Lesson 7.1 Scale Diagrams and Enlargements

## Check

4. a) $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{8}{2}$

$$
=4
$$

The scale factor is 4 .
b) $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{3}{2}$

$$
=1.5
$$

The scale factor is 1.5 .
5. To determine the side length of each scale diagram, multiply the side length of the original square by the scale factor.
a)

| Side Length of <br> Original Square | Scale <br> Factor | Side Length of Scale <br> Diagram |
| :---: | :---: | :---: |
| 12 cm | 3 | $12 \mathrm{~cm} \times 3=36 \mathrm{~cm}$ |
| 82 mm | $\frac{5}{2}$ | $82 \mathrm{~mm} \times \frac{5}{2}=205 \mathrm{~mm}$ |
| 1.55 cm | 4.2 | $1.55 \mathrm{~cm} \times 4.2=6.51 \mathrm{~cm}$ |
| 45 mm | 3.8 | $45 \mathrm{~mm} \times 3.8=171 \mathrm{~mm}$ |
| 0.8 cm | 12.5 | $0.8 \mathrm{~cm} \times 12.5=10 \mathrm{~cm}$ |

## Apply

6. To determine the dimensions of each enlargement, multiply the surfboard dimensions by the scale factor.
a) $17.5 \mathrm{~cm} \times 12=210 \mathrm{~cm}$
$12.5 \mathrm{~cm} \times 12=150 \mathrm{~cm}$
The dimensions of the enlargement are 210 cm by 150 cm .
b) $17.5 \mathrm{~cm} \times 20=350 \mathrm{~cm}$
$12.5 \mathrm{~cm} \times 20=250 \mathrm{~cm}$
The dimensions of the enlargement are 350 cm by 250 cm .
c) Write $\frac{7}{2}$ as 3.5 .
$17.5 \mathrm{~cm} \times 3.5=61.25 \mathrm{~cm}$
61.25 rounded to the nearest centimetre is 61 .
$12.5 \mathrm{~cm} \times 3.5=43.75 \mathrm{~cm}$
43.75 rounded to the nearest centimetre is 44 .

The dimensions of the enlargement are 61 cm by 44 cm .
d) Write $\frac{17}{4}$ as 4.25 .
$17.5 \mathrm{~cm} \times 4.25=74.375 \mathrm{~cm}$
74.375 rounded to the nearest centimetre is 74 .
$12.5 \mathrm{~cm} \times 4.25=53.125 \mathrm{~cm}$
53.125 rounded to the nearest centimetre is 53 .

The dimensions of the enlargement are 74 cm by 53 cm .
7. Length on scale diagram is about 4.8 cm , which is 48 mm .

The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length of salmon fry }}=\frac{48 \mathrm{~mm}}{30 \mathrm{~mm}}$

$$
=1.6
$$

The scale factor is about 1.6.
8. Length on scale diagram is 15 mm .

The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length of head of pin }}=\frac{15 \mathrm{~mm}}{2 \mathrm{~mm}}$

$$
=7.5
$$

The scale factor is 7.5 .
9. Length on picture is 1 cm .
$1 \mathrm{~cm} \times 2.5=2.5 \mathrm{~cm}$

Each length in the scale diagram is 2.5 cm .

10.

11. Determine if corresponding widths and lengths are proportional.
a) Diagram A: $\frac{\text { Width of scale diagram }}{\text { Width of shaded diagram }}=\frac{1}{1}$

$$
=1
$$

$$
\frac{\text { Length of scale diagram }}{\text { Length of shaded diagram }}=\frac{3}{2}
$$

$$
=1.5
$$

The ratios of the corresponding sides are not equal, so diagram $A$ is not a scale diagram of the shaded shape.

$$
\text { Diagram B: } \begin{aligned}
\frac{\text { Width of scale diagram }}{\text { Width of shaded diagram }} & =\frac{2}{1} \\
& =2 \\
\frac{\text { Length of scale diagram }}{\text { Length of shaded diagram }} & =\frac{3}{3} \\
& =1
\end{aligned}
$$

The ratios of the corresponding sides are not equal, so diagram $B$ is not a scale diagram of the shaded shape.

Diagram C: $\frac{\text { Width of scale diagram }}{\text { Width of shaded diagram }}=\frac{2}{1}$

$$
=2
$$

$$
\frac{\text { Length of scale diagram }}{\text { Length of shaded diagram }}=\frac{6}{3}
$$

$$
=2
$$

The ratios of the corresponding sides are equal, so diagram $C$ is a scale diagram of the shaded shape.
i) The scale factor is 2 .
ii) Each side of diagram C is 2 times the length of the corresponding side on the original diagram.

Diagram D: $\frac{\text { Width of scale diagram }}{\text { Width of shaded diagram }}=\frac{2}{1}$

$$
=2
$$

$\frac{\text { Length of scale diagram }}{\text { Length of shaded diagram }}=\frac{5}{3}$
The ratios of the corresponding sides are not equal, so diagram $D$ is not a scale diagram of the shaded shape.
b) Label each side length of the diagram and determine which of $A, B, C, D$ have proportional corresponding lengths.


Diagram A: $\frac{\text { Length of a on scale diagram }}{\text { Length of } a \text { on original diagram }}=\frac{3}{2}$

$$
\begin{aligned}
\frac{\text { Length of } b \text { on scale diagram }}{\text { Length of } b \text { on original diagram }} & =\frac{4}{4} \\
& =1
\end{aligned}
$$

The ratios of the corresponding sides are not equal, so diagram $A$ is not a scale diagram of the shaded shape.

