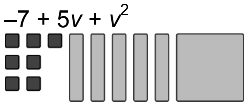
b) $7v + 5 - v^2$



c) $5v + v^2 - 7$

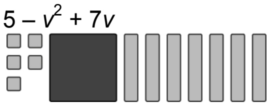


d) $-7 + 5v + v^2$



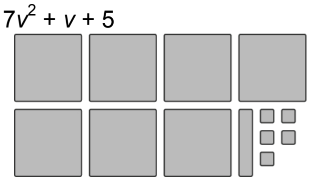
This matches polynomial c.

e) $5 - v^2 + 7v$



This matches polynomial b.

f) $7v^2 + v + 5$



Polynomials b and e are equivalent.
They can be represented with the same algebra tiles and both can be written as $-v^2 + 7v + 5$.
Polynomials c and d are equivalent.
They can be represented with the same algebra tiles and both can be written as $v^2 + 5v - 7$.

17. An expression that contains the square root of a variable is not a polynomial.
So, for example, $2\sqrt{x}$ is not a polynomial.

18. a) i) $-2x - 3x^2 + 4$

Variable: *x*; degree: 2; number of terms: 3; coefficients: –2, –3

ii) $m^2 + m$

Variable: *m*; degree: 2; number of terms: 2; coefficients: 1, 1

b) Build the polynomial. Since it is a binomial, the polynomial will have two terms $\square + \square$. The polynomial is in variable *c*, of degree 2: $3c^2 + \square$. The constant term is –5: $3c^2 - 5$. So, a possible solution is $3c^2 - 5$.

c) Look for a polynomial equivalent to $3c^2 - 5$.
Rewrite the polynomial, but in ascending order: $-5 + 3c^2$.
$3c^2 - 5$ and $-5 + 3c^2$ are equivalent because they can be represented by the same algebra tiles. They have identical terms with the same degree, only the order is different.

19. a) $-8d^2 - 3d - 4$ $\quad$ $-3d - 4 - 8d^2$
$-8d^2 - 4 - 3d$ $\quad$ $-4 - 8d^2 - 3d - 4 - 3d - 8d^2$
$-3d - 8d^2 - 4$ $\quad$ $-4 - 3d - 8d^2$
Using an organized list, I know there are 6 possible arrangements of the three terms.

b) $-8d^2 - 3d - 4$ is in descending order because the exponents of the variable decrease from left to right. It is useful to write a polynomial in this form because you can immediately see the degree and the constant.

Take It Further

20. a) i) Substitute $s = 25$ in $0.4s + 0.02s^2$.

$$\begin{aligned} 0.4(25) + 0.02(25)^2 &= 10 + 12.5 \\ &= 22.5 \end{aligned}$$

The stopping distance of the car is 22.5 m.

ii) Substitute $s = 50$ in $0.4s + 0.02s^2$.

$$\begin{aligned} 0.4(50) + 0.02(50)^2 &= 20 + 50 \\ &= 70 \end{aligned}$$

The stopping distance of the car is 70 m.

iii) Substitute $s = 100$ in $0.4s + 0.02s^2$.

$$\begin{aligned} 0.4(100) + 0.02(100)^2 &= 40 + 200 \\ &= 240 \end{aligned}$$

The stopping distance of the car is 240 m.

b) No, doubling the speed more than doubles the stopping distance. From part a, when the speed is 25 km/h, the stopping distance is 22.5 m, and when the speed is 50 km/h, the stopping distance is 70 m; this is a stopping distance that is 3 times as far. The relationship between speed and stopping distance is not a linear relationship.

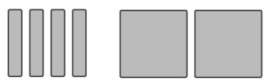
Lesson 5.2 Like Terms and Unlike Terms Practice (pages 222–224)

Check

4. a) $3d$ $-5d$

b) $3d$ and $-5d$ are like terms because they can both be modelled by algebra tiles of the same size and shape. Each term has the same variable, d, raised to the same exponent, 1.

5. a) $4p$ $2p^2$



b) $4p$ and $2p^2$ are not like terms because they cannot be modelled by algebra tiles of the same size and shape. Each term has the same variable, p, but the exponents are different.

Apply

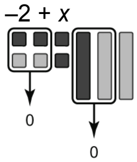
6. $-3x$, $3x$, and $7x$ are like $8x$; these terms have the same variable, x, raised to the same exponent, 1. Each term can be modelled with x-tiles.

7. $-n^2$, $2n^2$, and $5n^2$ are like $-2n^2$; these terms have the same variable, n, raised to the same exponent, 2. Each term can be modelled with x^2-tiles.

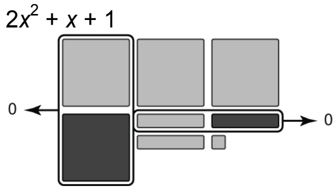
8. I group like tiles and remove zero pairs. The remaining tiles represent:

a) $x + 4$

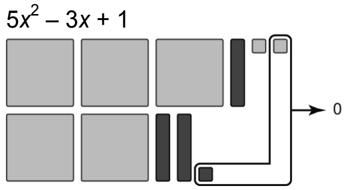
b) $-2 + x$



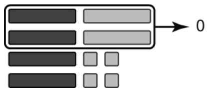
c) $2x^2 + x + 1$



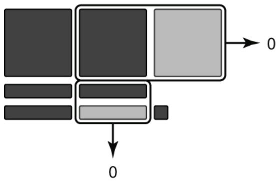
d) $5x^2 - 3x + 1$



e) $-2x + 4$



f) $-x^2 - 2x - 1$



9. Use a table.

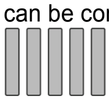
Tile Model	Symbolic Record	Simplified Polynomial
a	$x^2 + x^2 + 1$	$2x^2 + 1$
b	$x^2 - 2x - 5 + x + 2 - x^2$	$-x - 3$
c	$-x^2 + 2$	$-x^2 + 2$
d	$x^2 - x^2 + 4x + 3 - x^2 - 2x - 3$	$-x^2 + 2x$
e	$2x^2 + 4 - 3x - 3 + 3x$	$2x^2 + 1$
f	$-x - 3$	$-x - 3$

Polynomials a and e are equivalent since both simplify to $2x^2 + 1$.

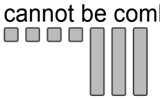
Polynomials b and f are equivalent since both simplify to $-x - 3$.

Polynomials c and d are equivalent since both simplify to $-x^2 + 2x$.

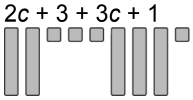
10. When you look at the tiles for $2x + 3x$, you can see that they are tiles of the same size and shape, and so can be combined. Two x-tiles and three x-tiles make five x-tiles. The correct answer is $5x$.



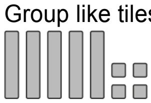
When you look at the tiles for $4 + 3x$, you can see that they are tiles of different size and shape, and so cannot be combined. $4 + 3x$ cannot be simplified any further.



11. a) $2c + 3 + 3c + 1$

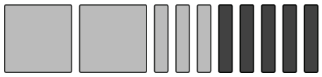


Group like tiles.

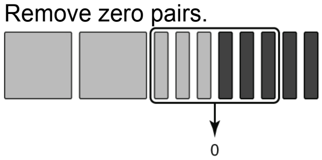


The combined tiles represent $5c + 4$.

b) $2x^2 + 3x - 5x$

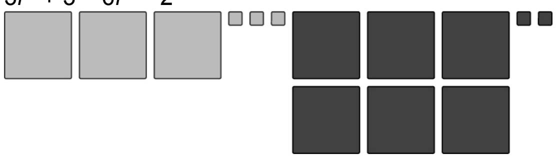


Remove zero pairs.

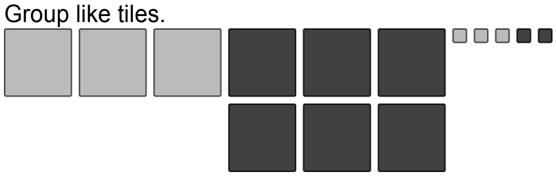


The combined tiles represent $2x^2 - 2x$.

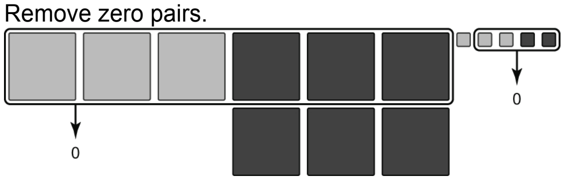
c) $3f^2 + 3 - 6f^2 - 2$



Group like tiles.

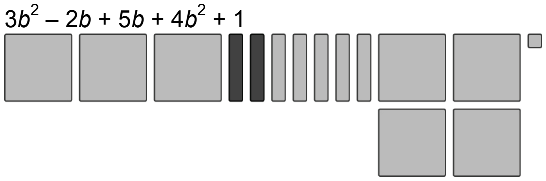


Remove zero pairs.

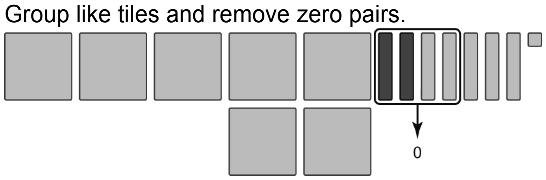


The combined tiles represent $-3f^2 + 1$.

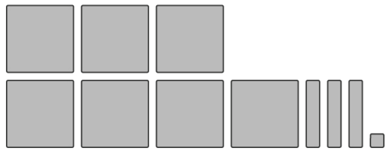
d) $3b^2 - 2b + 5b + 4b^2 + 1$



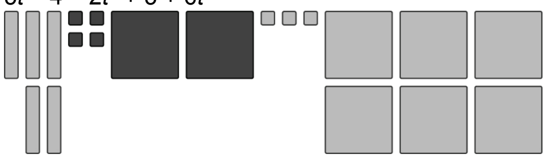
Group like tiles and remove zero pairs.



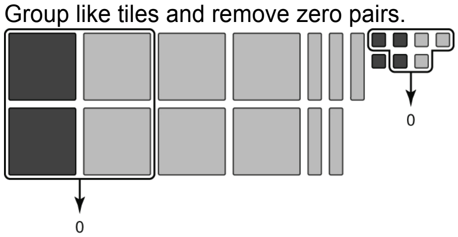
The combined tiles represent $7b^2 + 3b + 1$.



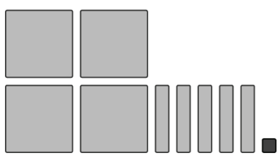
e) $5t - 4 - 2t^2 + 3 + 6t^2$



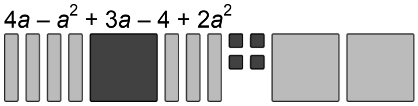
Group like tiles and remove zero pairs.



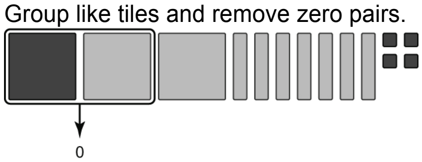
The combined tiles represent $4t^2 + 5t - 1$.



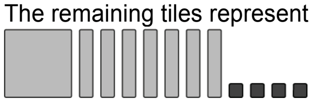
f) $4a - a^2 + 3a - 4 + 2a^2$



Group like tiles and remove zero pairs.



The remaining tiles represent $a^2 + 7a - 4$.



12. a) $2m + 4 - 3m - 8$ Group like terms.
$= 2m - 3m + 4 - 8$ Add the coefficients of like terms.
$= -m - 4$

b) $4 - 5x + 6x - 2$ Group like terms.
$= -5x + 6x + 4 - 2$ Add the coefficients of like terms.
$= x + 2$

c) $3g - 6 - 2g + 9$ Group like terms.
$= 3g - 2g - 6 + 9$ Add the coefficients of like terms.
$= g + 3$

d) $-5 + 1 + h - 4h$ Add the coefficients of like terms.
$= -4 - 3h$

e) $-6n - 5n - 4 - 7$ Add the coefficients of like terms.
$= -11n - 11$

f) $3s - 4s - 5 - 6$ Add the coefficients of like terms.
$= -s - 11$

13. a) $6 - 3x + x^2 + 9 - x$ Group like terms.
$= x^2 - 3x - x + 6 + 9$ Add the coefficients of like terms.
$= x^2 - 4x + 15$

b) $5m - 2m^2 - m^2 + 5m$ Group like terms.
$= -2m^2 - m^2 + 5m + 5m$ Add the coefficients of like terms.
$= -3m^2 + 10m$

c) $5x - x^2 + 3x + x^2 - 7$ Group like terms.
$= -x^2 + x^2 + 5x + 3x - 7$ Add the coefficients of like terms.
$= 8x - 7$

d) $3p^2 - 2p + 4 + p^2 + 3$ Group like terms.
$= 3p^2 + p^2 - 2p + 4 + 3$ Add the coefficients of like terms.
$= 4p^2 - 2p + 7$

e) $a^2 - 2a - 4 + 2a - a^2 + 4$ Group like terms.
$= a^2 - a^2 - 2a + 2a - 4 + 4$ Add the coefficients of like terms.
$= 0$

f) $-6x^2 + 17x - 4 - 3x^2 + 8 - 12x$ Group like terms.
$= -6x^2 - 3x^2 + 17x - 12x - 4 + 8$ Add the coefficients of like terms.
$= -9x^2 + 5x + 4$

14. a) $3x^2 + 5y - 2x^2 - 1 - y$ Group like terms.
$= 3x^2 - 2x^2 + 5y - y - 1$ Combine like terms.
$= x^2 + 4y - 1$

b) $pq - 1 - p^2 + 5p - 5pq - 2p$ Group like terms.
$= -p^2 + 5p - 2p + pq - 5pq - 1$ Combine like terms.
$= -p^2 + 3p - 4pq - 1$

c) $5x^2 + 3xy - 2y - x^2 - 7x + 4xy$ Group like terms.
$= 5x^2 - x^2 + 3xy + 4xy - 2y - 7x$ Combine like terms.
$= 4x^2 + 7xy - 2y - 7x$

d) $3r^2 - rs + 5s + r^2 - 2rs - 4s$ Group like terms.
$= 3r^2 + r^2 - rs - 2rs + 5s - 4s$ Combine like terms.
$= 4r^2 - 3rs + s$

e) $4gh + 7 - 2g^2 - 3gh - 11 + 6g$ Group like terms.
$= -2g^2 + 6g + 4gh - 3gh + 7 - 11$ Combine like terms.
$= -2g^2 + 6g + gh - 4$

f) $-5s + st - 4s^2 - 12st + 10s - 2s^2$ Group like terms.
$= -4s^2 - 2s^2 - 5s + 10s + st - 12st$ Combine like terms.
$= -6s^2 + 5s - 11st$

15. Use a table, then look for matching polynomials.

Part	Polynomial	Simplified Polynomial
a	$1 + 5x$	$5x + 1$
b	$6 - 2x + x^2 - 1 - x + x^2$	$2x^2 - 3x + 5$
c	$4x^2 - 7x + 1 - 7x^2 + 2x + 3$	$-3x^2 - 5x + 4$
d	$4 - 5x - 3x^2$	$-3x^2 - 5x + 4$
e	$2x^2 - 3x + 5$	$2x^2 - 3x + 5$
f	$3x + 2x^2 + 1 - 2x^2 + 2x$	$5x + 1$

Polynomials a and f are equivalent since both can be written as $5x + 1$.
Polynomials b and e are equivalent since both can be written as $2x^2 - 3x + 5$.
Polynomials c and d are equivalent since both can be written as $-3x^2 - 5x + 4$.

16. Work backward. Start with $-2a^2 + 4a - 8$ and introduce zero pairs of like terms. For example:

i) $-2a^2 + 4a - 8$
$= 3a^2 - 3a^2 - 2a^2 + 4a - 8$
$= 3a^2 - 5a^2 + 4a - 8$

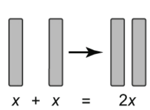
ii) $-2a^2 + 4a - 8$
$= -8a^2 + 8a^2 - 2a^2 - 4a + 8a - 8a - 8$
$= -8a^2 + 6a^2 + 4a - 8a - 8$

iii) $-2a^2 + 4a - 8$
$= -2a^2 + 4a - 7a + 7a - 8 + 10 - 10$
$= -2a^2 + 12a - 7a - 18 + 10$

Each result for i, ii, and iii is equivalent to $-2a^2 + 4a - 8$.

17. Look for a polynomial with 5 terms, that simplifies to 2 terms.
Start with the binomial. For example, $-4y^2 + 4y$. Introduce changes that don't affect the value, but do add terms. For example: $-4y^2 + 4y = -11y^2 + 7y^2 + 4y - 2 + 2$.
So, the polynomial $-11y^2 + 7y^2 + 4y - 2 + 2$ has 5 terms but only 2 terms, $-4y^2 + 4y$, when simplified.

18. a) When you simplify a polynomial, you add like terms by adding the numerical coefficients of the like terms.
If you were using algebra tiles, you would show two x-tiles; a simplified way to write this would be $2x$.
The answer x^2 indicates that you are using x^2-tiles, which is incorrect.
The correct answer for $x + x$ is $2x$.



b) i) $-2 + 4r - 2r + 3$ Group like terms.
$= 4r - 2r - 2 + 3$ Add the coefficients of like terms.
$= 2r + 1$

ii) $2t^2 - 3t + 4t^2 - 6t$ Group like terms.
$= 2t^2 + 4t^2 - 3t - 6t$ Add the coefficients of like terms.
$= 6t^2 - 9t$

iii) $3c^2 + 4c + 2 + c^2 + 2c + 1$ Group like terms.
$= 3c^2 + c^2 + 4c + 2c + 2 + 1$ Add the coefficients of like terms.
$= 4c^2 + 6c + 3$

iv) $15x^2 - 12xy + 5y + 10xy - 8y - 9x^2$ Group like terms.
$= 15x^2 - 9x^2 - 12xy + 10xy + 5y - 8y$ Add the coefficients of like terms.
$= 6x^2 - 2xy - 3y$

I know the answers are correct because the polynomials cannot be simplified further; they contain no like terms.

c) A polynomial can only be simplified if it contains like terms. So, to write a polynomial that cannot be simplified, I create a polynomial that has no like terms, for example, $-8d^2 - 3d - 4$.