Diagram B: $\frac{\text { Length of } a \text { on scale diagram }}{\text { Length of } a \text { on original diagram }}=\frac{4}{2}$

$$
=2
$$

$$
\frac{\text { Length of } b \text { on scale diagram }}{\text { Length of } b \text { on original diagram }}=\frac{5}{4}
$$

The ratios of the corresponding sides are not equal, so diagram $B$ is not a scale diagram of the shaded shape.

$$
\text { Diagram C: } \begin{aligned}
\frac{\text { Length of } a \text { on scale diagram }}{\text { Length of } a \text { on original diagram }} & =\frac{3}{2} \\
& =1.5 \\
\frac{\text { Length of } b \text { on scale diagram }}{\text { Length of } b \text { on original diagram }} & =\frac{6}{4} \\
& =\frac{3}{2} \\
& =1.5 \\
\frac{\text { Length of } c \text { on scale diagram }}{\text { Length of } c \text { on original diagram }} & =\frac{1.5}{1} \\
& =1.5 \\
\frac{\text { Length of } d \text { on scale diagram }}{\text { Length of } d \text { on original diagram }} & =\frac{4.5}{3} \\
& =1.5 \\
\frac{\text { Length of } e \text { on scale diagram }}{\text { Length of } e \text { on original diagram }} & =\frac{1.5}{1} \\
& =1.5 \\
\frac{\text { Length of } f \text { on scale diagram }}{\text { Length of } f \text { on original diagram }} & =\frac{1.5}{1} \\
& =1.5
\end{aligned}
$$

The ratios of the corresponding sides are equal, so diagram C is a scale diagram of the shaded shape. i) The scale factor is 1.5 .
ii) Each side of diagram C is 1.5 times the length of the corresponding side on the original diagram.

$$
\text { Diagram D: } \begin{aligned}
\frac{\text { Length of } a \text { on scale diagram }}{\text { Length of } a \text { on original diagram }} & =\frac{3}{2} \\
& =1.5 \\
\frac{\text { Length of } b \text { on scale diagram }}{\text { Length of } b \text { on original diagram }} & =\frac{6}{4} \\
& =\frac{3}{2} \\
& =1.5 \\
\frac{\text { Length of } c \text { on scale diagram }}{\text { Length of } c \text { on original diagram }} & =\frac{1.5}{1} \\
& =1.5 \\
\frac{\text { Length of } d \text { on scale diagram }}{\text { Length of } d \text { on original diagram }} & =\frac{4.5}{3} \\
& =1.5 \\
\frac{\text { Length of } e \text { on scale diagram }}{\text { Length of } e \text { on original diagram }} & =\frac{1.5}{1} \\
\frac{\text { Length of } f \text { on scale diagram }}{\text { Length of } f \text { on original diagram }} & =\frac{1.5}{1} \\
& =1.5
\end{aligned}
$$

The ratios of the corresponding sides are equal, so diagram D is a scale diagram of the shaded shape. i) The scale factor is 1.5 .
ii) Each side of diagram D is 1.5 times the length of the corresponding side on the original diagram.
12. a) To calculate the scale factor, the units of length must be the same.
$16 \mathrm{~m}=16000 \mathrm{~mm}$
The scale factor is: $\begin{aligned} \frac{\text { Length on scale diagram }}{\text { Length on original diagram }} & =\frac{16000 \mathrm{~mm}}{50 \mathrm{~mm}} \\ & =320\end{aligned}$
The scale factor of the enlargement is 320 .
b) To determine the height of the penguin on screen, multiply the height of the penguin by the scale factor. $35 \mathrm{~mm} \times 320=11200 \mathrm{~mm}$, or 11.2 m
The penguin on the screen is 11.2 m high.
13. For example: photos of red blood cells, magnified 3100 times. The width of the blood cell in the magnified photo was 2 cm . The scale factor indicates that the width on the original was too small to view with the naked eye.
14. To determine the length of each line segment in the scale diagram, measure the length of each line segment in the original diagram, then multiply each length by the scale factor, 2.5.

15. a) The coordinates of the vertices of the enlargement are: $O(3,3), A(3,12), B(15,3)$

b) Yes, here are 2 more possible enlargements of $\triangle \mathrm{OAB}$. $O(3,-6), A(3,3), B(15,-6)$



## Take It Further

16. a) Multiply the width of a human hair by the scale factor.

200 microns $\times 400=80000$ microns, or 0.08 m , or 8 cm , or 80 mm
b) To calculate the scale factor, the units of length must be the same. 4 microns $=0.0004 \mathrm{~cm}$
The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{5 \mathrm{~cm}}{0.0004 \mathrm{~cm}}$

$$
=12500
$$

The scale factor is 12500 .

## Lesson 7.2 Scale Diagrams and Reductions

## Check

4. a) $\frac{25}{1000}=\frac{1}{40}$

$$
=0.025
$$

b) $\frac{5}{125}=\frac{1}{25}$

$$
=0.04
$$

c) $\frac{2}{1000}=\frac{1}{500}$

$$
=0.002
$$

d) $\frac{3}{180}=\frac{1}{60}$

$$
\doteq 0.0167
$$

5. a) The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{1}{5}$

$$
=0.2
$$

b) The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{3}{4}$

$$
=0.75
$$

6. To determine the scale factor, divide the diameter of the reduction by the diameter of the actual circle.
a)

| Diameter of <br> Actual Circle | Diameter of <br> Reduction | Scale Factor |
| :---: | :---: | :---: |
| 50 cm | 30 cm | $\frac{30 \mathrm{~cm}}{50 \mathrm{~cm}}=\frac{3}{5}$ |
| 30 cm | 20 cm | $\frac{20 \mathrm{~cm}}{30 \mathrm{~cm}}=\frac{2}{3}$ |
| 126 cm | 34 cm | $\frac{34 \mathrm{~cm}}{126 \mathrm{~cm}}=\frac{17}{63}$ |
| 5 m | 2 cm | $\frac{2 \mathrm{~cm}}{500 \mathrm{~cm}}=\frac{1}{250}$ |
| 4 km | 300 m | $\frac{300 \mathrm{~m}}{4000 \mathrm{~m}}=\frac{3}{40}$ |

## Apply

7. Measure the length of one square on the reduced grid and one square on the original grid.

The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{2.5 \mathrm{~mm}}{5 \mathrm{~mm}}$

$$
=\frac{1}{2} \text {, or } 0.5
$$

The scale factor is $\frac{1}{2}$, or 0.5 .
8. Determine if the ratios of corresponding lengths are equal.

Rectangle A:
$\frac{\text { length of longer side on rectangle } A}{\text { length of longer side on large rectangle }}=\frac{4}{12}$

$$
\doteq 0.33
$$

$\frac{\text { length of shorter side on rectangle A }}{\text { length of shorter side on large rectangle }}=\frac{2}{8}$

$$
=0.25
$$

Since the fractions are not equal, rectangle A is not a reduction of the large rectangle.
Rectangle B:
$\frac{\text { length of longer side on rectangle } B}{\text { length of longer side on large rectangle }}=\frac{4}{12}$

$$
\doteq 0.33
$$

$\frac{\text { length of shorter side on rectangle B }}{\text { length of shorter side on large rectangle }}=\frac{3}{8}$

$$
=0.375
$$

Since the fractions are not equal, rectangle $B$ is not a reduction of the large rectangle.
Rectangle C:
$\frac{\text { length of longer side on rectangle } C}{\text { length of longer side on large rectangle }}=\frac{3}{12}$

$$
=0.25
$$

length of shorter side on rectangle C $=0.25$
$\frac{\text { length of shorter side on rectangle C }}{\text { length of shorter side on large rectangle }}=\frac{2}{8}$

$$
=0.25
$$

Since the fractions are equal, pairs of corresponding lengths are proportional, so rectangle $C$ is a reduction of the large rectangle.
9. Determine if corresponding lengths are proportional.

$$
\text { Polygons A and B: } \begin{aligned}
\frac{\text { length of longer side on } B}{\text { length of longer side on } A} & =\frac{2}{6} \\
& =\frac{1}{3} \\
& \frac{\text { length of shorter side on } B}{\text { length of shorter side on } A}
\end{aligned}=\frac{1}{3}
$$

Polygons $A$ and $B$ have proportional corresponding lengths. $B$ is a reduction of $A$ and the scale factor is $\frac{1}{3}$.
The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{1}{3}$
10. Label each length in the diagram. Determine if corresponding lengths are proportional.

Polygons $A$ and $B: \begin{aligned} & \frac{\text { length of side } a \text { on } B}{\text { length of side } a \text { on } A}=\frac{3}{5} \\ & \\ & \begin{array}{l}\text { length of side } b \text { on } B \\ \text { length of side } b \text { on } A\end{array}=\frac{3}{5} \\ & \frac{\text { length of side } c \text { on } B}{\text { length of side } c \text { on } A}=\frac{1.5}{3}\end{aligned}$

$$
\begin{aligned}
& \text { Polygons } B \text { and } C: \frac{\text { length of side } a \text { on } C}{\text { length of side } a \text { on } B}=\frac{2}{3} \\
& \frac{\text { length of side } b \text { on } C}{\text { length of side } b \text { on } B}=\frac{2}{3} \\
& \frac{\text { length of side } c \text { on } C}{\text { length of side } c \text { on } B}=\frac{1}{1.5} \\
& \frac{\text { length of side } d \text { on } C}{\text { length of side } d \text { on } B}=\frac{1}{1.5} \\
& \frac{\text { length of side } e \text { on } C}{\text { length of side } e \text { on } B}=\frac{1}{1.5} \\
& \frac{\text { length of side } f \text { on } C}{\text { length of side } f \text { on } B}=\frac{1}{1.5}
\end{aligned}
$$

Polygons B and C have proportional corresponding lengths.
$C$ is a reduction of $B$ and the scale factor is $\frac{2}{3}$.
11. To determine the corresponding length on the scale diagram, multiply the original length by the scale factor.
a) $75 \mathrm{~cm} \times \frac{1}{3}=25 \mathrm{~cm}$

The corresponding length on the scale diagram is 25 cm .
b) $60 \mathrm{~cm} \times \frac{3}{50}=3.6 \mathrm{~cm}$

The corresponding length on the scale diagram is 3.6 cm .
c) $200 \mathrm{~cm} \times 0.05=10 \mathrm{~cm}$

The corresponding length on the scale diagram is 10 cm .
d) $8 \mathrm{~m}=800 \mathrm{~cm}$
$800 \mathrm{~cm} \times 0.02=16 \mathrm{~cm}$
The corresponding length on the scale diagram is 16 cm .
e) $12 \mathrm{~km}=1200000 \mathrm{~cm}$
$1200000 \mathrm{~cm} \times 0.00004=48 \mathrm{~cm}$
The corresponding length on the scale diagram is 48 cm .
12. a)

b)

13. a) The scale is $1: 400$; this means that 1 cm on the diagram represents 400 cm on the outdoor hockey rink.

So, the scale factor is $\frac{1}{400}$.
Each net is 1.82 m long. I multiply by the scale factor to determine how long each net would be on the diagram.
$182 \mathrm{~cm} \times \frac{1}{400}=0.455 \mathrm{~cm}$, or 4.55 mm
Each net would be about 4.55 mm on the diagram.
b) I measured the width of the rink on the scale diagram; it is 4 cm .

Each actual measure is 400 times as great as the measure on the scale diagram.
So, the actual width of the rink is: $4 \mathrm{~cm} \times 400=1600 \mathrm{~cm}$
The width of the rink is 1600 cm ; that is 16 m .
14. Multiply each dimension by the scale factor to determine the dimensions of the scale drawing.

Court length: $\frac{1}{200} \times 18 \mathrm{~m}=0.09 \mathrm{~m}$, or 9 cm
Court width: $\frac{1}{200} \times 9 \mathrm{~m}=0.045 \mathrm{~m}$, or 4.5 cm

15. Multiply each dimension by the scale factor to determine the dimensions of the scale drawing. Field length: $0.002 \times 99 \mathrm{~m}=0.198 \mathrm{~m}$, or 19.8 cm
Field width: $0.002 \times 54 \mathrm{~m}=0.108 \mathrm{~m}$, or 10.8 cm

16. Answers will vary depending on the size of your classroom.

For example:
The classroom is about 10 m by 12 m .
I will use $0.5-\mathrm{cm}$ grid paper. The grid on the grid paper is 20 cm high and 16 cm wide. I will choose a scale factor of 0.015 so that my diagram covers most of the grid.
I get these lengths for my scale diagram of the classroom:
length of shorter side of classroom: $10 \mathrm{~m} \times 0.015=0.15 \mathrm{~m}$, or 15 cm
length of longer side of classroom: $12 \mathrm{~m} \times 0.015=0.18 \mathrm{~m}$, or 18 cm
desk length: $1 \mathrm{~m} \times 0.015=0.015 \mathrm{~m}$, or 1.5 cm
desk width: $0.66 \mathrm{~m} \times 0.015=0.0099 \mathrm{~m}$, or about 1 cm
door width: $0.8 \mathrm{~m} \times 0.015=0.012 \mathrm{~m}$, or 1.2 cm
window: $6.7 \mathrm{~m} \times 0.015=0.1005 \mathrm{~m}$, or about 10 cm
blackboard: $4.7 \mathrm{~m} \times 0.015=0.0705 \mathrm{~m}$, or about 7 cm
teacher's desk width: $1 \mathrm{~m} \times 0.015=0.015 \mathrm{~m}$, or 1.5 cm
teacher's desk length: $1.3 \mathrm{~m} \times 0.015=0.0195 \mathrm{~m}$, or about 2 cm
My Classrooom