19. a) The dimensions of the rectangle are $5x$ and x. So, the perimeter of the rectangle is:
$5x + x + 5x + x = 12x$

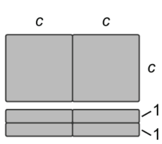
b) The dimensions of the rectangle are $2x$ and 2.
So, the perimeter of the rectangle is: $2x + 2 + 2x + 2 = 4x + 4$

c) The dimensions of the rectangle are $3x$ and $2x$.
So, the perimeter of the rectangle is: $3x + 2x + 3x + 2x = 10x$

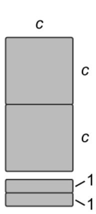
d) The dimensions of the rectangle are $4x$ and 3.
So, the perimeter of the rectangle is: $4x + 3 + 4x + 3 = 8x + 6$

20. a) For perimeter $6c + 4$:

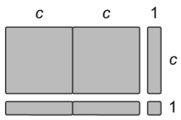
$3c + 2 + 3c + 2 = 6c + 4$



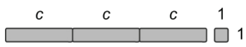
$2c + (c + 2) + 2c + (c + 2) = 6c + 4$



$c + (2c + 2) + c + (2c + 2) = 6c + 4$



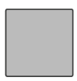
$(2c + 1) + (c + 1) + (2c + 1) + (c + 1) = 6c + 4$



$(3c + 1) + 1 + (3c + 1) + 1 = 6c + 4$

I can make 5 rectangles with perimeter $6c + 4$.

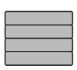
b) For perimeter $4d$:



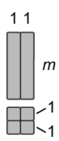
$d + d + d + d = 4d$

I can make 1 rectangle with perimeter $4d$.

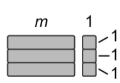
c) For perimeter $8 + 2m$:



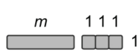
$m + 4 + m + 4 = 8 + 2m$



$2 + (m + 2) + 2 + (m + 2) = 8 + 2m$



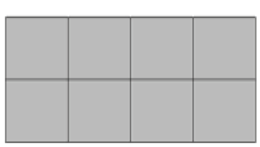
$(m + 1) + 3 + (m + 1) + 3 = 8 + 2m$



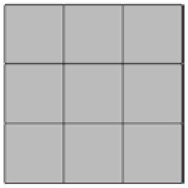
$(m + 3) + 1 + (m + 3) + 1 = 8 + 2m$

I can make 4 rectangles with perimeter $8 + 2m$.

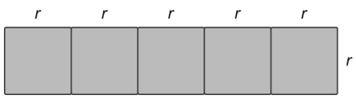
d) For perimeter $12r$:



$4r + 2r + 4r + 2r = 12r$



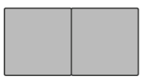
$3r + 3r + 3r + 3r = 12r$



$5r + r + 5r + r = 12r$

I can make 3 rectangles with perimeter $12r$.

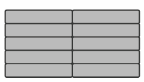
e) For perimeter $6s$:



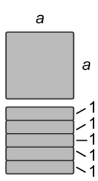
$2s + s + 2s + s = 6s$

I can make 1 rectangle with perimeter $6s$.

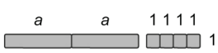
f) For perimeter $4a + 10$:



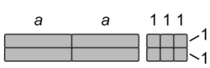
$2a + 5 + 2a + 5 = 4a + 10$



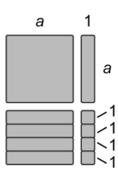
$a + (a + 5) + a + (a + 5) = 4a + 10$



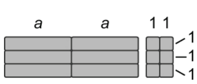
$(2a + 4) + 1 + (2a + 4) + 1 = 4a + 10$



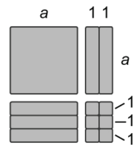
$(2a + 3) + 2 + (2a + 3) + 2 = 4a + 10$



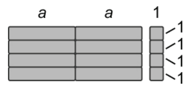
$(a + 1) + (a + 4) + (a + 1) + (a + 4) = 4a + 10$



$(2a + 2) + 3 + (2a + 2) + 3 = 4a + 10$



$(a + 2) + (a + 3) + (a + 2) + (a + 3) = 4a + 10$

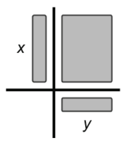


$(2a + 1) + 4 + (2a + 1) + 4 = 4a + 10$

I can make 8 rectangles with perimeter $4a + 10$.

Take It Further

21. An *xy*-tile is a rectangle with length of the *x*-tile and width of the *y*-tile. The area of the rectangle is then $x \times y$, or xy.



22. The perimeter of the shape is the sum of the measures of all sides.
I calculate the side length that is not labelled:
$3x – x$, or $2x$.

Then, the perimeter is:

$x + y + 2x + 2y + 3x + 3y$	Group like terms.
$= x + 2x + 3x + y + 2y + 3y$	Combine like terms.
$= 6x + 6y$	

The perimeter of the shape is $6x + 6y$.

Lesson 5.3 **Adding Polynomials** **Practice (pages 228–230)**

Check

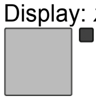
3. a) $(3x + 5) + (-2x + 2) = 3x + 5 - 2x + 2$ Group like terms.
$= 3x - 2x + 5 + 2$ Combine like terms.
$= x + 7$

b) $(-2x^2 + 4x - 2) + (2x^2 + 4x + 8) = -2x^2 + 4x - 2 + 2x^2 + 4x + 8$ Group like terms.
$= -2x^2 + 2x^2 + 4x + 4x - 2 + 8$ Combine like terms.
$= 8x + 6$

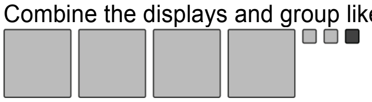
c) $(3x^2 - 6x + 4) + (-x^2 - 4x + 2) = 3x^2 - 6x + 4 - x^2 - 4x + 2$ Group like terms.
$= 3x^2 - x^2 - 6x - 4x + 4 + 2$ Combine like terms.
$= 2x^2 - 10x + 6$

4. Display: $3x^2 + 2$

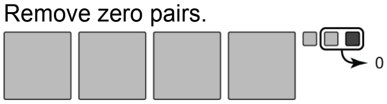
Display: $x^2 - 1$



Combine the displays and group like tiles.

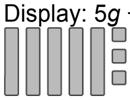


Remove zero pairs.

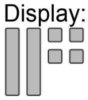


The remaining tiles represent the sum: $4x^2 + 1$.

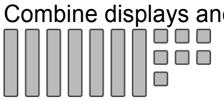
5. a) Display: $5g + 3$



Display: $2g + 4$



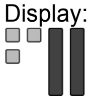
Combine displays and group like tiles.



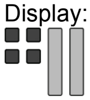
The combined tiles represent $7g + 7$.

So, $(5g + 3) + (2g + 4) = 5g + 3 + 2g + 4$
$= 5g + 2g + 3 + 4$
$= 7g + 7$

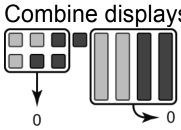
b) Display: $3 - 2j$



Display: $-4 + 2j$



Combine displays, group like tiles, and remove zero pairs.



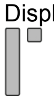
The remaining tiles represent -1.

So, $(3 - 2j) + (-4 + 2j) = 3 - 2j - 4 + 2j$

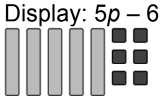
$= -2j + 2j + 3 - 4$

$= -1$

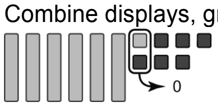
c) Display: $p + 1$



Display: $5p - 6$



Combine displays, group like tiles, and remove zero pairs.



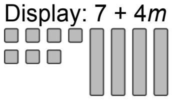
The remaining tiles represent $6p - 5$.

So, $(p + 1) + (5p - 6) = p + 1 + 5p - 6$

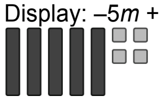
$= p + 5p + 1 - 6$

$= 6p - 5$

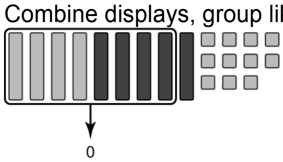
d) Display: $7 + 4m$



Display: $-5m + 4$



Combine displays, group like tiles, and remove zero pairs.



The remaining tiles represent $-m + 11$.

So, $(7 + 4m) + (-5m + 4) = 7 + 4m - 5m + 4$

$= 4m - 5m + 7 + 4$

$= -m + 11$

6. a)

$$\begin{array}{r} 2x+4 \\ +\,3x-5 \\ \hline 5x-1 \end{array}$$

Add the coefficients of like terms.

b)

$$\begin{array}{r} 3x^2 + 5x \\ +\,-2x^2 - 8x \\ \hline x^2 - 3x \end{array}$$

Add the coefficients of like terms.

c)

$$\begin{array}{r} 3x^2+5x+7 \\ +\,-8x^2-3x+5 \\ \hline -5x^2+2x+12 \end{array}$$

Add the coefficients of like terms.

7. Responses may vary, depending on personal choice. For example, I prefer to add vertically; it is easier to see the like terms that are being combined. Or, I prefer to add horizontally; it is easier to group like terms.

Apply

8. a) $(6x + 3) + (3x + 4)$ Remove brackets.
$= 6x + 3 + 3x + 4$ Group like terms.
$= 6x + 3x + 3 + 4$ Combine like terms.
$= 9x + 7$

b) $(5b - 4) + (2b + 9)$ Remove brackets.
$= 5b - 4 + 2b + 9$ Group like terms.
$= 5b + 2b - 4 + 9$ Combine like terms.
$= 7b + 5$

c) $(6 - 3y) + (-3 - 2y)$ Remove brackets.
$= 6 - 3y - 3 - 2y$ Group like terms.
$= -3y - 2y - 3 + 6$ Combine like terms.
$= -5y + 3$

d) $(-n + 7) + (3n - 2)$ Remove brackets.
$= -n + 7 + 3n - 2$ Group like terms.
$= -n + 3n - 2 + 7$ Combine like terms.
$= 2n + 5$

e) $(-4s - 5) + (6 - 3s)$ Remove brackets.
$= -4s - 5 + 6 - 3s$ Group like terms.
$= -4s - 3s - 5 + 6$ Combine like terms.
$= -7s + 1$

f) $(1 - 7h) + (-7h - 1)$ Remove brackets.
$= 1 - 7h - 7h - 1$ Group like terms.
$= -7h - 7h + 1 - 1$ Combine like terms.
$= -14h$

g) $(8m + 4) + (-9 + 3m)$ Remove brackets.
$= 8m + 4 - 9 + 3m$ Group like terms.
$= 8m + 3m + 4 - 9$ Combine like terms.
$= 11m - 5$

h) $(-8m - 4) + (9 - 3m)$ Remove brackets.
$= -8m - 4 + 9 - 3m$ Group like terms.
$= -8m - 3m - 4 + 9$ Combine like terms.
$= -11m + 5$

9. I removed the brackets, grouped like terms, then combined like terms.

a) $(4m^2 + 4m - 5) + (2m^2 - 2m + 1) = 4m^2 + 4m - 5 + 2m^2 - 2m + 1$
$= 4m^2 + 2m^2 + 4m - 2m - 5 + 1$
$= 6m^2 + 2m - 4$

b) I removed the brackets, grouped like terms, then combined like terms.
$(3k^2 - 3k + 2) + (-3k^2 - 3k + 2) = 3k^2 - 3k + 2 - 3k^2 - 3k + 2$
$= 3k^2 - 3k^2 - 3k - 3k + 2 + 2$
$= -6k + 4$

c) I removed the brackets, grouped like terms, then combined like terms.
$(-7p - 3) + (p^2 + 5) = -7p - 3 + p^2 + 5$
$= p^2 - 7p - 3 + 5$
$= p^2 - 7p + 2$

d) I removed the brackets, grouped like terms, then combined like terms.
$(9 - 3t) + (9t + 3t^2 - 6t) = 9 - 3t + 9t + 3t^2 - 6t$
$= 3t^2 - 3t + 9t - 6t + 9$
$= 3t^2 + 9$

e) I removed the brackets, grouped like terms, then combined like terms.
$(3x^2 - 2x + 3) + (2x^2 + 4) = 3x^2 - 2x + 3 + 2x^2 + 4$
$= 3x^2 + 2x^2 - 2x + 3 + 4$
$= 5x^2 - 2x + 7$

f) I added vertically.

$$\begin{array}{r} 3x^2 - 7x + 5 \\ + \underline{-6x^2 + 6x + 8} \\ -3x^2 - x + 13 \end{array}$$

Add the coefficients of like terms.

g) I added vertically.

$$\begin{array}{r} x^2 - 7x + 6 \\ + \underline{-6x^2 + 6x + 10} \\ -5x^2 - x + 16 \end{array}$$

Add the coefficients of like terms.

h) I added vertically.

$$\begin{array}{r} r^2 - 3r + 1 \\ + \underline{-3r^2 + 4r + 5} \\ -2r^2 + r + 6 \end{array}$$

Add the coefficients of like terms.

10. The perimeter is the sum of the measures of all sides.

a) i)

$$\begin{array}{r} 2n + 1 \\ + \quad n + 5 \\ + \underline{2n + 5} \\ 5n + 11 \end{array}$$

Add the coefficients of like terms.

The perimeter is $5n + 11$.

ii)

$$\begin{array}{r} 7r+2 \\ +7r+2 \\ +7r+2 \\ +\underline{7r+2} \\ 28r+8 \end{array}$$

Add the coefficients of like terms.

The perimeter is $28r + 8$.

iii)

$$\begin{array}{r} 6t+5 \\ +\ 2t+1 \\ +\ 6t+5 \\ +\ \underline{2t+1} \\ 16t+12 \end{array}$$

Add the coefficients of like terms.

The perimeter is $16t + 12$.

iv)

$$\begin{array}{r} 3f+1 \\ +\ f+2 \\ +3f+1 \\ +\ \underline{f+2} \\ 8f+6 \end{array}$$

Add the coefficients of like terms.

The perimeter is $8f + 6$.

b) i) Choose a value for n, such as $n = 1$.
Write the addition sentence:
$2n + 1 + n + 5 + 2n + 5 = 5n + 11$
Substitute $n = 1$.

Left side:	Right side:
$2n + 1 + n + 5 + 2n + 5$	$5n + 11$
$= 2(1) + 1 + 1 + 5 + 2(1) + 5$	$= 5(1) + 11$
$= 16$	$= 16$

Since the left side equals the right side, the polynomial for the perimeter is correct.

ii) Choose a value for r, such as $r = 2$.
Write the addition sentence:
$7r + 2 + 7r + 2 + 7r + 2 + 7r + 2 = 28r + 8$
Substitute $r = 2$.

Left side:	Right side:
$7r + 2 + 7r + 2 + 7r + 2 + 7r + 2$	$28r + 8$
$= 7(2) + 2 + 7(2) + 2 + 7(2) + 2 + 7(2) + 2$	$= 28(2) + 8$
$= 64$	$= 64$

Since the left side equals the right side, the polynomial for the perimeter is correct.

iii) Choose a value for t, such as $t = 1$.
Write the addition sentence:
$6t + 5 + 2t + 1 + 6t + 5 + 2t + 1 = 16t + 12$
Substitute $t = 1$.

Left side:	Right side:
$6t + 5 + 2t + 1 + 6t + 5 + 2t + 1$	$16t + 12$
$= 6(1) + 5 + 2(1) + 1 + 6(1) + 5 + 2(1) + 1$	$= 16(1) + 12$
$= 28$	$= 28$

Since the left side equals the right side, the polynomial for the perimeter is correct.

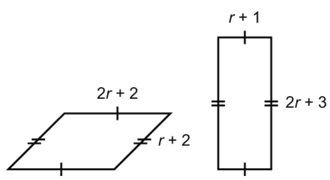
iv) Choose a value for f, such as $f = 2$.
Write the addition sentence:
$3f + 1 + f + 2 + 3f + 1 + f + 2 = 8f + 6$
Substitute $f = 2$.

Left side:	Right side:
$3f + 1 + f + 2 + 3f + 1 + f + 2$	$8f + 6$
$= 3(2) + 1 + 2 + 2 + 3(2) + 1 + 2 + 2$	$= 8(2) + 6$
$= 22$	$= 22$

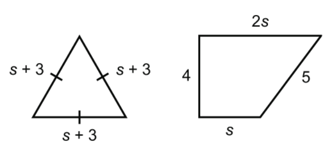
Since the left side equals the right side, the polynomial for the perimeter is correct.

11. Answers may vary. For example:

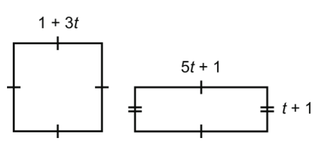
a) The perimeter is $8 + 6r$.
$8 + 6r = (2r + 2) + (r + 2) + (2r + 2) + (r + 2)$
$8 + 6r = (r + 1) + (2r + 3) + (r + 1) + (2r + 3)$



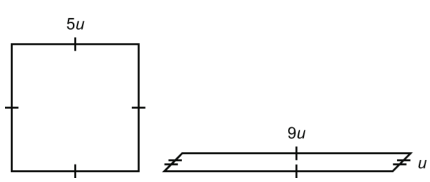
b) The perimeter is $3s + 9$.
$3s + 9 = (s + 3) + (s + 3) + (s + 3)$
$3s + 9 = 4 + 2s + 5 + s$



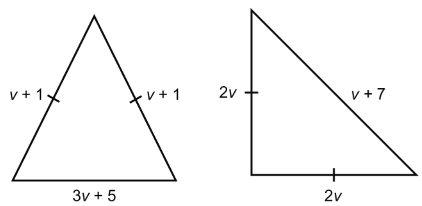
c) The perimeter is $4 + 12t$.
$4 + 12t = (1 + 3t) + (1 + 3t) + (1 + 3t) + (1 + 3t)$
$4 + 12t = (5t + 1) + (t + 1) + (5t + 1) + (t + 1)$



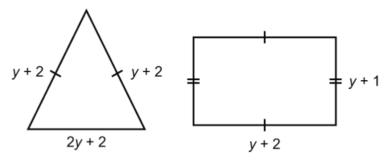
d) The perimeter is $20u$.
$20u = 5u + 5u + 5u + 5u$
$20u = 9u + u + 9u + u$



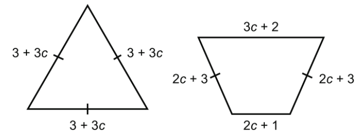
e) The perimeter is $7 + 5v$.
$7 + 5v = (v + 1) + (v + 1) + (3v + 5)$
$7 + 5v = 2v + 2v + (v + 7)$



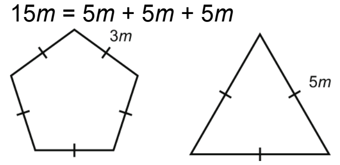
f) The perimeter is $4y + 6$.
$4y + 6 = (y + 2) + (y + 2) + (2y + 2)$
$4y + 6 = (y + 1) + (y + 2) + (y + 1) + (y + 2)$



g) The perimeter is $9 + 9c$.
$9 + 9c = (3 + 3c) + (3 + 3c) + (3 + 3c)$
$9 + 9c = (3c + 2) + (2c + 3) + (2c + 1) + (2c + 3)$



h) The perimeter is $15m$.
$15m = 3m + 3m + 3m + 3m + 3m$
$15m = 5m + 5m + 5m$



12. The student is not correct; the error was made when combining like terms. $-7x - 5x = -12x$, not $-2x$; $3 + 9 = 12$, not 1.
The correct solution:
$(4x^2 - 7x + 3) + (-x^2 - 5x + 9)$
$= 4x^2 - 7x + 3 - x^2 - 5x + 9$
$= 4x^2 - x^2 - 7x - 5x + 3 + 9$
$= 3x^2 - 12x + 12$

13. a) The algebra tiles model $-2x^2 + 2x + 1$, so I need to determine two polynomials with a sum of $-2x^2 + 2x + 1$. I think of two terms that, when added together, have the sum $-2x^2$. I choose $-3x^2$ and x^2. Then I think of two terms with a sum of $2x$; I choose $-3x$ and $5x$. Finally, I think of two constant terms with a sum of 1; I choose 5 and -4.
So, the two polynomials could be: $(-3x^2 - 3x + 5) + (x^2 + 5x - 4)$

b) Some examples of polynomials that have a sum of $-2x^2 + 2x + 1$:
$(-5x^2 + x + 5) + (3x^2 + x - 4)$;
$(-7x^2 - 3x + 9) + (5x^2 + 5x - 8)$;
$(-x^2 + 10x + 5) + (-x^2 - 8x - 4)$;
$(-3x^2 + 9x - 5) + (x^2 - 7x + 6)$;
$(10x^2 - 3x + 15) + (-12x^2 + 5x - 14)$
There are an infinite number of possibilities.