17. Answers will vary. For example:

My room is 5.5 m by 4 m .
I chose a scale factor of 0.02 .
room length: $5.5 \mathrm{~m} \times 0.02=0.11 \mathrm{~m}$, or 11 cm
room width: $4 \mathrm{~m} \times 0.02=0.08 \mathrm{~m}$, or 8 cm bed length: $2.45 \mathrm{~m} \times 0.02=0.049 \mathrm{~m}$, or 4.9 cm bed width: $1.12 \mathrm{~m} \times 0.02=0.0224 \mathrm{~m}$, or 2.24 cm dresser length: $1 \mathrm{~m} \times 0.02=0.02 \mathrm{~m}$, or 2 cm dresser width: $0.7 \mathrm{~m} \times 0.02=0.014 \mathrm{~m}$, or 1.4 cm desk length: $1.15 \mathrm{~m} \times 0.02=0.023 \mathrm{~m}$, or 2.3 cm desk width: $0.66 \mathrm{~m} \times 0.02=0.0132 \mathrm{~m}$, or 1.32 cm bookshelf length: $0.95 \mathrm{~m} \times 0.02=0.019 \mathrm{~m}$, or 1.9 cm bookshelf width: $0.25 \mathrm{~m} \times 0.02=0.005 \mathrm{~m}$, or 0.5 cm closet length: $2 \mathrm{~m} \times 0.02=0.04 \mathrm{~m}$, or 4 cm closet width: $1.06 \mathrm{~m} \times 0.02=0.0212 \mathrm{~m}$, or 2.12 cm shelves length: $0.89 \mathrm{~m} \times 0.02=0.0178 \mathrm{~m}$, or 1.78 cm shelves width: $0.42 \mathrm{~m} \times 0.02=0.0084 \mathrm{~m}$, or 0.84 cm

18. Answers may vary. For example:

A house plan in a newspaper advertisement has a scale of $\frac{1}{100}$, which means that each length on the diagram is $\frac{1}{100}$ of the length in the house; for example, a room dimension of 3.5 cm on the plan represents $100 \times 3.5 \mathrm{~cm}=350 \mathrm{~cm}$, or 3.5 m , which is the corresponding dimension in the house.
19. a) I measure the length of the scale diagram; it is 150 cm . The length of the room is 7.5 m , or 750 cm .

So, the scale factor is: $\frac{15 \mathrm{~cm}}{750 \mathrm{~cm}}=\frac{1}{50}$, or 1:50
b) i) I measure the dimensions of the ping pong table on the diagram; the length is 5.5 cm and the width is 3 cm . Then I multiply each dimension by the scale factor to get the actual length and width.
Length: $5.5 \mathrm{~cm} \times 50=275 \mathrm{~cm}$, or 2.75 m
Width: $3 \mathrm{~cm} \times 50=150 \mathrm{~cm}$, or 1.5 m
The dimensions of the ping pong table are 2.75 m by 1.5 m .
ii) I measure the dimensions of the pool table on the diagram; the length is 5 cm and the width is 2.5 cm . Then I multiply each dimension by the scale factor to get the actual length and width.

Length: $5 \mathrm{~cm} \times 50=250 \mathrm{~cm}$, or 2.5 m
Width: $2.5 \mathrm{~cm} \times 50=125 \mathrm{~cm}$, or 1.25 m
The dimensions of the pool table are 2.5 m by 1.25 m .
c) I measure the size of the flat screen television on the diagram to be 3 cm . I then multiply this by the scale factor to get its actual size.
$3 \mathrm{~cm} \times 50=150 \mathrm{~cm}$, or 1.5 m
d) The length of the room is 7.5 m .

I measure the width of the room on the diagram to be 12 cm . I then multiply by the scale factor to get the actual width of the room.
$12 \mathrm{~cm} \times 50=600 \mathrm{~cm}$, or 6 m
To determine the cost of moulding, I first calculate the perimeter of the room.
$2 \times(7.5 \mathrm{~m}+6 \mathrm{~m})=27 \mathrm{~m}$
Since moulding costs $\$ 4.99 / \mathrm{m}$, the cost will be $\$ 4.99 / \mathrm{m} \times 27 \mathrm{~m}=\$ 134.73$.
It will cost $\$ 134.73$ to place moulding around the ceiling of the room.
20. a) The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{28 \mathrm{~cm}}{7000 \mathrm{~cm}}$

$$
=0.004
$$

b) $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{24 \mathrm{~cm}}{\text { Length on original diagram }}$

$$
\begin{aligned}
\text { So, length on original diagram } & =\frac{24 \mathrm{~cm}}{0.004} \\
& =6000 \mathrm{~cm}, \text { or } 60 \mathrm{~m}
\end{aligned}
$$

The wingspan on the 747 plane is 60 m .
c) $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}=\frac{7.6 \mathrm{~cm}}{\text { Length on original diagram }}$

$$
=0.004
$$

So, length on original diagram $=\frac{7.6 \mathrm{~cm}}{0.004}$

$$
=1900 \mathrm{~cm}, \text { or } 19 \mathrm{~m}
$$

The height of the tail on the 747 plane is 19 m .

## Take It Further

21. Students' answers may vary. For example:

I chose a scale factor of 0.000000001 so my diagram will fit on one piece of paper.
Approximate diameters on scale diagram:
Earth: $12760 \mathrm{~km} \times 0.000000001=0.00001276 \mathrm{~km}$, or 1.3 cm Jupiter: $142800 \mathrm{~km} \times 0.000000001=0.0001428 \mathrm{~km}$, or 14.3 cm Mars: $6790 \mathrm{~km} \times 0.000000001=0.00000679 \mathrm{~km}$, or 0.7 cm Mercury: $4880 \mathrm{~km} \times 0.000000001=0.00000488 \mathrm{~km}$, or 0.5 cm
Neptune: $49500 \mathrm{~km} \times 0.000000001=0.0000495 \mathrm{~km}$, or 5 cm
Saturn: $120600 \mathrm{~km} \times 0.000000001=0.0001206 \mathrm{~km}$, or 12.1 cm
Uranus: $51120 \mathrm{~km} \times 0.000000001=0.00005112 \mathrm{~km}$, or 5.1 cm
Venus: $12100 \mathrm{~km} \times 0.000000001=0.0000121 \mathrm{~km}$, or 1.2 cm


## Check

4. Solve each proportion.
a) $\frac{A B}{8}=\frac{3}{2}$

$$
\begin{aligned}
& 8 \times \frac{A B}{8}=8 \times \frac{3}{2} \\
& A B=\frac{8 \times 3}{2} \\
& A B=12
\end{aligned}
$$

b) $\frac{\mathrm{BC}}{25}=\frac{12}{15}$

$$
\begin{aligned}
& 25 \times \frac{B C}{25}=25 \times \frac{12}{15} \\
& B C=\frac{25 \times 12}{15} \\
& B C=20
\end{aligned}
$$

c) $\frac{C D}{4}=\frac{63}{28}$

$$
4 \times \frac{C D}{4}=4 \times \frac{63}{28}
$$

$$
C D=\frac{4 \times 63}{28}
$$

$$
C D=9
$$

d) $\frac{D E}{7}=\frac{24}{30}$

$$
\begin{aligned}
& 7 \times \frac{D E}{7}=7 \times \frac{24}{30} \\
& D E=\frac{7 \times 24}{30} \\
& D E=5.6
\end{aligned}
$$

5. Solve each proportion.
a) $\frac{x}{2.5}=\frac{7.5}{1.5}$

$$
2.5\left(\frac{x}{2.5}\right)=2.5\left(\frac{7.5}{1.5}\right)
$$

$$
x=12.5
$$

b) $\frac{y}{21.4}=\frac{23.7}{15.8}$

$$
\begin{aligned}
& 21.4\left(\frac{y}{21.4}\right)=21.4\left(\frac{23.7}{15.8}\right) \\
& y=32.1
\end{aligned}
$$

c) $\frac{z}{12.5}=\frac{0.8}{1.2}$

$$
\begin{aligned}
& 12.5\left(\frac{z}{12.5}\right)=12.5\left(\frac{0.8}{1.2}\right) \\
& z=8 . \overline{3}
\end{aligned}
$$

d) $\frac{a}{0.7}=\frac{1.8}{24}$

$$
0.7\left(\frac{a}{0.7}\right)=0.7\left(\frac{1.8}{24}\right)
$$

$$
a=0.0525
$$

6. Similar quadrilaterals will have the same shape. So, determine if ABCD is similar to QPMN, if EFGH is similar to UVWX, and if IJKL is similar to QRST.

For quadrilaterals $A B C D$ and QPMN, determine the ratio of corresponding sides.

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{QP}}=\frac{2}{4}=\frac{1}{2} \\
& \frac{\mathrm{BC}}{\mathrm{PM}}=\frac{4}{8}=\frac{1}{2} \\
& \frac{\mathrm{CD}}{\mathrm{MN}}=\frac{6}{12}=\frac{1}{2} \\
& \frac{\mathrm{DA}}{\mathrm{NQ}}=\frac{4}{8}=\frac{1}{2} \\
& \frac{\mathrm{AB}}{\mathrm{QP}}=\frac{\mathrm{BC}}{\mathrm{PM}}=\frac{\mathrm{CD}}{\mathrm{MN}}=\frac{\mathrm{DA}}{\mathrm{NQ}}=\frac{1}{2} .
\end{aligned}
$$

This shows that all sides are proportional. Upon inspection, we can see that $\angle \mathrm{A}=\angle \mathrm{Q}, \angle \mathrm{B}=\angle \mathrm{P}$, $\angle \mathrm{C}=\angle \mathrm{M}, \angle \mathrm{D}=\angle \mathrm{N}$. So, quadrilateral $\mathrm{ABCD} \sim$ quadrilateral QPMN.

For quadrilaterals EFGH and UVWX, determine the ratio of corresponding sides.
$\frac{E F}{U V}=\frac{4}{1}=4$
$\frac{\mathrm{FG}}{\mathrm{VW}}=\frac{2}{1}=2$
These numbers show that the corresponding sides are not proportional. So, EFGH and UVWX are not similar.

Quadrilaterals IJKL and QRST are squares. The measure of each angle in a square is $90^{\circ}$. So, for any two squares, their corresponding angles are equal. So, $\angle \mathrm{I}=\angle \mathrm{Q}, \angle \mathrm{J}=\angle \mathrm{R}, \angle \mathrm{K}=\angle \mathrm{S}, \angle \mathrm{L}=\angle \mathrm{T}$.
As well, since the quadrilaterals are squares, all fours sides are equal and $\frac{\mathrm{IJ}}{\mathrm{QR}}=\frac{\mathrm{JK}}{\mathrm{RS}}=\frac{\mathrm{KL}}{\mathrm{ST}}=\frac{\mathrm{LI}}{\mathrm{TQ}}=2$.
IJKL is similar to QRST
7.

8.


## Apply

9. The measure of each angle in a rectangle is $90^{\circ}$. So, for any two rectangles, their corresponding angles are equal.
For each pair of rectangles, determine the ratios of corresponding sides. Since the opposite sides of a rectangle are equal, we only need to check the ratios of corresponding lengths and corresponding widths.

For rectangles ABCD and EFGH:

$$
\begin{aligned}
\frac{\mathrm{AB}}{\mathrm{EF}} & =\frac{5.4}{7.5} & \frac{\mathrm{BC}}{\mathrm{FG}} & =\frac{1.9}{2.5} \\
& =0.72 & & =0.76
\end{aligned}
$$

These numbers show that the corresponding sides are not proportional.
So, rectangles $A B C D$ and EFGH are not similar.
For rectangles $A B C D$ and IJKM:

$$
\begin{aligned}
\frac{\mathrm{AB}}{\mathrm{IJ}} & =\frac{5.4}{4.8} & \frac{\mathrm{BC}}{\mathrm{JK}} & =\frac{1.9}{1.6} \\
& =1.125 & & =1.1875
\end{aligned}
$$

These numbers show that the corresponding sides are not proportional.
So, rectangles ABCD and IJKM are not similar.