14. The other polynomial is $8m^2 + 8m - 4$.
To determine the polynomial, I worked backward.
To get the term $12m^2$: there is $4m^2$ in the first polynomial, so I need to add $8m^2$ to have a sum of $12m^2$.
To get the term $2m$: there is $-6m$ in the first polynomial, so I need to add $8m$ to have a sum of $2m$.
To get the constant 4: there is 8 in the first polynomial, so I need to add -4 to have a sum of 4.

15. Add a polynomial to $3x^2 + 7x + 2$ to get each sum.

a) $5x^2 + 10x + 1$

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline 5x^2 + 10x + 1 \end{array}$$

By inspection, the polynomial must be $2x^2 + 3x - 1$.

b) $2x^2 + 5x + 8$

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline 2x^2 + 5x + 8 \end{array}$$

By inspection, the polynomial must be $-x^2 - 2x + 6$.

c) $4x^2 + 3x$

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline 4x^2 + 3x \end{array}$$

By inspection, the polynomial must be $x^2 - 4x - 2$.

d) $-x^2 + x - 1$

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline -x^2 + x - 1 \end{array}$$

By inspection, the polynomial must be $-4x^2 - 6x - 3$.

e) $2x + 3$

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline 2x + 3 \end{array}$$

By inspection, the polynomial must be $-3x^2 - 5x + 1$.

f) 4

Think: $$\begin{array}{r} 3x^2 + 7x + 2 \\ +\ \square x^2 + \square x + \square \\ \hline 4 \end{array}$$

By inspection, the polynomial must be $-3x^2 - 7x + 2$.

16. a) $-5x^2 - 3x + 1$; in order to have each term have a sum of zero, you need to add the opposite of each term.

$$\begin{aligned}(5x^2 + 3x - 1) + (-5x^2 - 3x + 1) &= 5x^2 + 3x - 1 - 5x^2 - 3x + 1\\ &= 5x^2 - 5x^2 + 3x - 3x - 1 + 1\\ &= 0\end{aligned}$$

b) The coefficients are opposite numbers; this will be true for all polynomials with a sum of zero. The only way to arrive at a sum of zero is to add the opposite numerical value.

17. a)

$(3x^2 - 2y^2 + xy) + (-2xy - 2y^2 - 3x^2)$	Remove brackets.
$= 3x^2 - 2y^2 + xy - 2xy - 2y^2 - 3x^2$	Group like terms.
$= 3x^2 - 3x^2 - 2y^2 - 2y^2 + xy - 2xy$	Combine like terms.
$= -4y^2 - xy$	

b)

$(-5q^2 + 3p - 2q + p^2) + (4p + q + pq)$	Remove brackets.
$= -5q^2 + 3p - 2q + p^2 + 4p + q + pq$	Group like terms.
$= -5q^2 + p^2 + 3p + 4p - 2q + q + pq$	Combine like terms.
$= -5q^2 + p^2 + 7p - q + pq$	

c)

$(3mn + m^2 - 3n^2 + 5m) + (7n^2 - 8n + 10)$	Remove brackets.
$= 3mn + m^2 - 3n^2 + 5m + 7n^2 - 8n + 10$	Group like terms.
$= m^2 - 3n^2 + 7n^2 + 3mn + 5m - 8n + 10$	Combine like terms.
$= m^2 + 4n^2 + 3mn + 5m - 8n + 10$	

d)

$(3 - 8f + 5g - f^2) + (2g^2 - 3f + 4g - 5)$	Remove brackets.
$= 3 - 8f + 5g - f^2 + 2g^2 - 3f + 4g - 5$	Group like terms.
$= -f^2 + 2g^2 - 8f - 3f + 5g + 4g + 3 - 5$	Combine like terms.
$= -f^2 + 2g^2 - 11f + 9g - 2$	

Take It Further

18. a) Two sides of a triangle are $4x - 3y$ and $2x + y$.
$(4x - 3y) + (2x + y) = 6x - 2y$
The perimeter of the triangle is $9x + 2$.
Determine the polynomial you add to $6x - 2y$ to get $9x + 2$.

Think: $6x$ and what gives a sum of $9x$? The answer is $3x$.
There is no y-term in $9x + 2$, so eliminate $-2y$ by adding $2y$; $-2y + 2y = 0$
There is a constant in $9x + 2$, so add 2 to get the constant 2.
The third side of the triangle is $3x + 2y + 2$.

b) Choose a value for x, such as $x = 1$. Choose a value for y, such as $y = 2$.
Write the addition sentence:
$4x - 3y + 2x + y + 3x + 2y + 2 = 9x + 2$
Substitute $x = 1$ and $y = 2$.

Left side:	Right side:
$4x - 3y + 2x + y + 3x + 2y + 2$	$9x + 2$
$= 4(1) - 3(2) + 2(1) + 2 + 3(1) + 2(2) + 2$	$= 9(1) + 2$
$= 4 - 6 + 2 + 2 + 3 + 4 + 2$	$= 11$
$= 11$	

Since the left side equals the right side, the polynomial for the third side of the triangle is correct.

19. The perimeter of an isosceles triangle is $5y + 3x + 7$.
I find three polynomials that add up to $5y + 3x + 7$. Since the triangle is isosceles, two polynomials are equivalent. Some possible side lengths are:
$(2y + 2) + (2y + 2) + (3x + y + 3)$;
$(y + 3) + (y + 3) + (3y + 3x + 1)$;
$(2y + x) + (2y + x) + (x + y + 7)$;
$(3) + (3) + (5y + 3x + 1)$
There are an infinite number of possibilities.

Lesson 5.4 Subtracting Polynomials Practice (pages 234–236)

Check

4. a) $(-2x^2 + 4x - 2) - (-x^2 + 3x - 1)$ Subtract each term.

$= -2x^2 + 4x - 2 - (-x^2) - (+3x) - (-1)$ Add the opposite terms.

$= -2x^2 + 4x - 2 + x^2 - 3x + 1$ Collect like terms.

$= -2x^2 + x^2 + 4x - 3x - 2 + 1$ Combine like terms.

$= -x^2 + x - 1$

b) $(x^2 - 5x - 4) - (x^2 - 4x - 2)$ Subtract each term.

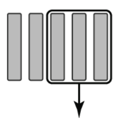
$= x^2 - 5x - 4 - (x^2) - (-4x) - (-2)$ Add the opposite terms.

$= x^2 - 5x - 4 - x^2 + 4x + 2$ Collect like terms.

$= x^2 - x^2 - 5x + 4x - 4 + 2$ Combine like terms.

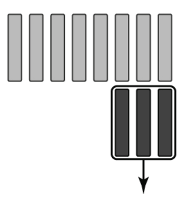
$= -x - 2$

5. a) Display $5r$ and remove tiles for $3r$.



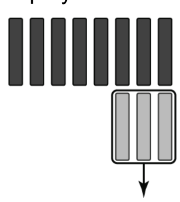
The remaining tiles represent $2r$.

b) Display $5r$. To subtract $-3r$, add 3 zero pairs of r-tiles. Remove tiles for $-3r$.



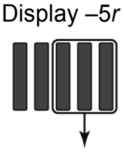
The remaining tiles represent $8r$.

c) Display $-5r$. To subtract $3r$, add 3 zero pairs of r-tiles. Remove tiles for $3r$.



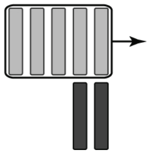
The remaining tiles represent $-8r$.

d) Display $-5r$ and remove tiles for $-3r$.



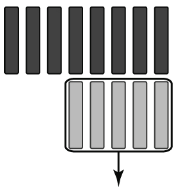
The remaining tiles represent $-2r$.

e) Display 3*r*. To subtract 5*r*, add 2 zero pairs of *r*-tiles. Remove tiles for 5*r*.



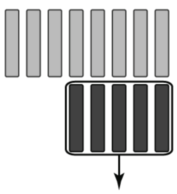
The remaining tiles represent –2*r*.

f) Display –3*r*. To subtract 5*r*, add 5 zero pairs of *r*-tiles. Remove tiles for 5*r*.



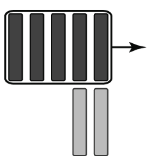
The remaining tiles represent –8*r*.

g) Display 3*r*. To subtract –5*r*, add 5 zero pairs of *r*-tiles. Remove tiles for –5*r*.



The remaining tiles represent 8*r*.

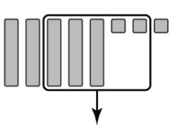
h) Display –3*r*. To subtract –5*r*, add 2 zero pairs of *r*-tiles. Remove tiles for –5*r*.



The remaining tiles represent 2*r*.

Apply

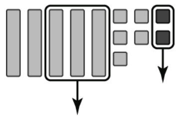
6. **a)** Display 5*x* + 3. Take away three *x*-tiles and two 1-tiles.



The remaining tiles represent 2x + 1.

$(5x + 3) – (3x + 2)$
$= 5x + 3 – (3x) – (+2)$
$= 5x + 3 – 3x – 2$
$= 5x – 3x + 3 – 2$
$= 2x + 1$

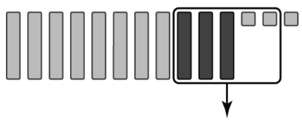
b) Display $5x + 3$. Take away three x-tiles. To take away two –1-tiles, add 2 zero pairs of 1-tiles.



The remaining tiles represent $2x + 5$.

$(5x + 3) – (3x – 2)$
$= 5x + 3 – (3x) – (–2)$
$= 5x + 3 – 3x + 2$
$= 5x – 3x + 3 + 2$
$= 2x + 5$

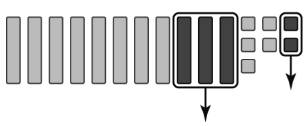
c) Display $5x + 3$. To take away three $–x$-tiles, add 3 zero pairs of x-tiles. Take away two 1-tiles.



The remaining tiles represent $8x + 1$.

$(5x + 3) – (–3x + 2)$
$= 5x + 3 – (–3x) – (+2)$
$= 5x + 3 + 3x – 2$
$= 5x + 3x + 3 – 2$
$= 8x + 1$

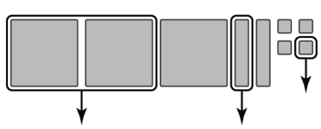
d) Display $5x + 3$. To take away three $–x$-tiles, add 3 zero pairs of x-tiles. To take away two –1-tiles, add 2 zero pairs of 1-tiles.



The remaining tiles represent $8x + 5$.

$(5x + 3) – (–3x – 2)$
$= 5x + 3 – (–3x) – (–2)$
$= 5x + 3 + 3x + 2$
$= 5x + 3x + 3 + 2$
$= 8x + 5$

7. a) Display $3s^2 + 2s + 4$. Take away two s^2-tiles, one s-tile, and one 1-tile.



The remaining tiles represent $s^2 + s + 3$.

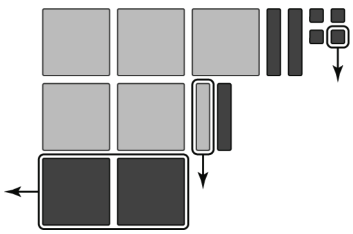
$(3s^2 + 2s + 4) – (2s^2 + s + 1)$
$= 3s^2 + 2s + 4 – (2s^2) – (+s) – (+1)$
$= 3s^2 + 2s + 4 – 2s^2 – s – 1$
$= 3s^2 – 2s^2 + 2s – s + 4 – 1$
$= s^2 + s + 3$

b) Display $3s^2 - 2s + 4$. Take away two s^2-tiles, one $-s$-tile, and one 1-tile.

The remaining tiles represent $s^2 - s + 3$.

$(3s^2 - 2s + 4) - (2s^2 - s + 1)$
$= 3s^2 - 2s + 4 - (2s^2) - (-s) - (+1)$
$= 3s^2 - 2s + 4 - 2s^2 + s - 1$
$= 3s^2 - 2s^2 - 2s + s + 4 - 1$
$= s^2 - s + 3$

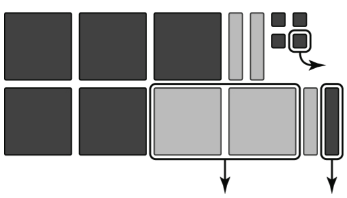
c) Display $3s^2 - 2s - 4$. To take away two $-s^2$-tiles, add 2 zero pairs of s^2-tiles. To subtract one s-tile, add 1 zero pair of s-tiles. Then take away one –1-tile.



The remaining tiles represent $5s^2 - 3s - 3$.

$(3s^2 - 2s - 4) - (-2s^2 + s - 1)$
$= 3s^2 - 2s - 4 - (-2s^2) - (+s) - (-1)$
$= 3s^2 - 2s - 4 + 2s^2 - s + 1$
$= 3s^2 + 2s^2 - 2s - s - 4 + 1$
$= 5s^2 - 3s - 3$

d) Display $-3s^2 + 2s - 4$. To subtract two s^2-tiles, add 2 zero pairs of s^2-tiles. To subtract one $-s$-tile, add 1 zero pair of s-tiles. Then remove one –1-tile.



The remaining tiles represent $-5s^2 + 3s - 3$.

$(-3s^2 + 2s - 4) - (2s^2 - s - 1)$
$= -3s^2 + 2s - 4 - (2s^2) - (-s) - (-1)$
$= -3s^2 + 2s - 4 - 2s^2 + s + 1$
$= -3s^2 - 2s^2 + 2s + s - 4 + 1$
$= -5s^2 + 3s - 3$

8. I used the properties of integers to subtract. I subtracted each term, added opposite terms, collected like terms, then combined like terms.

a) $(3x + 7) – (–2x – 2) = 3x + 7 – (–2x) – (–2)$
$= 3x + 7 + 2x + 2$
$= 3x + 2x + 7 + 2$
$= 5x + 9$

To check, add the difference to the second polynomial:
$(–2x – 2) + (5x + 9) = –2x – 2 + 5x + 9$
$= –2x + 5x – 2 + 9$
$= 3x + 7$

The sum is equal to the first polynomial. So, the difference is correct.

b) $(b^2 + 4b) – (–3b^2 + 7b) = b^2 + 4b – (–3b^2) – (+7b)$
$= b^2 + 4b + 3b^2 – 7b$
$= b^2 + 3b^2 + 4b – 7b$
$= 4b^2 – 3b$

To check, add the difference to the second polynomial:
$(–3b^2 + 7b) + (4b^2 – 3b) = –3b^2 + 7b + 4b^2 – 3b$
$= –3b^2 + 4b^2 + 7b – 3b$
$= b^2 + 4b$

The sum is equal to the first polynomial. So, the difference is correct.

c) $(–3x + 5) – (4x + 3) = –3x + 5 – 4x – 3$
$= –3x – 4x + 5 – 3$
$= –7x + 2$

To check, add the difference to the second polynomial:
$(4x + 3) + (–7x + 2) = 4x + 3 + (–7x) + 2$
$= 4x – 7x + 3 + 2$
$= –3x + 5$

The sum is equal to the first polynomial. So, the difference is correct.

d) $(4 – 5p) – (–7p + 3) = 4 – 5p – (–7p) – (+3)$
$= 4 – 5p + 7p – 3$
$= –5p + 7p + 4 – 3$
$= 2p + 1$

To check, add the difference to the second polynomial:
$(–7p + 3) + (2p + 1) = –7p + 3 + 2p + 1$
$= –7p + 2p + 3 + 1$
$= –5p + 4$, or $4 – 5p$

The sum is equal to the first polynomial. So, the difference is correct.

e) $(6x^2 + 7x + 9) – (4x^2 + 3x + 1) = 6x^2 + 7x + 9 – (4x^2) – (+3x) – (+1)$
$= 6x^2 + 7x + 9 – 4x^2 – 3x – 1$
$= 6x^2 – 4x^2 + 7x – 3x + 9 – 1$
$= 2x^2 + 4x + 8$

To check, add the difference to the second polynomial:
$(4x^2 + 3x + 1) + (2x^2 + 4x + 8) = 4x^2 + 3x + 1 + 2x^2 + 4x + 8$
$= 4x^2 + 2x^2 + 3x + 4x + 1 + 8$
$= 6x^2 + 7x + 9$

The sum is equal to the first polynomial. So, the difference is correct.
So, the difference is correct.

f) $(12m^2 - 4m + 7) - (8m^2 + 3m - 3) = 12m^2 - 4m + 7 - (8m^2) - (+3m) - (-3)$
$= 12m^2 - 4m + 7 - 8m^2 - 3m + 3$
$= 12m^2 - 8m^2 - 4m - 3m + 7 + 3$
$= 4m^2 - 7m + 10$
To check, add the difference to the second polynomial:
$(8m^2 + 3m - 3) + (4m^2 - 7m + 10) = 8m^2 + 3m - 3 + 4m^2 - 7m + 10$
$= 8m^2 + 4m^2 + 3m - 7m - 3 + 10$
$= 12m^2 - 4m + 7$
The sum is equal to the first polynomial. So, the difference is correct.

g) $(-4x^2 - 3x - 11) - (x^2 - 4x - 15) = -4x^2 - 3x - 11 - (x^2) - (-4x) - (-15)$
$= -4x^2 - 3x - 11 - x^2 + 4x + 15$
$= -4x^2 - x^2 - 3x + 4x - 11 + 15$
$= -5x^2 + x + 4$
To check, add the difference to the second polynomial:
$(x^2 - 4x - 15) + (-5x^2 + x + 4) = x^2 - 4x - 15 - 5x^2 + x + 4$
$= x^2 - 5x^2 - 4x + x - 15 + 4$
$= -4x^2 - 3x - 11$
The sum is equal to the first polynomial. So, the difference is correct.

h) $(1 - 3r + r^2) - (4r + 5 - 3r^2) = 1 - 3r + r^2 - (4r) - (+5) - (-3r^2)$
$= 1 - 3r + r^2 - 4r - 5 + 3r^2$
$= r^2 + 3r^2 - 3r - 4r + 1 - 5$
$= 4r^2 - 7r - 4$
To check, add the difference to the second polynomial:
$(4r + 5 - 3r^2) + (4r^2 - 7r - 4) = 4r + 5 - 3r^2 + 4r^2 - 7r - 4$
$= -3r^2 + 4r^2 + 4r - 7r + 5 - 4$
$= r^2 - 3r + 1$
The sum is equal to the first polynomial. So, the difference is correct.