For rectangles EFGH and IJKM:

$$
\begin{aligned}
\frac{\mathrm{EF}}{\mathrm{IJ}} & =\frac{7.5}{4.8} & \frac{\mathrm{FG}}{\mathrm{JK}} & =\frac{2.5}{1.6} \\
& =1.5625 & & =1.5625
\end{aligned}
$$

These numbers show that the corresponding sides are proportional.
So, rectangles EFGH and IJKM are similar.
10. a) i) Each side length is twice the length of the corresponding side in the original polygon. Corresponding angles are equal.

ii) Each side length is one-half the length of the corresponding side in the original polygon. Corresponding angles are equal.

b) i) Each side length is twice the length of the corresponding side in the original polygon. Corresponding angles are equal.

ii) Each side length is one-half the length of the corresponding side in the original polygon. Corresponding angles are equal.

11. a) No; from inspection, I can tell that some corresponding angles are not equal.
b) Label the diagrams and determine the ratios of corresponding sides.


$$
\frac{\mathrm{AB}}{\mathrm{JK}}=\frac{2}{1}=2 \quad \frac{\mathrm{BC}}{\mathrm{KL}}=\frac{4}{2}=2
$$

These numbers show that corresponding sides are proportional. So, the polygons are similar.
12.

a) i) Compare the side lengths of rectangles A and B :
$\frac{\text { width of rectangle } B}{\text { width of rectangle } A}=\frac{6}{3}$, or 2
$\frac{\text { length of rectangle } B}{\text { length of rectangle } A}=\frac{8}{4}$, or 2
Corresponding sides are proportional, and corresponding angles are equal (all angles are $90^{\circ}$ ).
So, rectangles $A$ and $B$ are similar.

Compare the side lengths of rectangles A and C :
$\frac{\text { width of rectangle } C}{\text { width of rectangle } A}=\frac{9}{3}$, or 3
$\frac{\text { length of rectangle } C}{\text { length of rectangle } A}=\frac{12}{4}$, or 3
Corresponding sides are proportional, and corresponding angles are equal (all angles are $90^{\circ}$ ).
So, rectangles $A$ and $C$ are similar.
Compare the side lengths of rectangles $B$ and $C$ :
$\frac{\text { width of rectangle } C}{\text { width of rectangle } B}=\frac{9}{6}$, or $\frac{3}{2}$
$\frac{\text { length of rectangle } C}{\text { length of rectangle } B}=\frac{12}{8}$, or $\frac{3}{2}$
Corresponding sides are proportional, and corresponding angles are equal (all angles are $90^{\circ}$ ).
So, rectangles B and C are similar.
Compare the side lengths of rectangles $A$ and $D$ :
$\frac{\text { width of rectangle } D}{\text { width of rectangle } A}=\frac{12}{3}$, or 4
$\frac{\text { length of rectangle } D}{\text { length of rectangle } A}=\frac{15}{4}$, or 3.75
Corresponding sides are not proportional.
So, rectangle D is not similar to the other rectangles.
ii) I multiply the dimensions of rectangle $D$ by the same scale factor to determine the dimensions of a similar rectangle.
width of 1 st rectangle: 12 units $\times \frac{1}{3}=4$ units
length of 1 st rectangle: 15 units $\times \frac{1}{3}=5$ units
width of 2 nd rectangle: 12 units $\times \frac{1}{2}=6$ units
length of 2 nd rectangle: 15 units $\times \frac{1}{2}=7.5$ units

b) Use the scale factors from part a) i).

Diagonal of rectangle $B$ has length:
$2 \times 5$ units $=10$ units
Diagonal of rectangle $C$ has length:
$3 \times 5$ units $=15$ units

## PEARSON MMS 9 UNIT 7

Similarity and Transformations
13. a)

b) Since the doors are proportional, the ratios of corresponding lengths are equal.

Let $x$ represent the width of the doll's house door. Then,

$$
\begin{aligned}
& \frac{25}{200}=\frac{x}{75} \\
& x=\frac{75 \times 25}{200}=9.375
\end{aligned}
$$

The width of the doll's house door is about 9.4 cm .
14. Not all corresponding angles are equal. So, the pentagons are not similar.
15. a) i)

ii)

iii)

b) Yes; all regular polygons of the same type are similar because they have equal corresponding angles and proportional corresponding sides.

## Take It Further

16. Yes, all circles are similar since they have the same shape.
(The ratio of circumference to diameter is a constant: $\pi$ ).
17. Rectangles will vary. For example:


A rectangle with dimensions 2 units by 3 units is similar to a rectangle with dimensions 6 units by 9 units.
a) The ratio of corresponding sides is 1:3.
b) The area of the first rectangle is 6 square units. The area of the second rectangles is 54 square units. The ratio of the areas is $6: 54$, or 1:9.
c) The ratio of the areas is the square of the ratio of the corresponding sides.
d) I would test a different pair of similar shapes:

The ratio of the side lengths of the similar figures I drew in Practice question 10 part a is 4:1. The larger shape has area 120 square units and the smaller shape has area 7.5 square units. So, the ratio of the areas is $120: 7.5$, or $16: 1$. The ratio of the areas is the square of the ratio of the corresponding sides. Since the shapes in Practice question 10 were not regular polygons and had more than 4 sides, I think the relationship in part c will be true for all similar shapes.