9. a) The difference in cost can be represented by (full-colour cost) – (black-and-white cost):
$(4n + 2500) - (2n + 2100) = 4n + 2500 - (2n) - (+2100)$
$= 4n - 2n + 2500 - 2100$
$= 2n + 400$

b) Substitute $n = 3000$ in $2n + 400$.
$2(3000) + 400 = 6000 + 400$
$= 6400$
To print 3000 copies, colour printing, costs $6400 more than black-and-white printing.

10. a) Choose a value for x, such as $x = 4$.
Substitute $x = 4$ in the addition sentences.

Left side:	Right side:
$(2x^2 + 5x + 10) - (x^2 - 3)$	$x^2 + 8x + 10$
$= (2(4)^2 + 5(4) + 10) - ((4)^2 - 3)$	$= (4)2 + 8(4) + 10$
$= (32 + 20 + 10) - (13)$	$= 16 + 32 + 10$
$= 49$	$= 58$

Since the left side is not equal to the right side, the answer is incorrect.

b) The student did not subtract like terms; $5x$ and 3 does not equal $8x$; 3 must be added to 10 to give 13.
Corrected solution: $(2x^2 + 5x + 10) - (x^2 - 3)$
$= 2x^2 + 5x + 10 - x^2 + 3$
$= 2x^2 - x^2 + 5x + 10 + 3$
$= x^2 + 5x + 13$

11. For example:

$(-5x^2 - x + 16) - (3x^2 - 5x - 7)$	Subtract each term.
$= -5x^2 - x + 16 - (3x^2) - (-5x) - (-7)$	Add opposite terms.
$= -5x^2 - x + 16 - 3x^2 + 5x + 7$	Collect like terms.
$= -5x^2 - 3x^2 - x + 5x + 16 + 7$	Combine like terms.
$= -8x^2 + 4x + 23$	

To check, add the difference to the second polynomial:

$$\begin{aligned}(-8x^2 + 4x + 23) + (3x^2 - 5x - 7) &= -8x^2 + 4x + 23 + 3x^2 - 5x - 7\\ &= -8x^2 + 3x^2 + 4x - 5x + 23 - 7\\ &= -5x^2 - x + 16\end{aligned}$$

The sum is equal to the first polynomial.
So, the difference is correct.

12. a) The student did not add the opposite of each term in the 2nd polynomial being subtracted; the student only added the opposite of the first term.

b) Corrected solution:

$$\begin{aligned}&(2y^2 - 3y + 5) - (y^2 + 5y - 2)\\ &= 2y^2 - 3y + 5 - y^2 - 5y + 2\\ &= 2y^2 - y^2 - 3y - 5y + 5 + 2\\ &= y^2 - 8y + 7\end{aligned}$$

c) The answer can be checked by substitution, or by adding the difference to the second polynomial; if the sum is equal to the first polynomial, the answer is correct.

d) The student will need to focus on adding the opposite of each term in the second polynomial. Checking the answer will also help the student avoid errors.

13. The perimeter is the sum of the measures of all sides.

a) $$\begin{aligned}(6w + 14) - (2w + 3 + 2w + 3) &= 6w + 14 - (4w + 6)\\ &= 6w - 4w + 14 - 6\\ &= 2w + 8\end{aligned}$$

Since the remaining 2 lengths are equal, each unknown length is $w + 4$.

b) $$\begin{aligned}(7s + 7) - (3s + 2 + 3s + 2) &= 7s + 7 - (3s) - (+2) - (+3s) - (+2)\\ &= 7s + 7 - 3s - 2 - 3s - 2\\ &= 7s - 3s - 3s + 7 - 2 - 2\\ &= s + 3\end{aligned}$$

The unknown length is $s + 3$.

c) $$\begin{aligned}(10p + 8) - (p + 3 + p + 3) &= 10p + 8 - (2p + 6)\\ &= 10p + 8 - 2p - 6\\ &= 8p + 2\end{aligned}$$

Since the remaining 2 lengths are equal, each unknown length is $4p + 1$.

14. a) For example: $(8m^2 + 4m)$ and $(2m^2 - 5m)$

$$\begin{aligned}(8m^2 + 4m) - (2m^2 - 5m) &= 8m^2 + 4m - (2m^2) - (-5m)\\ &= 8m^2 - 2m^2 + 4m + 5m\\ &= 6m^2 + 9m\end{aligned}$$

b) $$\begin{aligned}(2m^2 - 5m) - (8m^2 + 4m) &= 2m^2 - 5m - (8m^2) - (+4m)\\ &= 2m^2 - 8m^2 - 5m - 4m\\ &= -6m^2 - 9m\end{aligned}$$

c) The coefficients of the like terms are opposites, because we changed the order of the terms when we subtracted. The two polynomials add to 0. This makes sense because, for example, 3 – 8 is the opposite of 8 – 3, and 3 – 8 + 8 – 3 = 0.

15. a) $(r^2 - 3rs + 5s^2) - (-2r^2 - 3rs - 5s^2)$	Subtract each term.
$= r^2 - 3rs + 5s^2 - (-2r^2) - (-3rs) - (-5s^2)$	Add the opposite terms.
$= r^2 - 3rs + 5s^2 + 2r^2 + 3rs + 5s^2$	Collect like terms.
$= r^2 + 2r^2 + 5s^2 + 5s^2 - 3rs + 3rs$	Combine like terms.
$= 3r^2 + 10s^2$	

b) $(-3m^2 + 4mn - n^2) - (5m^2 + 7mn + 2n^2)$	Subtract each term.
$= -3m^2 + 4mn - n^2 - (5m^2) - (+7mn) - (+2n^2)$	
$= -3m^2 + 4mn - n^2 - 5m^2 - 7mn - 2n^2$	Collect like terms.
$= -3m^2 - 5m^2 - n^2 - 2n^2 + 4mn - 7mn$	Combine like terms.
$= -8m^2 - 3n^2 - 3mn$	

c) $(5cd + 8c^2 - 7d^2) - (3d^2 + 6cd - 4c^2)$	Subtract each term.
$= 5cd + 8c^2 - 7d^2 - (3d^2) - (+6cd) - (-4c^2)$	Add the opposite term.
$= 5cd + 8c^2 - 7d^2 - 3d^2 - 6cd + 4c^2$	Collect like terms.
$= 8c^2 + 4c^2 - 7d^2 - 3d^2 + 5cd - 6cd$	Combine like terms.
$= 12c^2 - 10d^2 - cd$	

d) $(9e + 9f - 3e^2 + 4f^2) - (-f^2 - 2e^2 + 3f - 6e)$	Subtract each term.
$= 9e + 9f - 3e^2 + 4f^2 - (-f^2) - (-2e^2) - (+3f) - (-6e)$	Add the opposite terms.
$= 9e + 9f - 3e^2 + 4f^2 + f^2 + 2e^2 - 3f + 6e$	Collect like terms.
$= -3e^2 + 2e^2 + 4f^2 + f^2 + 9e + 6e + 9f - 3f$	Combine like terms.
$= -e^2 + 5f^2 + 15e + 6f$	

e) $(4jk - 7j - 2k + k^2) - (2j^2 + 3j - jk)$	Subtract each term.
$= 4jk - 7j - 2k + k^2 - (2j^2) - (+3j) - (-jk)$	Add the opposite term.
$= 4jk - 7j - 2k + k^2 - 2j^2 - 3j + jk$	Collect like terms.
$= -2j^2 + k^2 - 7j - 3j - 2k + 4jk + jk$	Combine like terms.
$= -2j^2 + k^2 - 10j - 2k + 5jk$	

16. a) The difference of two polynomials is $3x^2 + 4x - 7$.
If the given polynomial is the first polynomial, then:
$(-8x^2 + 5x - 4) - (\text{polynomial}) = 3x^2 + 4x - 7$
Subtract the difference from the given polynomial to get the other polynomial:

$$\begin{aligned}(-8x^2 + 5x - 4) - (3x^2 + 4x - 7) &= -8x^2 + 5x - 4 - (3x^2) - (+4x) - (-7)\\ &= -8x^2 + 5x - 4 - 3x^2 - 4x + 7\\ &= -8x^2 - 3x^2 + 5x - 4x - 4 + 7\\ &= -11x^2 + x + 3\end{aligned}$$

The other polynomial could be $-11x^2 + x + 3$.

If the given polynomial is the second polynomial, then:
$(\text{polynomial}) - (-8x^2 + 5x - 4) = 3x^2 + 4x - 7$
Add the difference to the given polynomial to get the other polynomial:

$$\begin{aligned}(\text{polynomial}) &= (3x^2 + 4x - 7) + (-8x^2 + 5x - 4)\\ &= 3x^2 - 8x^2 + 4x + 5x - 7 - 4\\ &= -5x^2 + 9x - 11\end{aligned}$$

The other polynomial could be $-5x^2 + 9x - 11$.

b) There are two possible answers to part a because the given polynomial could be either the first or second polynomial in the difference equation.

Take It Further

17. First, find the perimeter of each rectangle:

Perimeter of small rectangle:
$2x + 6 + x + 2 + 2x + 6 + x + 2 = 6x + 16$

Perimeter of large rectangle:
$2x + 1 + 4x + 3 + 2x + 1 + 4x + 3 = 12x + 8$

Then, subtract the perimeter of the small rectangle from the perimeter of the large rectangle to determine the difference in the perimeters of the rectangles:

$$\begin{aligned}(12x + 8) - (6x + 16) &= 12x + 8 - 6x - 16\\ &= 12x - 6x + 8 - 16\\ &= 6x - 8\end{aligned}$$

The difference is $6x - 8$.

18. The difference of two polynomials is $-4x^2 + 2x - 5$. So,

$$\begin{array}{r} \square x^2 + \square x + \square \\ -\ \square x^2 + \square x + \square \\ \hline -4x^2 + 2x - 5 \end{array}$$

We need any two coefficients that have a difference of –4 to get the first term; then we need any two coefficients that have a difference of 2 to get the second term; and for the constant term, we need to use any two numbers that have a difference of –5.

There are an infinite number of possibilities. For example,

$$\begin{aligned}(-8x^2 + 4x - 15) - (-4x^2 + 2x - 10) &= -8x^2 + 4x - 15 - (-4x^2) - (+2x) - (-10)\\ &= -8x^2 + 4x - 15 + 4x^2 - 2x + 10\\ &= -8x^2 + 4x^2 + 4x - 2x - 15 + 10\\ &= -4x^2 + 2x - 5\end{aligned}$$

$$\begin{aligned}(-12x^2 + 8x - 20) - (-8x^2 + 6x - 15) &= -12x^2 + 8x - 20 - (-8x^2) - (+6x) - (-15)\\ &= -12x^2 + 8x - 20 + 8x^2 - 6x + 15\\ &= -12x^2 + 8x^2 + 8x - 6x - 20 + 15\\ &= -4x^2 + 2x - 5\end{aligned}$$

Mid-Unit Review (page 237)

Lesson 5.1

1. **a)** $3m - 5$: variable m, 2 terms, coefficient 3, constant term –5, degree 1

b) $4r$: variable r, 1 term, coefficient 4, no constant term, degree 1

c) $x^2 + 4x + 1$: variable x, 3 terms, coefficients 1, 4, constant term 1, degree 2

2. Build the polynomial.
Since it is a trinomial, the polynomial will have three terms $\square + \square + \square$.
The polynomial is in variable m, of degree 2: $3m^2 + \square + \square$.
The constant term is –5: $3m^2 + \square - 5$.
A possible solution is $3m^2 - 4m - 5$.

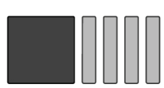
3. **a)** $-x^2 + 12$
The polynomial is a binomial, because it has two terms: $-x^2$ and 12
It is represented by two different kinds of algebra tiles.

b) $-2x^2 - 4x + 8$
The polynomial is a trinomial, because it has three terms: $-2x^2$, $-4x$, and 8
It is represented by three different kinds of algebra tiles.

c) $-4x$
The polynomial is a monomial, because it has one term: $-4x$
It is represented by only one kind of algebra tile.

4. **a)** $4n - 2$

b) $-t^2 + 4t$



c) $2d^2 + 3d + 2$

Lesson 5.2

5. **a)** $2x$ and $-5x$ are like terms; they have the same variable x, raised to the same exponent, 1, and they are represented by algebra tiles with the same shape and size.

b) 3 and $4g$ are not like terms; 3 is a constant term, $4g$ has a variable. 3 and $4g$ are represented by different kinds of algebra tiles.

c) 10 and 2 are like terms; both are constants, and are represented by algebra tiles with the same shape and size.

d) $2q^2$ and $-7q^2$ are like terms; they have the same variable q, raised to the same exponent, 2, and they are represented by algebra tiles with the same shape and size.

e) $8x^2$ and $3x$ are not like terms; the variable is raised to a different exponent in each term, and they are represented by different kinds of algebra tiles.

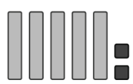
f) $-5x$ and $-5x^2$ are not like terms; the variable x is raised to a different exponent in each term. Each term is modelled with a different algebra tile.

6. To simplify, I collected like terms, then added the coefficients.

$3x^2 - 7 + 3 - 5x^2 - 3x + 5$
$= 3x^2 - 5x^2 - 3x - 7 + 3 + 5$
$= -2x^2 - 3x + 1$

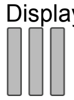
7. Renata and her friend are both correct. The polynomials are equivalent.
$4x^2 + 2x - 7$ can also be written as $-7 + 4x^2 + 2x$.
Their terms are ordered differently. Renata has written the polynomial in descending order.

8. No, Cooper is not correct. Display $5x - 2$ using algebra tiles.



$5x$ and -2 are not like terms, and cannot be combined.

Display $3x$ using algebra tiles.



$5x - 2$ and $3x$ are not equivalent polynomials. Their displays use different algebra tiles.

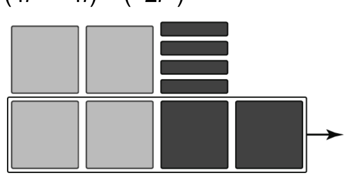
9. Use a table.

Part	Polynomial	Simplified Polynomial
a	$1 + 3x - x^2$	$-x^2 + 3x + 1$
b	$1 + 3x^2 - x^2 + 2x - 2x^2 + x - 2$	$3x - 1$
c	$x^2 - 3x^2 - 1$	$x^2 - 3x^2 - 1$
d	$6 + 6x - 6x^2 - 4x - 5 + 2x^2 + x^2 - 4$	$-3x^2 + 2x - 3$
e	$3x - 1$	$3x - 1$
f	$-3x^2 + 2x - 3$	$-3x^2 + 2x - 3$
g	$6x^2 - 6x - 6 + x - 5x^2 - 1 + 2x + 4$	$x^2 - 3x - 3$
h	$3x - x^2 + 1$	$-x^2 + 3x + 1$

Look for matching results.
Polynomials a and h are equivalent since both simplify to $-x^2 + 3x + 1$.
Polynomials b and e are equivalent since both simplify to $3x - 1$.
Polynomials d and f are equivalent since both simplify to $-3x^2 + 2x - 3$.

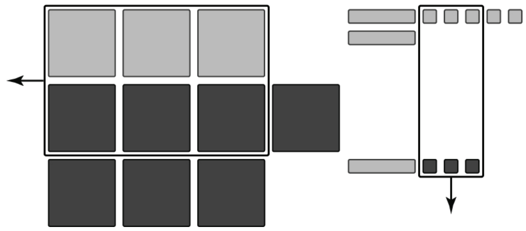
Lesson 5.3 and Lesson 5.4

10. a) $(4f^2 - 4f) + (-2f^2)$



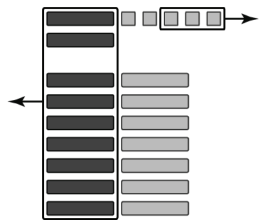
The remaining tiles represent $2f^2 - 4f$.

b) $(3r^2 + 2r + 5) + (-7r^2 + r - 3)$



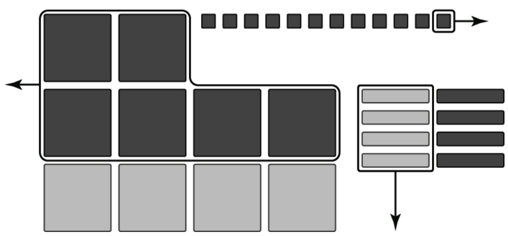
The remaining tiles represent $-4r^2 + 3r + 2$.

c) $(-2v + 5) - (-9v + 3)$



The remaining tiles represent $7v + 2$.

d) $(-2g^2 - 12) - (-6g^2 + 4g - 1)$



The remaining tiles represent $4g^2 - 4g - 11$.

11. a) $(3w^2 + 17w) + (12w^2 - 3w)$ Remove brackets.
$= 3w^2 + 17w + 12w^2 - 3w$ Group like terms.
$= 3w^2 + 12w^2 + 17w - 3w$ Combine like terms.
$= 15w^2 + 14w$

b) $(5m^2 - 3) + (m^2 + 3)$ Remove brackets.
$= 5m^2 - 3 + m^2 + 3$ Group like terms.
$= 5m^2 + m^2 - 3 + 3$ Combine like terms.
$= 6m^2$

c) $(-3h - 12) - (-9h - 6)$ Subtract each term.
$= -3h - 12 - (-9h) - (-6)$ Add the opposite terms.
$= -3h - 12 + 9h + 6$ Collect like terms.
$= -3h + 9h - 12 + 6$ Combine like terms.
$= 6h - 6$

d) $(6a^2 + 2a - 2) + (-7a^2 + 4a + 11)$ Remove brackets.
$= 6a^2 + 2a - 2 - 7a^2 + 4a + 11$ Group like terms.
$= 6a^2 - 7a^2 + 2a + 4a - 2 + 11$ Combine like terms.
$= -a^2 + 6a + 9$

e) $(3y^2 + 9y + 7) - (2y^2 - 4y + 13)$ Subtract each term.
$= 3y^2 + 9y + 7 - (2y^2) - (-4y) - (+13)$ Add the opposite term.
$= 3y^2 + 9y + 7 - 2y^2 + 4y - 13$ Collect like terms.
$= 3y^2 - 2y^2 + 9y + 4y + 7 - 13$ Combine like terms.
$= y^2 + 13y - 6$

f) $(-14 + 3p^2 + 2p) - (-5p + 10 - 7p^2)$ Subtract each term.
$= -14 + 3p^2 + 2p - (-5p) - (+10) - (-7p^2)$ Add the opposite terms.
$= -14 + 3p^2 + 2p + 5p - 10 + 7p^2$ Collect like terms.
$= 3p^2 + 7p^2 + 2p + 5p - 14 - 10$ Combine like terms.
$= 10p^2 + 7p - 24$

12. a) Subtract $5x^2 + 3x - 2$ from $7x^2 + 5x + 1$:

$$
\begin{aligned}
(7x^2 + 5x + 1) - (5x^2 + 3x - 2) &= 7x^2 + 5x + 1 - (5x^2) - (+3x) - (-2) \\
&= 7x^2 + 5x + 1 - 5x^2 - 3x + 2 \\
&= 7x^2 - 5x^2 + 5x - 3x + 1 + 2 \\
&= 2x^2 + 2x + 3
\end{aligned}
$$

The polynomial $2x^2 + 2x + 3$ must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$.
To check, add the difference to the second polynomial:

$$
\begin{aligned}
(5x^2 + 3x - 2) + (2x^2 + 2x + 3) &= 5x^2 + 3x - 2 + 2x^2 + 2x + 3 \\
&= 5x^2 + 2x^2 + 3x + 2x - 2 + 3 \\
&= 7x^2 + 5x + 1
\end{aligned}
$$

The sum is equal to the first polynomial.
So, the difference is correct.

b) Subtract $7x^2 + 5x + 1$ from $5x^2 + 3x - 2$:

$$
\begin{aligned}
(5x^2 + 3x - 2) - (7x^2 + 5x + 1) &= 5x^2 + 3x - 2 - (7x^2) - (+5x) - (+1) \\
&= 5x^2 + 3x - 2 - 7x^2 - 5x - 1 \\
&= 5x^2 - 7x^2 + 3x - 5x - 2 - 1 \\
&= -2x^2 - 2x - 3
\end{aligned}
$$

The polynomial $-2x^2 - 2x - 3$ must be subtracted from $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$.
To check, add the difference to the second polynomial:

$$
\begin{aligned}
(7x^2 + 5x + 1) + (-2x^2 - 2x - 3) &= 7x^2 + 5x + 1 - 2x^2 - 2x - 3 \\
&= 7x^2 - 2x^2 + 5x - 2x + 1 - 3 \\
&= 5x^2 + 3x - 2
\end{aligned}
$$

The sum is equal to the first polynomial.
So, the difference is correct.

Lesson 5.5 **Multiplying and Dividing a Polynomial by a Constant** **Practice (pages 246–248)**

Check

3. **a)** The display shows 5 rows of four 1-tiles. So, (4)(5) = 20

b) The display shows 3 x-tiles. So, (3)(x) = 3x

c) The display shows 2 rows of one x-tile and two 1-tiles. So, $2(x + 2) = 2x + 4$

d) The display shows 3 rows of three x-tiles and two 1-tiles. So, $3(3x + 2) = 9x + 6$

4. **a)** $20 \div 5 = 4$

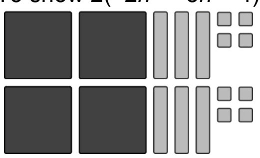
b) $3x \div 3 = x$

c) $(2x + 4) \div 2 = x + 2$

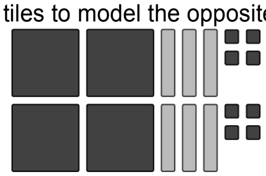
d) $(9x + 6) \div 3 = 3x + 2$

5. **a)** The display shows 2 rows of two n^2-tiles, three n-tiles, and four 1-tiles. This shows $2(2n^2 – 3n + 4)$, which is the product in part ii.

b) To show $2(–2n^2 + 3n + 4)$, display 2 rows of two $–n^2$-tiles, three n-tiles, and four 1-tiles.



To show $–2(2n^2 – 3n + 4)$, display 2 rows of two n^2-tiles, three $–n$-tiles, and four 1-tiles. Then, flip all the tiles to model the opposite.



6. **c)** The display shows eight t-tiles and twelve –1-tiles arranged as 4 equal rows of two t-tiles and three –1-tiles. So, this models $\frac{8t - 12}{4}$, which is the quotient in part c.

Apply

7. **a)** **i)** $3(5r) = 15r$ **ii)** $–3(5r) = –15r$

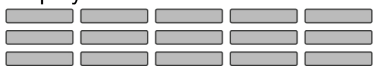
iii) $(5r)(3) = 15r$ **iv)** $–5(3r) = –15r$

v) $–5(–3r) = 15r$ **vi)** $(–3r)(5) = –15r$

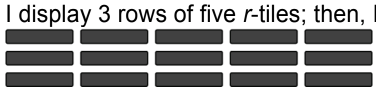
b) The products are either 15r or –15r. The product is positive when the terms being multiplied are the same sign and negative when the terms being multiplied are opposite signs.

c) I can use algebra tiles for all products in part a.

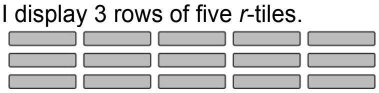
i) I display 3 rows of five *r*-tiles.



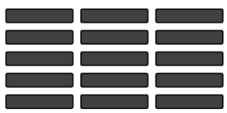
ii) I display 3 rows of five *r*-tiles; then, I flip the tiles to model the opposite.



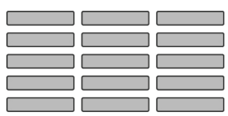
iii) I display 3 rows of five *r*-tiles.



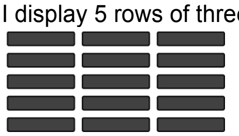
iv) I display 5 rows of three *r*-tiles; then, I flip the tiles to model the opposite.



v) I display 5 rows of three –*r*-tiles; then, I flip the tiles to model the opposite.



vi) I display 5 rows of three –*r*-tiles.



8. a) i) $\frac{12k}{4} = \frac{12}{4} \times k$

$= 3k$

ii) $(-12k) \div 4 = \frac{(-12)}{4} \times k$

$= (-3) \times k$

$= -3k$

iii) $\frac{12k}{-4} = \frac{12}{(-4)} \times k$

$= (-3) \times k$

$= -3k$

iv) $(-12k) \div (-4) = \frac{(-12)}{(-4)} \times k$

$= 3 \times k$

$= 3k$

b) All of the quotients are either $3k$ or $-3k$. The quotient is negative when the dividend and the divisor have different signs and positive when the dividend and the divisor have the same signs.

c) I can use algebra tiles for quotients i and ii in part a.

i) To model $\frac{12k}{4}$ I display twelve k-tiles arranged in 4 equal rows. In each row, there are three k-tiles.

ii) To model (–12k) ÷ 4, I display twelve –k-tiles arranged in 4 equal rows. In each row, there are three –k-tiles.

The divisions in parts iii and iv cannot be easily modelled with algebra tiles. In part iii, we cannot divide twelve k-tiles into a negative number of equal groups. Instead, I could multiply the numerator and the denominator by (–1) to get an equivalent fraction: $\frac{-12k}{4}$. This can be modelled with twelve –k-tiles arranged in 4 equal groups of three –k-tiles.

In part iv, I could divide the divisor and the dividend by (–1) to get the equivalent division expression $12k \div 4$. This can be modelled with twelve k-tiles arranged in 4 equal groups of three k-tiles.

9. a) $2(3v^2 + 2v + 4) = 2(3v^2) + 2(2v) + 2(4)$ Use the distributive property.

$= 6v^2 + 4v + 8$

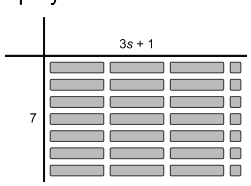
b) $5(m^2 + 3) = 5(m^2) + 5(3)$ Use the distributive property.

$= 5m^2 + 15$

10. a) $(6v^2 + 4v + 8) \div 2 = 3v^2 + 2v + 4$

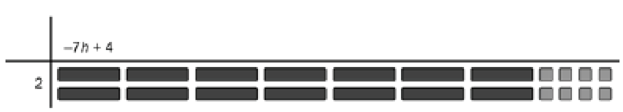
b) $(5m^2 + 15) \div 5 = m^2 + 3$

11. a) Display 7 rows of three s-tiles and one 1-tile.

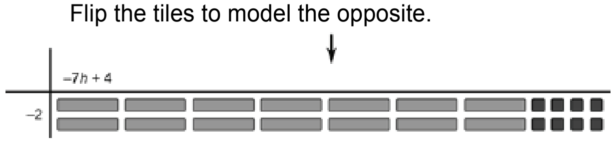


There are twenty-one s-tiles and seven 1-tiles.
So, $7(3s + 1) = 21s + 7$

b) Display 2 rows of seven –h tiles and four 1-tiles.



Flip the tiles to model the opposite.

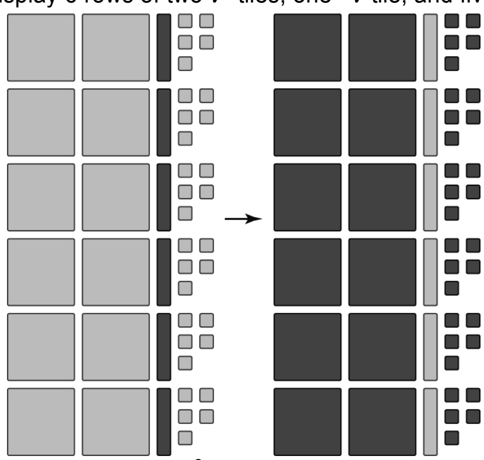


There are fourteen h-tiles and eight –1-tiles.
So, $-2(-7h + 4) = 14h - 8$

c) Display 2 rows of three $-p^2$-tiles, two $-p$-tiles, and one 1-tile.

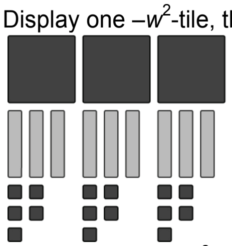
There are six $-p^2$-tiles, four $-p$-tiles, and two 1-tiles.
So, $2(-3p^2 - 2p + 1) = -6p^2 - 4p + 2$

d) Display 6 rows of two v^2-tiles, one $-v$-tile, and five 1-tiles. Then, flip the tiles to model the opposite.



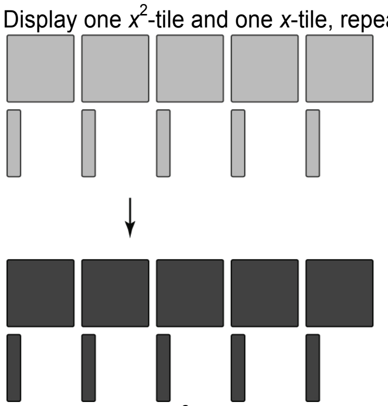
There are twelve $-v^2$-tiles, six v-tiles, and thirty -1-tiles.
So, $-6(2v^2 - v + 5) = -12v^2 + 6v - 30$

e) Display one $-w^2$-tile, three w-tiles, and five -1-tiles, repeated 3 times.



There are three $-w^2$-tiles, nine w-tiles, and fifteen -1-tiles.
So, $(-w^2 + 3w - 5)(3) = -3w^2 + 9w - 15$

f) Display one x^2-tile and one x-tile, repeated 5 times. Then, flip the tiles to model the opposite.



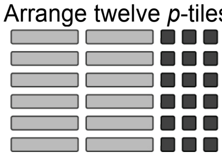
There are five $-x^2$-tiles and five $-x$-tiles.
So, $(x^2 + x)(-5) = -5x^2 - 5x$

12. The student did not multiply correctly. The errors are: $(–2)(–r) = 2r$, not $(–2r)$, and $(–2)(7)$ is $–14$, not $–16$.

Correct solution:

$$\begin{aligned}-2(4r^2 - r + 7) &= -2(4r^2) - 2(-r) - 2(7)\\ &= -8r^2 + 2r - 14\end{aligned}$$

13. a) Arrange twelve p-tiles and eighteen $–1$-tiles in 6 equal rows.



In each row, there are two p-tiles and three $–1$-tiles.

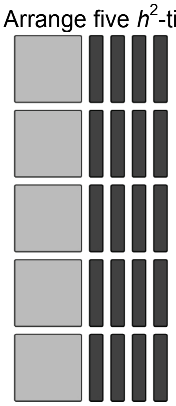
So, $\dfrac{12p-18}{6} = 2p - 3$

b) Arrange six $–q^2$-tiles and ten $–1$-tiles in 2 equal rows.

In each row, there are three $–q^2$-tiles and five $–1$-tiles.

So, $\dfrac{-6q^2-10}{2} = -3q^2 - 5$

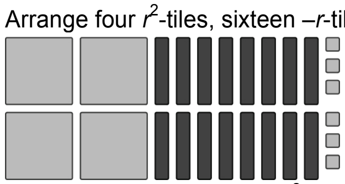
c) Arrange five h^2-tiles and twenty $–h$-tiles in 5 equal rows.



In each row, there is one h^2-tile and four $–h$-tiles.

So, $\dfrac{5h^2-20h}{5} = h^2 - 4h$

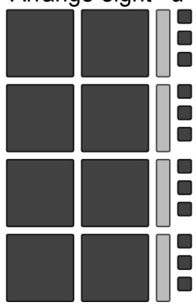
d) Arrange four r^2-tiles, sixteen $–r$-tiles, and six 1-tiles in 2 equal rows.



In each row, there are two r^2-tiles, eight $–r$-tiles, and three 1-tiles.

So, $\dfrac{4r^2-16r+6}{2} = 2r^2 - 8r + 3$

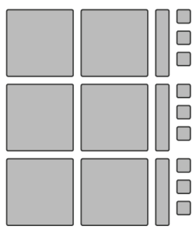
e) Arrange eight $-a^2$-tiles, four a-tiles, and twelve –1-tiles in 4 equal rows.



In each row, there are two $-a^2$-tiles, one a-tile and three –1-tiles.

So, $\dfrac{-8a^2+4a-12}{4} = -2a^2 + a - 3$

f) Arrange six x^2-tiles, three x-tiles, and nine 1-tiles in 3 equal rows.



There are two x^2-tiles, one x-tile, and three 1-tiles in each row.

So, $\dfrac{6x^2+3x+9}{3} = 2x^2 + x + 3$

14. The negative sign of the divisor should apply to all denominators; it is missing in the second and third fractions.
So, $-28m$ divided by –7 gives $+4m$, not $-4m$. Also, 7 divided by –7 simplifies to –1, not 0.
$2m^2 - 4m$ cannot be simplified to $-2m$ because $2m^2$ and $4m$ are unlike terms.
Correct solution:

$$(-14m^2 - 28m + 7) \div (-7)$$
$$= \frac{-14m^2}{-7} + \frac{-28m}{-7} + \frac{7}{-7}$$
$$= 2m^2 + 4m - 1$$

15. I used the distributive property to determine each product.

a) $3(-4u^2 + 16u + 8) = -3(-4u^2) + (-3)(16u) + (-3)(8)$
$= 12u^2 - 48u - 24$

b) $12(2m^2 - 3m) = 12(2m^2) + 12(-3m)$
$= 24m^2 - 36m$

c) $(5t^2 + 2t)(-4) = 5t^2(-4) + 2t(-4)$
$= -20t^2 - 8t$

d) $(-6s^2 - 5s - 7)(-5) = (-6s^2)(-5) + (-5s)(-5) + (-7)(-5)$
$= 30s^2 + 25s + 35$

e) $4(-7y^2 + 3y - 9) = 4(-7y^2) + 4(3y) + 4(-9)$
$= -28y^2 + 12y - 36$

f) $10(8n^2 - n - 6) = 10(8n^2) + 10(-n) + 10(-6)$
$= 80n^2 - 10n - 60$

16. a) $\dfrac{24d^2 - 12}{12}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{24d^2}{12} + \dfrac{-12}{12}$ Simplify each fraction.

$= 2d^2 - 1$

b) $\dfrac{8x + 4}{4}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{8x}{4} + \dfrac{4}{4}$ Simplify each fraction.

$= 2x + 1$

c) $\dfrac{-10 + 4m^2}{-2}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{-10}{-2} + \dfrac{4m^2}{-2}$ Simplify each fraction.

$= 5 - 2m^2$

d) $(25 - 5n) \div (-5)$ Write the quotient expression as a fraction.

$= \dfrac{25 - 5n}{5}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{25}{-5} + \dfrac{-5n}{-5}$ Simplify each fraction.

$= -5 + n$

e) $(-14k^2 + 28k - 49) \div 7$ Write the quotient expression as a fraction.

$= \dfrac{-14k^2 + 28k - 49}{7}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{-14k^2}{7} + \dfrac{28k}{7} + \dfrac{-49}{7}$ Simplify each fraction.

$= -2k^2 + 4k - 7$

f) $\dfrac{30 - 36d^2 + 18d}{-6}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{30}{-6} + \dfrac{-36d^2}{-6} + \dfrac{18d}{-6}$ Simplify each fraction.

$= -5 + 6d^2 - 3d$

g) $\dfrac{-26c^2 + 39c - 13}{-13}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{-26c^2}{-13} + \dfrac{39c}{-13} + \dfrac{-13}{-13}$ Simplify each fraction.

$= 2c^2 - 3c + 1$

17. Use the distributive property to expand brackets.

a) No; $5(j + 4) = 5(j) + 5(4)$, or $5j + 20$, not $5j^2 + 4$.

b) No; $3x(x + 7) = 3x(x) + 3x(7)$, or $3x^2 + 21x$, not $10x^2$.

c) Yes; $5(-2 + 3x) = 5(-2) + 5(3x)$, or $-10 + 15x$, which is equivalent to $15x - 10$.

d) No; $-3(-4x - 1) = -3(-4x) + (-3)(-1)$, or $12x + 3$, not $12x^2 - 3x$.

e) No; $-5(3x^2 - 7x + 2) = -5(3x^2) + (-5)(-7x) + (-5)(2)$, or $-15x^2 + 35x - 10$, not $-15x^2 + 12x - 10$.

f) Yes; $2x(-3x - 7) = 2x(-3x) + (2x)(-7)$, or $-6x^2 - 14x$.