## Check

4. a) Similar; the corresponding angles are equal:

$$
\angle \mathrm{P}=\angle \mathrm{N}=30^{\circ}, \angle \mathrm{Q}=\angle \mathrm{M}=70^{\circ}, \angle \mathrm{R}=\angle \mathrm{H}=80^{\circ}
$$

b) Similar; the corresponding sides are proportional:
$\frac{S T}{J H}=\frac{3}{6}=\frac{1}{2}$
$\frac{\mathrm{TU}}{\mathrm{HG}}=\frac{5}{10}=\frac{1}{2}$
$\frac{U S}{G J}=\frac{4}{8}=\frac{1}{2}$
So, $\frac{S T}{J H}=\frac{T U}{H G}=\frac{U S}{G J}$
c) Similar; the corresponding angles are equal:
$\angle C=\angle R=70^{\circ}$
$\angle \mathrm{E}=\angle \mathrm{Q}=60^{\circ}$
$\angle \mathrm{D}=\angle \mathrm{P}=180^{\circ}-60^{\circ}-70^{\circ}=50^{\circ}$
d) Not similar; the corresponding sides are not proportional:
$\frac{\mathrm{DE}}{\mathrm{TS}}=\frac{2}{4}=\frac{1}{2}$
$\frac{\mathrm{FD}}{\mathrm{VT}}=\frac{4}{8}=\frac{1}{2}$
$\frac{E F}{S V}=\frac{5}{9}$
So, $\frac{D E}{T S}=\frac{F D}{V T}=\frac{1}{2}$ but $\frac{E F}{S V}=\frac{5}{9}$
5. a) $\Delta \mathrm{HGF} \sim \Delta \mathrm{HJK}$; the corresponding angles are equal:
$\angle \mathrm{GHF}=\angle \mathrm{JHK}=59^{\circ}$
$\angle \mathrm{G}=\angle \mathrm{J}=88^{\circ}$
$\angle \mathrm{F}=\angle \mathrm{K}=33^{\circ}$
b) $\triangle \mathrm{CED} \sim \triangle \mathrm{CAB}$; the corresponding sides are proportional:
$\frac{C E}{C A}=\frac{3}{6}=\frac{1}{2}$
$\frac{\mathrm{ED}}{\mathrm{AB}}=\frac{6}{12}=\frac{1}{2}$
$\frac{D C}{B C}=\frac{5}{10}=\frac{1}{2}$
So, $\frac{C E}{C A}=\frac{E D}{A B}=\frac{D C}{B C}=\frac{1}{2}$
c) $\triangle \mathrm{QMN} \sim \triangle \mathrm{QRP}$; the corresponding angles are equal:
$\angle \mathrm{MQN}=\angle \mathrm{RQP}=39^{\circ}$
$\angle \mathrm{M}=\angle \mathrm{R}=66^{\circ}$
$\angle \mathrm{N}=\angle \mathrm{P}=180^{\circ}-66^{\circ}-39^{\circ}=75^{\circ}$

## Apply

6. Since each pair of triangles is similar, the ratios of the lengths of corresponding sides is equal.
a) $\frac{A B}{D E}=\frac{A C}{D F}$

$$
\begin{aligned}
& \frac{A B}{12}=\frac{5}{10} \\
& A B=\frac{12 \times 5}{10} \\
& A B=6
\end{aligned}
$$

b) $\frac{A B}{G B}=\frac{B J}{B F}$

$$
\frac{\mathrm{AB}}{20}=\frac{12}{15}
$$

$$
A B=\frac{20 \times 12}{15}
$$

$$
A B=16
$$

c) $\frac{A B}{C D}=\frac{A E}{C E}$

$$
\begin{aligned}
& \frac{A B}{2.0}=\frac{2.5+7.5}{2.5} \\
& \frac{A B}{2.0}=\frac{10.0}{2.5} \\
& A B=\frac{2.0 \times 10.0}{2.5} \\
& A B=8.0
\end{aligned}
$$

7. Let $x$ represent the height of the flagpole.

Then the right triangle with leg lengths 1.6 m and 2.0 m is similar to the right triangle with leg lengths $x$ and 16 m . Since the triangles are similar, the ratios of the lengths of corresponding sides are equal.
$\frac{x}{1.6}=\frac{16}{2.0}$
$x=\frac{1.6 \times 16}{2.0}$
$x=12.8$
The flagpole is 12.8 m tall.
8. We chose the school.
a) I am 1.5 m tall.

My shadow is 1.2 m long.
The shadow of the school at the same time is 8 m long.

b) $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$
c) Since the triangles are similar, the ratios of the lengths of corresponding sides are equal.
$\frac{A B}{1.5}=\frac{8}{1.2}$
$A B=\frac{1.5 \times 8}{1.2}$
$A B=10$
So, the height of the school is 10 m .
9. a) Let $x$ represent the height of the first tree.

Then the right triangle with leg lengths 1.7 m and 2.4 m is similar to the right triangle with leg lengths $x$ and 10.8 m . Since the triangles are similar, the ratios of the lengths of corresponding sides are equal.
$\frac{x}{1.7}=\frac{10.8}{2.4}$
$x=\frac{1.7 \times 10.8}{2.4}$
$x=7.65$
So, the height of the first tree is 7.65 m .
b) Let $y$ represent the height of the second tree.

Then the right triangle with leg lengths 1.7 m and 0.8 m is similar to the right triangle with leg lengths $y$ and 12.8 m . Since the triangles are similar, the ratios of the lengths of corresponding sides are equal.

$$
\frac{y}{1.7}=\frac{12.8}{0.8}
$$

$$
y=\frac{1.7 \times 12.8}{0.8}
$$

$$
y=27.2
$$

So, the height of the second tree is 27.2 m .
10. a)

b) Let $x$ represent the height of the building.

Then the right triangle with leg lengths 3 m and 4 m is similar to the right triangle with leg lengths 16 m and $x$.

$$
\begin{aligned}
\frac{x}{4} & =\frac{16}{3} \\
x & =\frac{4 \times 16}{3} \\
x & =21 . \overline{3}
\end{aligned}
$$

So, the height of the building is about 21.3 m .
11. Let $y$ represent the length of Lac Lalune.

Then the right triangle with leg lengths 30 m and 40 m is similar to the right triangle with leg lengths $y$ and 140 m .
$\frac{y}{30}=\frac{140}{40}$
$y=\frac{30 \times 140}{40}$
$y=105$
So, the length of the lake is 105 m .
12. $\triangle R S T \sim \triangle R P Q$
$\frac{P Q}{S T}=\frac{P R}{S R}$
$\frac{P Q}{15}=\frac{110}{20}$
$P Q=\frac{15 \times 110}{20}$
$P Q=82.5$
So, the distance across the river is 82.5 m .