18. a) i) $(3p)(4) = 12p$

ii) $\dfrac{-21x}{3} = -7x$

iii) $$\begin{aligned}(3m^2 - 7)(-4) &= (3m^2)(-4) + (-7)(-4)\\ &= -12m^2 + 28\end{aligned}$$

iv) $$\begin{aligned}\frac{-2f^2 + 14f - 8}{2} &= \frac{-2f^2}{2} + \frac{14f}{2} + \frac{-8}{2}\\ &= -f^2 + 7f - 4\end{aligned}$$

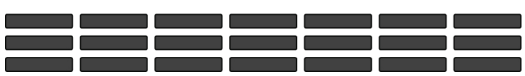
v) $$\begin{aligned}(6y^2 - 36y) \div (-6) &= \frac{6y^2}{-6} + \frac{-36y}{-6}\\ &= -y^2 + 6y\end{aligned}$$

vi) $$\begin{aligned}(-8n + 2 - 3n^2)(3) &= (-8n)(3) + 2(3) - 3n^2(3)\\ &= -24n + 6 - 9n^2\end{aligned}$$

b) The products and quotients in parts i, ii, iii, iv, and vi can be modelled with algebra tiles. The quotient in part v cannot be modelled because we cannot use the algebra tile model to show division by a negative number.

c) i) To model $(3p)(4)$, display three p-tiles repeated 4 times:

ii) To model $\dfrac{-21x}{3}$, display twenty-one $-x$-tiles arranged in 3 equal rows.



19. a) i) $$\begin{aligned}2(2x + 1) &= 2(2x) + 2(1)\\ &= 4x + 2\\ 3(2x + 1) &= 3(2x) + 3(1)\\ &= 6x + 3\\ 4(2x + 1) &= 4(2x) + 4(1)\\ &= 8x + 4\\ 5(2x + 1) &= 5(2x) + 5(1)\\ &= 10x + 5\end{aligned}$$

ii) $2(1 - 2x) = 2(1) + 2(-2x)$
$= 2 - 4x$
$3(1 - 2x) = 3(1) + 3(-2x)$
$= 3 - 6x$
$4(1 - 2x) = 4(1) + 4(-2x)$
$= 4 - 8x$
$5(1 - 2x) = 5(1) + 5(-2x)$
$= 5 - 10x$

b) i) Each time, the coefficient of the *x*-term increases by 2 while the constant term increases by 1.
ii) Each time, the coefficient of the *x*-term decreases by 2 while the constant term increases by 1.

c) i) $12x + 6$; $14x + 7$; $16x + 8$

ii) $6 - 12x$; $7 - 14x$; $8 - 16x$.
The products are correct because they follow the patterns described in part b.

d) i) To predict the preceding products, subtract 2 from the coefficient of the *x*-term and 1 from the constant term each time: $2x + 1$; 0; $-2x - 1$

ii) To predict the preceding products, add two to the coefficient of the x-term and subtract 1 from the constant term each time: $1 - 2x$; 0; $-1 + 2x$

20. a) I know that an equilateral triangle has 3 equal sides. So, I divide the perimeter by 3 to determine the length of one side of the triangle.

$$\frac{15a^2 + 21a + 6}{3} = \frac{15a^2}{3} + \frac{21a}{3} + \frac{6}{3}$$
$$= 5a^2 + 7a + 2$$

The polynomial the represents the length of one side is $5a^2 + 7a + 2$.

b) Substitute $a = 4$ in $5a^2 + 7a + 2$.
$5(4)^2 + 7(4) + 2 = 5(16) + 28 + 2$
$= 110$
The length of one side is 110 cm.

21. a) The perimeter of a square is 4 times the side length.
Square A has side length $4s + 1$.
Square A perimeter: $4(4s + 1) = 4(4s) + 4(1)$
$= 16s + 4$
Square B has side length $3(4s + 1)$.
Square B perimeter: $4[3(4s + 1)] = 12(4s + 1)$
$= 12(4s) + 12(1)$
$= 48s + 12$

b) $(48s + 12) - (16s + 4) = 48s + 12 - 16s - (+4)$
$= 48s - 16s + 12 - 4$
$= 32s + 8$

22. a) $2(2x^2 - 3xy + 7y^2) = 2(2x^2) + 2(-3xy) + 2(7y^2)$
$= 4x^2 - 6xy + 14y^2$

b) $-4(pq + 3p^2 + 3q^2) = -4(pq) + (-4)(3p^2) + (-4)(3q^2)$
$= -4pq - 12p^2 - 12q^2$

c) $(-2gh + 6h^2 - 3g^2 - 9g)(3) = (-2gh)(3) + 6h^2(3) + (-3g^2)(3) + (-9g)(3)$
$= -6gh + 18h^2 - 9g^2 - 27g$

d) $5(-r^2 + 8rs - 3s^2 - 5s + 4r) = 5(-r^2) + 5(8rs) + 5(-3s^2) + 5(-5s) + 5(4r)$
$= -5r^2 + 40rs - 15s^2 - 25s + 20r$

e) $-2(4t^2 - 3v^2 + 19tv - 6v - t) = -2(4t^2) + (-2)(-3v^2) + (-2)(19tv) + (-2)(-6v) + (-2)(-t)$
$= -8t^2 + 6v^2 - 38tv + 12v + 2t$

23. a) $(3n^2 - 12mn + 6m^2) \div 3 = \dfrac{3n^2}{3} + \dfrac{-12mn}{3} + \dfrac{6m^2}{3}$
$= n^2 - 4mn + 2m^2$

b) $\dfrac{-6rs - 16r - 4s}{-2} = \dfrac{-6rs}{-2} + \dfrac{-16r}{-2} + \dfrac{-4s}{-2}$
$= 3rs + 8r + 2s$

c) $\dfrac{10gh - 30g^2 - 15h}{5} = \dfrac{10gh}{5} + \dfrac{-30g^2}{5} + \dfrac{-15h}{5}$
$= 2gh - 6g^2 - 3h$

d) $(12t^2 - 24ut - 48t) \div (-6) = \dfrac{12t^2}{-6} + \dfrac{-24ut}{-6} + \dfrac{-48t}{-6}$
$= -2t^2 + 4ut + 8t$

Take It Further

24. The area of a circle is given by the formula πr^2, where r is the radius.

For the large circle, substitute $r = 3x$ in the formula.

Area of large circle: $\pi(3x)^2 = \pi(3x)(3x)$
$= \pi(3)(3)(x)(x)$
$= 9\pi x^2$

For the small circle, substitute $r = x$.

Area of small circle: $\pi(x)^2 = \pi x^2$

To determine the shaded area in the diagram, subtract the area of the small circle from the area of the large circle.

$9\pi x^2 - \pi x^2 = 8\pi x^2$

A polynomial for the shaded area in the diagram is $8\pi x^2$.

Lesson 5.6 **Multiplying and Dividing a Polynomial by a Monomial** **Practice (pages 255–257)**

Check

4. Interpret each diagram to determine the multiplication sentence.

a) $(3c)(3c) = 9c^2$

b) $m(m + 3) = m^2 + 3m$

c) $2r(r + 2) = 2r^2 + 4r$

5. Start with the area, and one side length.

a) $9c^2 \div 3c = 3c$

b) $(m^2 + 3m) \div m = m + 3$

c) $(2r^2 + 4r) \div 2r = r + 2$

6. The display models $2n(2n + 1)$. This matches part c.

7. **a)** $3x(2x + 1) = 3x(2x) + 3x(1)$
$= 6x^2 + 3x$

b) $4x(2x + 7) = 4x(2x) + 4x(7)$
$= 8x^2 + 28x$

8. **a)** $(6x^2 + 3x) \div (3x) = 2x + 1$

b) $(8x^2 + 28x) \div (4x) = 2x + 7$

Apply

9. **a)** **i)** $(3m)(4m) = 12m^2$

ii) $(–3m)(4m) = –12m^2$

iii) $(3m)(–4m) = –12m^2$

iv) $(–3m)(–4m) = 12m^2$

v) $(4m)(3m) = 12m^2$

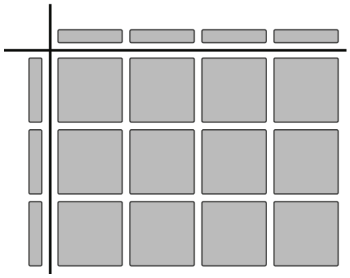
vi) $(4m)(–3m) = –12m^2$

b) There are only two answers in part a, $12m^2$ and $–12m^2$, because the products have the same two factors, $3m$ and $4m$. The signs of the coefficients all differs.

c) Algebra tiles can be used to model all the products.

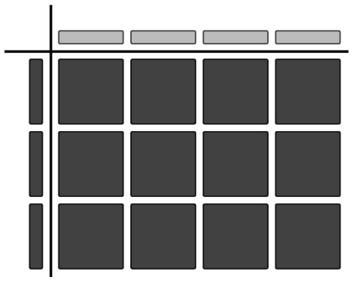
i) $(3m)(4m)$

Make a rectangle with dimensions $3m$ and $4m$. Twelve m^2-tiles fill the rectangle.



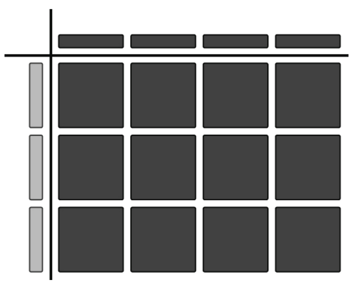
ii) $(–3m)(4m)$

Make a rectangle with guiding tiles: three $–m$-tiles along one dimension; four m-tiles along the other dimension. Twelve $–m^2$-tiles fill the rectangle.



iii) $(3m)(–4m)$

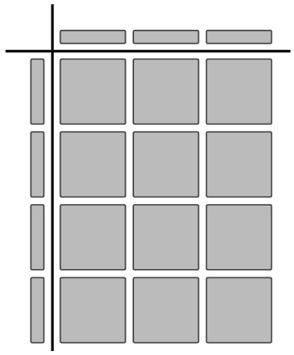
Make a rectangle with guiding tiles: four $–m$-tiles along one dimension; three m-tiles along the other dimension. Twelve $–m^2$-tiles fill the rectangle.



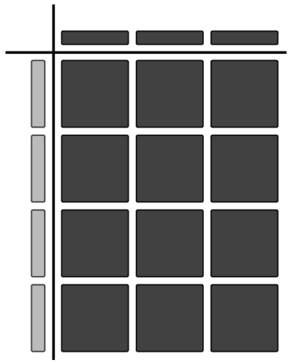
iv) $(–3m)(–4m)$

Make a rectangle with guiding tiles: four $–m$-tiles along one dimension; three $–m$-tiles along the other dimension. Twelve m^2-tiles fill the rectangle.

v) $(4m)(3m)$
Make a rectangle with guiding tiles: four m-tiles along one dimension; three m-tiles along the other dimension. Twelve m^2-tiles fill the rectangle.



vi) $(4m)(-3m)$
Make a rectangle with guiding tiles: four m-tiles along one dimension; three $-m$-tiles along the other dimension. Twelve $-m^2$-tiles fill the rectangle.



10. a) i) $\frac{12x}{2x} = \frac{12}{2} \times \frac{x}{x} = 6$

ii) $\frac{12x}{-2x} = \frac{12}{-2} \times \frac{x}{x} = -6$

iii) $\frac{-12x}{2x} = \frac{-12}{2} \times \frac{x}{x} = -6$

iv) $\frac{-12x}{-2x} = \frac{-12}{-2} \times \frac{x}{x} = 6$

v) $\frac{12x^2}{2x} = \frac{12}{2} \times \frac{x^2}{x} = 6x$

vi) $\frac{12x^2}{2x^2} = \frac{12}{2} \times \frac{x^2}{x^2} = 6$

vii) $\frac{-12x^2}{2x^2} = \frac{-12}{2} \times \frac{x^2}{x^2} = -6$

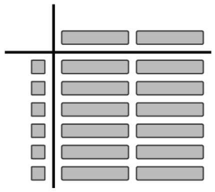
viii) $\frac{12x^2}{-2x^2} = \frac{12}{-2} \times \frac{x^2}{x^2} = -6$

b) Some quotients are equal because in fraction form they have the same numerator and denominator. The sign of the coefficient is the only thing that differs for some of the quotients.

c) Algebra tiles could be used for parts i through v.

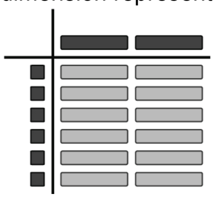
i) $\frac{12x}{2x}$

Arrange twelve x-tiles in a rectangle with one dimension $2x$. The guiding tiles along the other dimension represent 6.



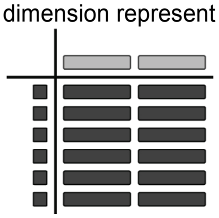
ii) $\frac{12x}{-2x}$

Arrange twelve x-tiles in a rectangle with one dimension $-2x$. The guiding tiles along the other dimension represent -6.



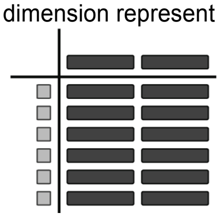
iii) $\frac{-12x}{2x}$

Arrange twelve $-x$-tiles in a rectangle with one dimension $2x$. The guiding tiles along the other dimension represent -6.



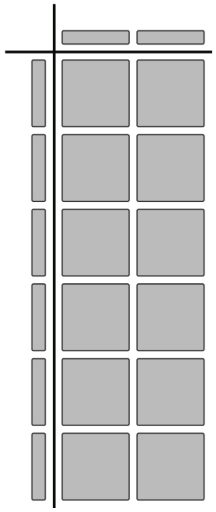
iv) $\frac{-12x}{-2x}$

Arrange twelve $-x$-tiles in a rectangle with one dimension $-2x$. The guiding tiles along the other dimension represent 6.



v) $\frac{12x^2}{2x}$

Arrange twelve x^2-tiles in a rectangle with one dimension $2x$. The guiding tiles along the other dimension represent $6x$.



11. a) $(2r)(-6r) = -12r^2$

b) $(-16n^2) \div (-8n) = \frac{(-16)}{(-8)} \times \frac{n^2}{n}$
$= 2n$

c) $(-5g)(7g) = -35g^2$

d) $\frac{40k}{-10k} = \frac{40}{(-10)} \times \frac{k}{k}$
$= -4$

e) $(9h)(3h) = 27h^2$

f) $\frac{48p^2}{12p} = \frac{48}{12} \times \frac{p^2}{p}$
$= 4p$

g) $18u^2 \div (-3u^2) = \frac{18}{(-3)} \times \frac{u^2}{u^2}$
$= -6$

h) $\frac{-24d^2}{-8d^2} = \frac{(-24)}{(-8)} \times \frac{d^2}{d^2}$
$= 3$

12. I used the distributive property to determine each product.

a) $2x(x + 6) = 2x(x) + (2x)(6)$
$= 2x^2 + 12x$

b) $3t(5t + 2) = 3t(5t) + (3t)(2)$
$= 15t^2 + 6t$

c) $-2w(3w - 5) = (-2w)(3w) + (-2w)(-5)$
$= -6w^2 + 10w$

d) $-x(2 + 8x) = (-x)(2) + (-x)(8x)$
$= -2x - 8x^2$

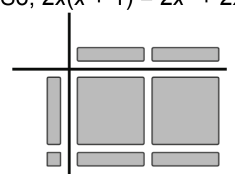
e) $3g(-5 - g) = 3g(-5) + 3g(-g)$
$= -15g - 3g^2$

f) $(4 + 3y)(2y) = 4(2y) + 3y(2y)$
$= 8y + 6y^2$

g) $(-7s - 1)(-y) = (-7s)(-y) + (-1)(-y)$
$= 7sy + y$

h) $(-3 + 6r)(2r) = (-3)(2r) + 6r(2r)$
$= -6r + 12r^2$

13. I use algebra tiles to make a rectangle with dimensions $2x$ and $x + 1$.
The guiding tiles along the top are $2x$ and the guiding tiles along the left side are $x + 1$.
Two x^2-tiles and two x-tiles fill the rectangle.
So, $2x(x + 1) = 2x^2 + 2x$.



14. The student calculated $(-2d)(-3d)$ as $-6d^2$ instead of $6d^2$, and wrote $- (9)(-3d)$ instead of $+ (9)(-3d)$ in the second line.
Correct solution:
$(-2d + 9)(-3d) = (-2d)(-3d) + (9)(-3d)$
$= 6d^2 - 27d$

15. a) i) Think multiplication: $3r \times \square = 3r^2 - 12r$
Since $3r \times r = 3r^2$ and $3r \times (-4) = -12r$, then $3r(r - 4) = 3r^2 - 12r$
So, $\dfrac{3r^2 - 12r}{3r} = r - 4$

ii) Write the quotient expression as the sum of two fractions.
$\dfrac{3r^2 - 12r}{3r} = \dfrac{3r^2}{3r} + \dfrac{-12r}{3r}$ Simplify each fraction.
$\dfrac{3r^2 - 12r}{3r} = r - 4$

b) Responses may vary, depending on personal choice. For example, I find thinking about multiplication to be easier, because using the distributive property is faster for me.
Or, I find writing the quotient expression as the sum of fractions to be easier, because I can see what I'm simplifying and I can keep track of my positive and negative signs.

16. I expressed each quotient expression as the sum of fractions, then simplified.

a) $\dfrac{10x^2+4x}{2x} = \dfrac{10x^2}{2x} + \dfrac{4x}{2x}$

$= 5x + 2$

b) $(6x^2 + 4x) \div x = \dfrac{6x^2+4x}{x}$

$= \dfrac{6x^2}{x} + \dfrac{4x}{x}$

$= 6x + 4$

c) $\dfrac{6y+3y^2}{3y} = \dfrac{6y}{3y} + \dfrac{3y^2}{3y}$

$= 2 + y$

d) $\dfrac{40x^2-16x}{8x} = \dfrac{40x^2}{8x} + \dfrac{-16x}{8x}$

$= 5x - 2$

e) $\dfrac{15g-10g^2}{5g} = \dfrac{15g}{5g} + \dfrac{-10g^2}{5g}$

$= 3 - 2g$

f) $\dfrac{-12k-24k^2}{3k} = \dfrac{-12k}{3k} + \dfrac{-24k^2}{3k}$

$= -4 - 8k$

g) $(24h^2 + 36h) \div (-4h) = \dfrac{24h^2+36h}{-4h}$

$= \dfrac{24h^2}{-4h} + \dfrac{36h}{-4h}$

$= -6h - 9$

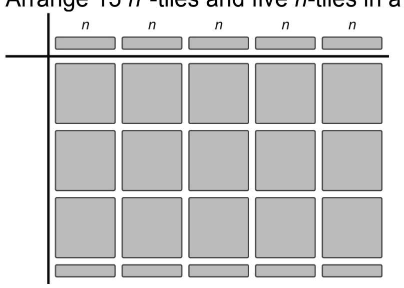
h) $(-8m^2 + 18m) \div (-2m) = \dfrac{-8m^2+18m}{-2m}$

$= \dfrac{-8m^2}{-2m} + \dfrac{18m}{-2m}$

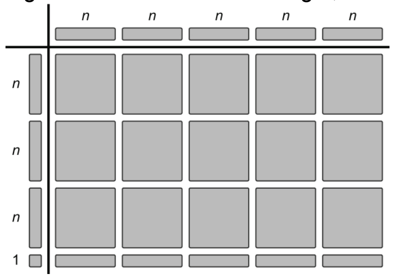
$= 4m - 9$

17. a) i) $\dfrac{15n^2+5n}{5n}$

Use algebra tiles to form a rectangle with guiding tiles:
Arrange 15 n^2-tiles and five n-tiles in a rectangle with one dimension $5n$.



Along the left side of the rectangle, there are 3 guiding n-tiles and 1 guiding 1-tile.



So, $\dfrac{15n^2 + 5n}{5n} = 3n + 1$

ii) $-3r(4 - 7r)$

Use the distributive property.

$-3r(4 - 7r) = -3r(4) + (-3r)(-7r)$

$= -12r + 21r^2$

iii) $(-16s^2 + 4s) \div (-2s)$

Think multiplication.

$(-2s) \times \square = (-16s^2 + 4s)$

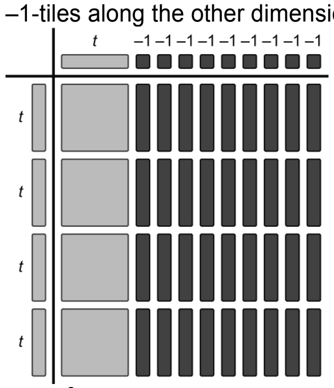
Since $(-2s) \times 8s = -16s^2$, and $(-2s) \times (-2) = 4s$

Then $(-2s) \times (8s - 2) = (-16s^2 + 4s)$

So, $(-16s^2 + 4s) \div (-2s) = 8s - 2$

iv) Use algebra tiles.

I can form a rectangle with guiding tiles: four t-tiles along one dimension, and one t-tile and nine -1-tiles along the other dimension.



Four t^2-tiles and 36 $-t$-tiles fill the rectangle.

So, $(t - 9)(4t) = 4t^2 - 36t$

b) i) Alternative strategy: Write the quotient expression as the sum of two fractions:

$$\frac{15n^2 + 5n}{5n} = \frac{15n^2}{5n} + \frac{5n}{5n}$$

Simplify each fraction.

$$\frac{15n^2 + 5n}{5n} = 3n + 1$$

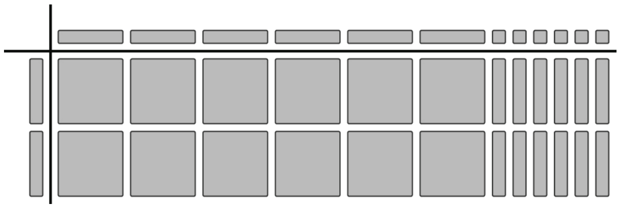
I prefer this strategy, since I can think of the division of each term separately, and make sure I have the correct signs. Or, I prefer the algebra tile model, where I form a rectangle with guiding tiles, because I can see the area and one dimension, and the answer is the number and type of tiles that form the other dimension.

iv) Alternative strategy: Use the distributive property.

$(t - 9)(4t) = (t)(4t) + (-9)(4t)$
$= 4t^2 - 36t$

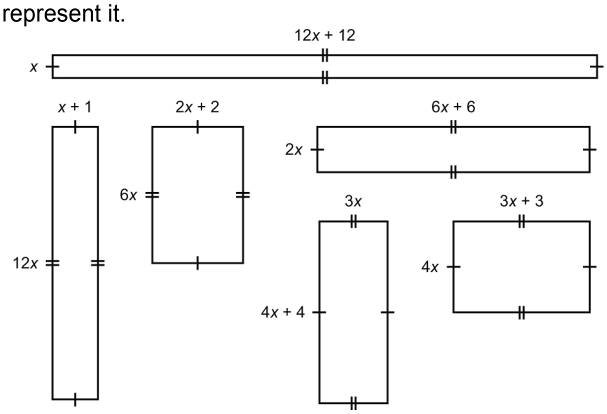
I prefer this strategy; I find it takes too much time to sketch the tiles, especially in this case, when you need 36 tiles. Using the distributive property is faster for me. Or, I prefer the algebra tile model, where I form a rectangle with guiding tiles, because I can see each dimension, and the answer is the number and type of tiles that fill the rectangle.

18. a) Arrange twelve x^2-tiles and twelve x-tiles in a rectangle with one dimension $2x$. The guiding tiles along the other dimension represent $6x + 6$.



$$\frac{12x^2 + 12x}{2x} = 6x + 6$$

b) Use guess and test and algebra tiles to make all possible rectangles that represent the polynomial $12x^2 + 12x$. Identify the dimensions of each algebra-tile rectangle, then sketch and label a rectangle to represent it.



$(12x^2 + 12x) \div x = 12x + 12$
$(12x^2 + 12x) \div 12x = x + 1$
$(12x^2 + 12x) \div 6x = 2x + 2$
$(12x^2 + 12x) \div 2x = 6x + 6$
$(12x^2 + 12x) \div 3x = 4x + 4$
$(12x^2 + 12x) \div 4x = 3x + 3$

19. a) The area of a rectangle is length × width.

Area of large rectangle: $(3s^2 + 2)(2s) = 3s^2(2s) + 2(2s)$
$= 6s^2 + 4s$

Area of small rectangle: $(s + 1)(2s) = s(2s) + 1(2s)$
$= 2s^2 + 2s$

b) To determine the area of the shaded region, subtract the area of the smaller rectangle from the area of the larger rectangle.

$(6s^2 + 4s) - (2s^2 + 2s) = 6s^2 + 4s - 2s^2 - 2s$
$= 6s^2 - 2s^2 + 4s - 2s$
$= 4s^2 + 2s$

c) Substitute $s = 2.5$ in $4s^2 + 2s$.
$$4(2.5)^2 + 2(2.5) = 25 + 5$$
$$= 30$$
The area is 30 cm^2.

20. I used the distributive property to determine each product.

a) $3m(2n + 4) = 3m(2n) + 3m(4)$
$= 6mn + 12m$

b) $(–5 + 3f)(–2g) = (–5)(–2g) + (3f)(–2g)$
$= 10g – 6fg$

c) $7m(–6p + 7m) = 7m(–6p) + 7m(7m)$
$= –42mp + 49m^2$

d) $(–8h – 3k)(4k) = (–8h)(4k) + (–3k)(4k)$
$= –32hk – 12k^2$

e) $(–2t + 3r)(4t) = (–2t)(4t) + 3r(4t)$
$= –8t^2 + 12rt$

f) $(–g)(8h – 5g) = (–g)(8h) + (–g)(–5g)$
$= –8gh + 5g^2$

21. I wrote each quotient expression as the sum of fractions, then simplified.

a) $(12x^2 + 6xy) \div 3x = \dfrac{12x^2 + 6xy}{3x}$
$$= \frac{12x^2}{3x} + \frac{6xy}{3x}$$
$$= 4x + 2y$$

b) $\dfrac{12gh + 6g}{2g} = \dfrac{12gh}{2g} + \dfrac{6g}{2g}$
$$= 6h + 3$$

c) $(–27p^2 + 36pq) \div 9p = \dfrac{-27p^2 + 36pq}{9p}$
$$= \frac{-27p^2}{9p} + \frac{36pq}{9p}$$
$$= –3p + 4q$$

d) $\dfrac{40rs - 35r}{-5r} = \dfrac{40rs}{-5r} + \dfrac{-35r}{-5r}$
$$= –8s + 7$$

e) $\dfrac{14n^2 + 42np}{-7n} = \dfrac{14n^2}{-7n} + \dfrac{42np}{-7n}$
$$= –2n – 6p$$

Take It Further

22. a) Divide the shape into two rectangles.
One rectangle has dimensions $7x$ by $5x$.
Its area is $(7x)(5x) = 35x^2$.
The other rectangle has dimensions $(7x - 3x)$ by $(12x - 5x)$, or $4x$ by $7x$.
Its area is $(4x)(7x) = 28x^2$.
Add the two areas to determine the area of the composite shape:
$35x^2 + 28x^2 = 63x^2$

23. a) A cube has 6 congruent faces. Its surface area is 6 times the area of one face. So, to find the area of a face, I divide the surface area by 6:
$54s^2 \div 6 = 9s^2$
The area of one face is $9s^2$.

b) To find the edge length, I think multiplication.
$\square \times \square = 9s^2$
Since $s \times s = s^2$ and $3 \times 3 = 9$, then $3s \times 3s = 9s^2$
So, the edge length of an edge is $3s$.

24. a) Use the distributive property.
$$\begin{aligned} 2\pi r(r + h) &= 2\pi r(r) + 2\pi r(h) \\ &= 2\pi r^2 + 2\pi rh \end{aligned}$$

b) Substitute $r = 5$ and $h = 3$ in $2\pi r(r + h)$.
$$\begin{aligned} 2\pi(5)(5+3) &= 2\pi(5)(8) \\ &= 80\pi \\ &\doteq 251.33 \end{aligned}$$
The surface area of the cylinder is about 251 cm^2.

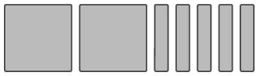
Substitute $r = 5$ and $h = 3$ in $2\pi r^2 + 2\pi rh$.
$$\begin{aligned} 2\pi(5)^2 + 2\pi(5)(3) &= 50\pi + 30\pi \\ &= 80\pi \\ &\doteq 251.33 \end{aligned}$$
The surface area of the cylinder is about 251 cm^2.

25.
$[(2x^2 - 8x + 3xy + 5)] + (24x^2 - 16x - 12xy)] \div 4x$ — Remove the square brackets.
$= (2x^2 - 8x + 3xy + 5 + 24x^2 - 16x - 12xy) \div 4x$ — Group like terms.
$= (2x^2 + 24x^2 - 8x - 16x + 3xy - 12xy + 5) \div 4x$ — Combine like terms.
$= (26x^2 - 24x - 9xy + 5) \div 4x$ — Write the quotient as a sum of 4 fractions.
$= \dfrac{26x^2}{4x} + \dfrac{-24x}{4x} + \dfrac{-9xy}{4x} + \dfrac{5}{4x}$ — Simplify each fraction.
$= \dfrac{13}{2}x - 6 - \dfrac{9}{4}y + \dfrac{5}{4x}$

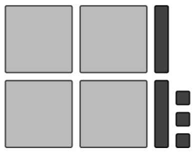
Review (pages 259–261)

Lesson 5.1

1. **a)** $2u^2 + 5u$



b) $4n^2 - 2n - 3$



2. **a)** Variable: *w,* coefficient: 4, constant term: –3

b) Variable: *v,* coefficient: 5, constant term: 3

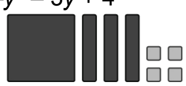
c) Variable: *y,* coefficients: 5, –1, constant term: –6

3. **a) i)** Binomial **ii)** First degree

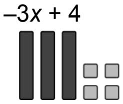
b) i) Monomial **ii)** Second degree

c) i) Trinomial **ii)** Second degree

4. **a)** $-y^2 - 3y + 4$



b) $-3x + 4$



5. Use a table.

Part	Polynomial	Simplified Polynomial
a	$-3x^2 + 3x - 11$	$-3x^2 + 3x - 11$
b	$3x^2 + 4x$	$3x^2 + 4x$
c	$-2 - x$	$-x - 2$
d	$7 + 5x$	$5x + 7$
e	$5x + 7$	$5x + 7$
f	$x - 2$	$x - 2$
g	$4x + 3x^2$	$3x^2 + 4x$
h	$3x - 11 - 3x^2$	$-3x^2 + 3x - 11$

Look for matching results.
Polynomials a and h are equivalent since both can be written $-3x^2 + 3x - 11$.
Polynomials b and g are equivalent since both can be written $3x^2 + 4x$.
Polynomials d and e are equivalent since both can be written $5x + 7$.

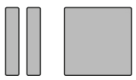
6. **a)** $4x + 3$; first degree

b) $2x^2 - 2x + 6$; second degree

c) $-x^2 - 9$; second degree

7. $2k = k + k$; $k^2 = k \times k$
$2k$ and k^2 are modelled using different types of algebra tiles.

$2k$ k^2



8. I rewrite each polynomial, but in descending order:
a) $-1 - 2h = -2h - 1$
b) $3j + 2j^2 - 4 = 2j^2 + 3j - 4$
c) $-5p + p^2 = p^2 - 5p$

Lesson 5.2

9. **a)** $5x^2$ and $-2x^2$ are like terms
b) $8x$, $5x$, and $-x$ are like terms; 8, –2, and 11 are like terms

10. Use a table then look for matches. Variables do not have to match.

Model	Symbolic Record	Simplified Polynomial	Matching Polynomial
A	$2x^2 + 3x - 2x^2 + 2 - x$	$2x + 2$	d; $2q + 2$
B	$-3x + 3 + x^2 + 2x$	$x^2 - x + 3$	a; $n^2 - n + 3$
C	$-3 - 2x^2 - 2x + x^2 + 2x$	$-x^2 - 3$	b; $-w^2 - 3$
D	$2x^2 - 3 - 2x + 4$	$2x^2 - 2x + 1$	e; $2r^2 - 2r + 1$
E	$-x^2 + x^2 - x^2 - 2x + x^2 - x + 1 - 1 + x$	$-2x$	c; $-2t$

11. Look for a polynomial with 5 terms, that simplifies to 3 terms.
Start with the trinomial. For example, $-x^2 + x + 8$. Introduce changes that don't affect the value, but do add terms. For example: $-x^2 + x + 8 = -x^2 + 3x - 2x + 3 + 5$.
So, the polynomial $-x^2 + 3x - 2x + 3 + 5$ has 5 terms but only 3 terms, $-x^2 + x + 8$, when simplified.

Answers will vary. For example:
$-x^2 + 3x - 2x + 3 + 5$ Combine like terms.
$= -x^2 + x + 8$

12. **a)** $3x + 4 - 2x - 8 + 3x - 3$ Group like terms.
$= 3x - 2x + 3x + 4 - 8 - 3$ Combine like terms.
$= 4x - 7$

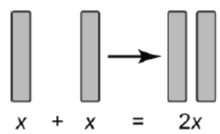
b) $4y^2 - 2y + 3y - 11y^2$ Group like terms.
$= 4y^2 - 11y^2 - 2y + 3y$ Combine like terms.
$= -7y^2 + y$

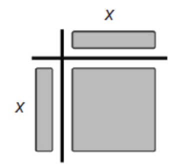
c) $2a^2 + 7a - 3 - 2a^2 - 4a + 6$ Group like terms.
$= 2a^2 - 2a^2 + 7a - 4a - 3 + 6$ Combine like terms.
$= 3a + 3$

d) $2a^2 + 3a + 3a^2 - a^2 - a - 4a^2$ Group like terms.
$= 2a^2 + 3a^2 - a^2 - 4a^2 + 3a - a$ Combine like terms.
$= 2a$

Lesson 5.3 and Lesson 5.4

13.





In the left diagram, $x + x$ is two x-tiles; each tile is x units long and 1 unit wide.
In the right diagram, the side length of the square is x and the area of the tile is $(x)(x) = x^2$. It is an x^2-tile.

14. a) $(-2x^2 + 3x - 4) + (-4x^2 + x - 3)$ Remove brackets.
$= -2x^2 + 3x - 4 - 4x^2 + x - 3$ Group like terms.
$= -2x^2 - 4x^2 + 3x + x - 4 - 3$ Combine like terms.
$= -6x^2 + 4x - 7$

b) $(3x^2 - 6x + 7) - (2x^2 - 2x + 3)$ Subtract each term.
$= 3x^2 - 6x + 7 - (2x^2) - (-2x) - (+3)$ Add the opposite term.
$= 3x^2 - 6x + 7 - 2x^2 + 2x - 3$ Collect like terms.
$= 3x^2 - 2x^2 - 6x + 2x + 7 - 3$ Combine like terms.
$= x^2 - 4x + 4$

15. a) $(p^2 + 3p + 5) + (3p^2 + p + 1)$ Remove brackets.
$= p^2 + 3p + 5 + 3p^2 + p + 1$ Group like terms.
$= p^2 + 3p^2 + 3p + p + 5 + 1$ Combine like terms.
$= 4p^2 + 4p + 6$

b) $(3q^2 + 3q + 7) - (2q^2 + q + 2)$
$= 3q^2 + 3q + 7 - (2q^2) - (+q) - (+2)$ Subtract each term.
$= 3q^2 + 3q + 7 - 2q^2 - q - 2$ Collect like terms.
$= 3q^2 - 2q^2 + 3q - q + 7 - 2$ Combine like terms.
$= q^2 + 2q + 5$

c) $(6 - 3r + 7r^2) - (9 + 4r + 3r^2)$
$= 6 - 3r + 7r^2 - (9) - (+4r) - (+3r^2)$ Subtract each term.
$= 6 - 3r + 7r^2 - 9 - 4r - 3r^2$ Collect like terms.
$= 7r^2 - 3r^2 - 3r - 4r + 6 - 9$ Combine like terms.
$= 4r^2 - 7r - 3$

d) $(5s + 3 - s^2) + (5 + 3s - 2s^2)$ Remove brackets.
$= 5s + 3 - s^2 + 5 + 3s - 2s^2$ Group like terms.
$= -s^2 - 2s^2 + 5s + 3s + 3 + 5$ Combine like terms.
$= -3s^2 + 8s + 8$

e) $(-4t^2 - 3t + 9) - (-2t^2 - 5t - 1)$ Subtract each term.
$= -4t^2 - 3t + 9 - (-2t^2) - (-5t) - (-1)$ Add the opposite terms.
$= -4t^2 - 3t + 9 + 2t^2 + 5t + 1$ Collect like terms.
$= -4t^2 + 2t^2 - 3t + 5t + 9 + 1$ Combine like terms.
$= -2t^2 + 2t + 10$

f) $(-9u^2 - 5) - (-3u^2 - 9)$ — Subtract each term.
$= -9u^2 - 5 - (-3u^2) - (-9)$ — Add the opposite terms.
$= -9u^2 - 5 + 3u^2 + 9$ — Collect like terms.
$= -9u^2 + 3u^2 - 5 + 9$ — Combine like terms.
$= -6u^2 + 4$

g) $(3a^2 + 5ab - 7b^2) + (3b^2 - 10ab - 7a^2)$ — Remove brackets.
$= 3a^2 + 5ab - 7b^2 + 3b^2 - 10ab - 7a^2$ — Group like terms.
$= 3a^2 - 7a^2 + 5ab - 10ab - 7b^2 + 3b^2$ — Combine like terms.
$= -4a^2 - 5ab - 4b^2$

h) $(10xy - 3y^2 + 2x) - (5y - 4x^2 + xy)$ — Subtract each term.
$= 10xy - 3y^2 + 2x - (5y) - (-4x^2) - (+xy)$ — Add the opposite term.
$= 10xy - 3y^2 + 2x - 5y + 4x^2 - xy$ — Collect like terms.
$= 4x^2 - 3y^2 + 10xy - xy + 2x - 5y$ — Combine like terms.
$= 4x^2 - 3y^2 + 9xy + 2x - 5y$

16. Subtract one polynomial from the sum to determine the other polynomial.

$$\begin{aligned}(15c + 6) - (3c - 7) &= 15c + 6 - (3c) - (-7)\\ &= 15c + 6 - 3c + 7\\ &= 15c - 3c + 6 + 7\\ &= 12c + 13\end{aligned}$$

The other polynomial is $12c + 13$.

17. Use a table. Add or subtract the polynomials as indicated, then simplify.

A	$(5x^2 - 2) + (2x^2 + 4) = 5x^2 - 2 + 2x^2 + 4$ $= 5x^2 + 2x^2 - 2 + 4$ $= 7x^2 + 2$	P	$4x^2 + 2x - 1$
B	$(x^2 - 3x) - (4x^2 - x) = x^2 - 3x - (4x^2) - (-x)$ $= x^2 - 3x - 4x^2 + x$ $= x^2 - 4x^2 - 3x + x$ $= -3x^2 - 2x$	Q	$7x^2 + 2$
C	$(x^2 + 2x + 3) + (3x^2 - 4) = x^2 + 2x + 3 + 3x^2 - 4$ $= x^2 + 3x^2 + 2x + 3 - 4$ $= 4x^2 + 2x - 1$	R	$x^2 + 2x - 1$
D	$(3x^2 - x + 2) - (2x^2 - 3x + 3) = 3x^2 - x + 2 - (2x^2) - (-3x) - (+3)$ $= 3x^2 - x + 2 - 2x^2 + 3x - 3$ $= 3x^2 - 2x^2 - x + 3x + 2 - 3$ $= x^2 + 2x - 1$	S	$-3x^2 - 2x$
E	$(-3x - 2) - (3x - 2) = -3x - 2 - (3x) - (-2)$ $= -3x - 2 - 3x + 2$ $= -3x - 3x - 2 + 2$ $= -6x$	T	$-6x$

A is equivalent to Q since both can be written $7x^2 + 2$.
B is equivalent to S since both can be written $-3x^2 - 2x$.
C is equivalent to P since both can be written $4x^2 + 2x - 1$.
D is equivalent to R since both can be written $x^2 + 2x - 1$.
E is equivalent to T since both can be written $-6x$.

18. a) The given polynomial is either the first or second polynomial in the subtraction question. If it is the first polynomial, subtract the difference from the given polynomial to get the second polynomial:

$$\begin{aligned}(-8d^2 - 5d + 1) - (3d^2 - 7d + 4) &= -8d^2 - 5d + 1 - (3d^2) - (-7d) - (+4)\\ &= -8d^2 - 5d + 1 - 3d^2 + 7d - 4\\ &= -8d^2 - 3d^2 - 5d + 7d + 1 - 4\\ &= -11d^2 + 2d - 3\end{aligned}$$

If it is the second polynomial, add the difference to the given polynomial to get the first polynomial:

$$\begin{aligned}(3d^2 - 7d + 4) + (-8d^2 - 5d + 1) &= 3d^2 - 7d + 4 + (-8d^2) + (-5d) + (+1)\\ &= 3d^2 - 7d + 4 - 8d^2 - 5d + 1\\ &= 3d^2 - 8d^2 - 7d - 5d + 4 + 1\\ &= -5d^2 - 12d + 5\end{aligned}$$

The possible solutions are $-11d^2 + 2d - 3$ and $-5d^2 - 12d + 5$.

b) There are two different answers.

19. The perimeter is the sum of the measures of all sides.

a) $$\begin{aligned}2(3a + 5) + 2(2a) &= 6a + 10 + 4a\\ &= 10a + 10\end{aligned}$$

Substitute $a = 3$ cm in $10a + 10$.

$$\begin{aligned}10(3) + 10 &= 30 + 10\\ &= 40\end{aligned}$$

The perimeter is 40 cm.

b) $$\begin{aligned}3(5a + 7) &= 3(5a) + 3(7)\\ &= 15a + 21\end{aligned}$$

Substitute $a = 3$ cm in $15a + 21$.

$$\begin{aligned}15(3) + 21 &= 45 + 21\\ &= 66\end{aligned}$$

The perimeter is 66 cm.

Lesson 5.5

20. a) $(4)(-x) = -4x$

b) $$\begin{aligned}2(2x + 3) &= 2(2x) + 2(3)\\ &= 4x + 6\end{aligned}$$

21. a) $$\begin{aligned}(-4x) \div 4 &= \frac{-4}{4} \times x\\ &= -x\end{aligned}$$

b) $$\begin{aligned}(4x + 6) \div 2 &= \frac{4x}{2} + \frac{6}{2}\\ &= 2x + 3\end{aligned}$$

22. a) $$\begin{aligned}10k \div 2 &= \frac{10}{2} \times k\\ &= 5k\end{aligned}$$

b) $5(-4x^2) = -20x^2$

c) $2(-3m + 4) = 2(-3m) + 2(4)$
$= -6m + 8$

d) $\frac{-6n^2}{3} = \frac{(-6)}{3} \times n^2$
$= -2n^2$

e) $-3(4s - 1) = (-3)(4s) + (-3)(-1)$
$= -12s + 3$

f) $\frac{9 - 12m}{3} = \frac{9}{3} + \frac{-12m}{3}$
$= 3 - 4m$

g) $5(-7 + 2x) = 5(-7) + 5(2x)$
$= -35 + 10x$

h) $-2(1 - 2n + 3n^2) = -2(1) + (-2)(-2n) + (-2)(3n^2)$
$= -2 + 4n - 6n^2$

i) $2(x + 3x^2) = 2(x) + 2(3x^2)$
$= 2x + 6x^2$

j) $(-6p^2 - 6p + 4) \div (-2) = \frac{-6p^2}{-2} + \frac{-6p}{-2} + \frac{4}{-2}$
$= 3p^2 + 3p - 2$

k) $\frac{15 - 21q + 6q^2}{-3} = \frac{15}{-3} + \frac{-21q}{-3} + \frac{6q^2}{-3}$
$= -5 + 7q - 2q^2$

l) $(2 + 5n - 7n^2)(-6) = 2(-6) + 5n(-6) + (-7n^2)(-6)$
$= -12 - 30n + 42n^2$

23. a) $(xy - x^2 + y^2)(-2) = xy(-2) + (-x^2)(-2) + (y^2)(-2)$ Use the distributive property.
$= -2xy + 2x^2 - 2y^2$
$= 2x^2 - 2xy - 2y^2$

b) $(12m^2 - 6n + 8m) \div (-2) = \frac{12m^2}{-2} + \frac{-6n}{-2} + \frac{8m}{-2}$ Write the quotient expression as the sum of 3 fractions
$= -6m^2 + 3n - 4m$

c) $\frac{-18pq + 3p^2 - 9q}{3} = \frac{-18pq}{3} + \frac{3p^2}{3} + \frac{-9q}{3}$ Write the quotient expression as the sum of 3 fractions.
$= -6pq + p^2 - 3q$

d) $4(2r^2 - 3r + 4s - 5s^2) = 4(2r^2) + 4(-3r) + 4(4s) + 4(-5s^2)$ Use the distributive property.
$= 8r^2 - 12r + 16s - 20s^2$

Lesson 5.6

24. a) $3x(2x + 3) = 6x^2 + 9x$

b) $5a(8a + 3) = 5a(8a) + 5a(3)$
$= 40a^2 + 15a$

25. a) $(6x^2 + 9x) \div 3x = 2x + 3$

b) $(40a^2 + 15a) \div 5a = 8a + 3$

26. a) $(7s)(2s) = 14s^2$

b) $(–3g)(–5g) = 15g^2$

c) $m(3m + 2) = m(3m) + m(2)$
$= 3m^2 + 2m$

d) $–5t(t – 3) = –5t(t) + (–5t)(–3)$
$= –5t^2 + 15t$

e) $7z(–4z – 1) = 7z(–4z) + 7z(–1)$
$= –28z^2 – 7z$

f) $(–3f – 5)(–2f) = (–3f)(–2f) + (–5)(–2f)$
$= 6f^2 + 10f$

g) $–5k(3 – k) = –5k(3) + (–5k)(–k)$
$= –15k + 5k^2$
$= 5k^2 – 15k$

h) $y(1 – y) = y(1) + y(–y)$
$= y – y^2$

27. a) Area of the outer rectangle:
$(6x)(3x) = 18x^2$
Area of the inner rectangle:
$(4x)(2x) = 8x^2$

b) To determine the area of the shaded region, subtract:
$18x^2 – 8x^2 = 10x^2$
The area of the shaded region is $10x^2$.

28. a) $24j \div (–6j) = \dfrac{24j}{-6j}$
$= \dfrac{24}{(-6)} \times \dfrac{j}{j}$
$= –4$

b) $\dfrac{24x}{3x} = \dfrac{24}{3} \times \dfrac{x}{x}$
$= 8$

c) $\dfrac{-36x^2}{-9x} = \dfrac{(-36)}{(-9)} \times \dfrac{x^2}{x}$

$= 4x$

d) $(-8a^2 - 12a) \div 4a = \dfrac{-8a^2}{4a} + \dfrac{-12a}{4a}$

$= \dfrac{(-8)}{4} \times \dfrac{a^2}{a} + \dfrac{(-12)}{4} \times \dfrac{a}{a}$

$= -2a - 3$

e) $(-8c + 4c^2) \div 4c = \dfrac{-8c}{4c} + \dfrac{4c^2}{4c}$

$= \dfrac{(-8)}{4} \times \dfrac{c}{c} + \dfrac{4}{4} \times \dfrac{c^2}{c}$

$= -2 + c$

f) $\dfrac{14y^2 - 21y}{-7y} = \dfrac{14y^2}{-7y} + \dfrac{-21y}{-7y}$

$= \dfrac{14}{(-7)} \times \dfrac{y^2}{y} + \dfrac{(-21)}{(-7)} \times \dfrac{y}{y}$

$= -2y + 3$

29. a) The area of the rectangular deck is length times width.
So, the width of the deck = area of the deck ÷ length of the deck

$(8d^2 + 20d) \div 4d = \dfrac{8d^2}{4d} + \dfrac{20d}{4d}$

$= \dfrac{8}{4} \times \dfrac{d^2}{d} + \dfrac{20}{4} \times \dfrac{d}{d}$

$= 2d + 5$

b) Substitute $d = 4$ in the polynomial expressions for the length, width, and area.
Length: $4(4) = 16$
Width: $2(4) + 5 = 13$
Area: $8(4)^2 + 20(4) = 208$
The dimensions are 16 m by 13 m, and the area is 208 m^2.

Practice Test (page 262)

1. **a)** $2t^2 - 6t + 4$

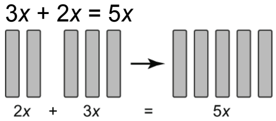
b) Degree: 2; trinomial

c) Constant term: 4, coefficient of the t^2- term: 2

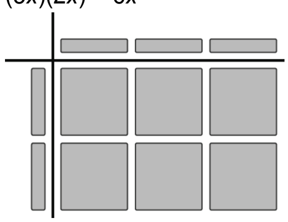
2. **a)** The perimeter is the sum of the measures of all sides:
$(d + 3) + 6 + (d + 3) + d + (6 - 2) + d + 2 = 4d + 18$

b) Substitute $d = 5$ in $4d + 18$.
$4(5) + 18 = 38$
The perimeter of the shape is 38 m.

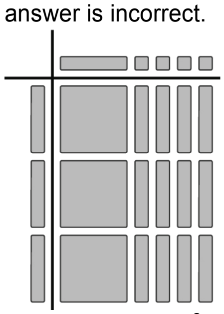
3. **a)** $3x + 2x = 5x$



b) $(3x)(2x) = 6x^2$



4. Form a rectangle with guiding tiles: one *r*-tile and three 1-tiles along one dimension; three *r*-tiles along the other dimension. Three r^2-tiles and twelve *r*-tiles fill the rectangle. So, $3r(r + 4) = 3r^2 + 12r$. The student's answer is incorrect.



$3r(r + 4) = 3r^2 + 12r$

5. **a)** $(15 - 3d) + (3 - 15d)$ Remove brackets.
$= 15 - 3d + 3 - 15d$ Group like terms.
$= -3d - 15d + 15 + 3$ Combine like terms.
$= -18d + 18$

b) $(9h + 3) - (9 - 3h^2)$ Remove brackets.
$= 9h + 3 - (9) - (-3h^2)$ Group like terms.
$= 9h + 3 - 9 + 3h^2$ Combine like terms.
$= 3h^2 + 9h - 6$

c) $(2y^2 + 5y - 6) + (-7y^2 + 2y - 6)$ Remove brackets
$= 2y^2 + 5y - 6 + (-7y^2) + 2y + (-6)$
$= 2y^2 + 5y - 6 - 7y^2 + 2y - 6$ Group like terms.
$= 2y^2 - 7y^2 + 5y + 2y - 6 - 6$ Combine like terms.
$= -5y^2 + 7y - 12$

d) $(7y^2 + y) - (3y - y^2)$ Remove brackets.
$= 7y^2 + y - (3y) - (-y^2)$ Add the opposite term.
$= 7y^2 + y - 3y + y^2$ Group like terms.
$= 7y^2 + y^2 + y - 3y$ Combine like terms.
$= 8y^2 - 2y$

6. a) $25m(3m - 2) = 25m(3m) - (25m)(2)$
$= 75m^2 - 50m$
I used the distributive property.

b) $-5(3v^2 - 2v - 1) = (-5)(3v^2) + (-5)(-2v) + (-5)(-1)$
$= -15v^2 + 10v + 5$
I used the distributive property.

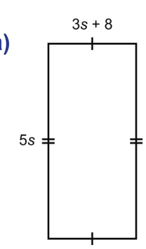
c) $(8x^2 - 4x) \div 2x = \dfrac{8x^2}{2x} + \dfrac{-4x}{2x}$
$= 4x - 2$
I wrote the quotient expression as the sum of two fractions.

d) $\dfrac{-6 + 3g^2 - 15g}{-3} = \dfrac{-6}{-3} + \dfrac{3g^2}{-3} + \dfrac{-15g}{-3}$
$= 2 - g^2 + 5g$
I wrote the quotient expression as the sum of three fractions.

7. a) Think:
$$\begin{array}{r} \square x^2 + \square x + \square \\ + \quad \square x^2 + \square x + \square \\ \hline 3x^2 - 4x - 2 \end{array}$$
There are many possible answers. For example, $(x^2 + x + 1) + (2x^2 - 5x - 3) = 3x^2 - 4x - 2$

b) Think:
$$\begin{array}{r} \square x^2 + \square x + \square \\ - \quad \square x^2 + \square x + \square \\ \hline 3x^2 - 4x - 2 \end{array}$$
There are many possible answers. For example, $(5x^2 + 2x + 2) - (2x^2 + 6x + 4) = 3x^2 - 4x - 2$

8. a)



b) $5s(3s + 8) = 15s^2 + 40s$
The area of the rectangle is $15s^2 + 40s$.

c) $5s + 5s + 3s + 8 + 3s + 8 = 16s + 16$
The perimeter of the rectangle is $16s + 16$.

Unit Problem Algebra Patterns on a 100-Chart (page 263)

I chose this 3 by 3 square:

53	54	55
63	64	65
73	74	75

53 + 64 + 75 = 192
55 + 64 + 73 = 192
The sum of the numbers in each diagonal is equal.

Then I chose this 3 by 3 square:

78	79	80
88	89	90
98	99	100

78 + 89 + 100 = 267
80 + 89 + 98 = 267
The sum of the numbers in each diagonal is equal.

In my first square, the number at the centre is 64, and the sum of the numbers in each diagonal is 192.
192 = 3 × 64
In my second square, the number at the centre is 89, and the sum of the numbers in each diagonal is 267.
267 = 3 × 89
Let x represent the number at the centre, and let s represent the sum.
$s = 3x$
The relationship between the number at the centre of any 3 by 3 square and the sum of the numbers in a diagonal is $s = 3x$.

In my first square, $x = 64$.
64 – 53 = 11; so, the number in the top left corner can be written as $x - 11$.
64 – 55 = 9; so, the number in the top right corner can be written as $x - 9$.
73 – 64 = 9; so, the number in the bottom left corner can be written as $x + 9$.
75 – 64 = 11; so, the number in the bottom right corner can be written as $x + 11$.

In my second square, $x = 89$.
89 – 78 = 11; so, the number in the top left corner can be written as $x - 11$.
89 – 80 = 9; so, the number in the top right corner can be written as $x - 9$.
98 – 89 = 9; so, the number in the bottom left corner can be written as $x + 9$.
100 – 89 = 11; so, the number in the bottom right corner can be written as $x + 11$.

$x - 11$		$x - 9$
	x	
$x + 9$		$x + 11$

Add the numbers in the first diagonal:

$$\begin{aligned}(x - 11) + x + (x + 11) &= x + x + x - 11 + 11 \\ &= 3x\end{aligned}$$

Add the numbers in the second diagonal:
$(x - 9) + x + (x + 9) = x + x + x - 9 + 9$
$= 3x$
This is the same as the relationship I found earlier; the sum of the numbers in a diagonal is 3 times the number at the centre of the 3 by 3 square.

I know that the sum of the numbers in a diagonal of a 3 by 3 square is $3x$.
To determine the number at the centre of the square, x, divide $3x$ by 3.

Since the sum of the numbers in a diagonal of a 3 by 3 square is $3x$, I predict the sum of the numbers in a diagonal of a 5 by 5 square is $5x$.
I predict the sum of the numbers in a diagonal of a 7 by 7 square is $7x$.

For a 5 by 5 square,

42	43	44	45	46
52	53	54	55	56
62	63	64	65	66
72	73	74	75	76
82	83	84	85	86

$64 - 42 = 22$; so, the number in the top left corner can be written as $x - 22$.
$86 - 65 = 22$; so, the number in the bottom right corner can be written as $x + 22$.

$x - 22$				$x - 18$
	$x - 11$		$x - 9$	
		x		
	$x + 9$		$x + 11$	
$x + 18$				$x + 22$

The sum of the numbers in a diagonal is:
$(x - 22) + (x - 11) + x + (x + 11) + (x + 22) = x + x + x + x + x + 22 - 22 - 11 + 11$
$= 5x$

For a 7 by 7 square, follow the same pattern. The sum of the numbers in a diagonal is:
$(x - 33) + (x - 22) + (x - 11) + x + (x + 11) + (x + 22) + (x + 33)$
$= x + x + x + x + x + x + x + 33 - 33 - 22 + 22 - 11 + 11$
$= 7x$

The sum of the numbers in a diagonal is one length of the square times the number at the center of the square.
To determine the number at the centre of a square, divide the sum of the numbers in a diagonal by one length of the square.