## Take It Further

13. The triangle formed by Phillipe's height and point $M$ is similar to the triangle formed by the tree's height and point M because the two triangles have two corresponding angles, which means that the third angles are also corresponding using the fact that the sum of the angles in a triangle is $180^{\circ}$.

Let $x$ represent the height of the tree.

$$
\begin{aligned}
\frac{x}{1.5} & =\frac{6.0}{1.7} \\
x & =\frac{1.5 \times 6.0}{1.7} \\
x & =5.3
\end{aligned}
$$

So, the height of the tree is about 5.3 m .
14. Let $y$ represent the height the ladder reaches up the wall.

Then the right triangle with leg lengths 1.4 m and $3.0 \mathrm{~m}-2.4 \mathrm{~m}=0.6 \mathrm{~m}$ is similar to the right triangle with leg lengths $y$ and 3.0 m .

$$
\begin{aligned}
\frac{y}{1.4} & =\frac{3.0}{0.6} \\
y & =\frac{1.4 \times 3.0}{0.6} \\
y & =7
\end{aligned}
$$

So, the ladder reaches 7 m up the wall.
15. The right triangle with leg lengths 16 m and $12 \mathrm{~m}+15 \mathrm{~m}+18 \mathrm{~m}=45 \mathrm{~m}$ is similar to the right triangle with leg lengths $x$ and 12 m .

$$
\begin{aligned}
\frac{x}{16} & =\frac{12}{45} \\
x & =\frac{16 \times 12}{45} \\
x & =4.2 \overline{6}
\end{aligned}
$$

The right triangle with leg lengths 16 m and $12 \mathrm{~m}+15 \mathrm{~m}+18 \mathrm{~m}=45 \mathrm{~m}$ is similar to the right triangle with leg lengths $y$ and $12 m+15 m=27 m$.

$$
\begin{aligned}
\frac{y}{16} & =\frac{27}{45} \\
y & =\frac{16 \times 27}{45} \\
y & =9.6
\end{aligned}
$$

So, $x$ is about 4.3 m high and $y$ is 9.6 m high.

## Mid-Unit Review

## Lesson 7.1

1. To determine the dimensions of the enlargement, multiply the photo dimensions by the scale factor.
$15 \mathrm{~cm} \times \frac{7}{5}=21 \mathrm{~cm}$
$10 \mathrm{~cm} \times \frac{7}{5}=14 \mathrm{~cm}$
The dimensions of the enlargement are 21 cm by 14 cm .
2. a) The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}$
$\frac{45 \mathrm{~mm}}{15 \mathrm{~mm}}=3$
$\frac{24 \mathrm{~mm}}{8 \mathrm{~mm}}=3$
The diagram is an enlargement with a scale factor of 3 .
b) Multiply the dimensions of the chip by the scale factor.
$15 \mathrm{~mm} \times 8=120 \mathrm{~mm}$
$8 \mathrm{~mm} \times 8=64 \mathrm{~mm}$
The scale diagram is a 12 cm by 6.4 cm rectangle.


## Lesson 7.2

3. a) I copied the polygon on $1-\mathrm{cm}$ grid paper.
b) To determine the side lengths of the scale diagram, I multiplied each length of the original diagram by the scale factor. The side lengths of the scale diagram are:
$5 \mathrm{~cm} \times \frac{3}{5}=3 \mathrm{~cm}$ (top and bottom)
$3 \mathrm{~cm} \times \frac{3}{5}=1.8 \mathrm{~cm}$ (left side)
$1 \mathrm{~cm} \times \frac{3}{5}=0.6 \mathrm{~cm}$ (right sides)
$2 \mathrm{~cm} \times \frac{3}{5}=1.2 \mathrm{~cm}$ (interior top and bottom)

4. The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}$
$\frac{3 \mathrm{~cm}}{6000 \mathrm{~cm}}=\frac{1}{2000}$, or 0.0005
The scale factor is $\frac{1}{2000}$, or 0.0005 .

## Lesson 7.3

5. a) Determine if any pair of quadrilaterals have corresponding sides proportional. For quadrilaterals $A B C D$ and $F G H E$, determine the ratio of corresponding sides.

$$
\frac{\mathrm{AB}}{\mathrm{FG}}=\frac{5.2}{5.8}=0.896 \ldots \quad \frac{\mathrm{BC}}{\mathrm{GH}}=\frac{4.5}{4.8}=0.9375
$$

Since these ratios are not equal, quadrilateral $A B C D$ is not similar to quadrilateral $F G H E$.
For quadrilaterals $A B C D$ and MKJN, determine the ratio of corresponding sides.

$$
\begin{array}{ll}
\frac{\mathrm{AB}}{\mathrm{MK}}=\frac{5.2}{7.8}=\frac{2}{3} & \frac{\mathrm{BC}}{\mathrm{KJ}}=\frac{4.5}{6.75}=\frac{2}{3} \\
\frac{\mathrm{CD}}{\mathrm{JN}}=\frac{3.6}{5.4}=\frac{2}{3} & \frac{\mathrm{DA}}{\mathrm{NM}}=\frac{4.8}{7.2}=\frac{2}{3}
\end{array}
$$

The ratios of corresponding sides are equal so quadrilateral $A B C D \sim$ quadrilateral MKJN.
For quadrilaterals FGHE and MKJN, determine the ratio of corresponding sides.

$$
\frac{\mathrm{FG}}{\mathrm{MK}}=\frac{5.8}{7.8}=0.743 \ldots \quad \frac{\mathrm{GH}}{\mathrm{KJ}}=\frac{4.8}{6.75}=0.7 \overline{1}
$$

Since these ratios are not equal, quadrilateral FGHE is not similar to quadrilateral MKJN.
b) Answers will vary.

For example:
This quadrilateral is similar to quadrilateral EFGH, because corresponding angles are equal, and corresponding sides are proportional.

$$
\frac{4.8}{9.6}=\frac{4.0}{8.0}=\frac{5.8}{11.6}=\frac{5.3}{10.6}=0.5
$$


6. I know the hexagons are similar, because the length of each side of this hexagon is 2 times the length of the corresponding side in the original hexagon, and the corresponding angles are equal.


## Lesson 7.4

7. a)

b) Let $t$ represent the height of the tree. Then the right triangle with leg lengths $t$ and 8 m is similar to the right triangle with leg lengths 2 m and 1.6 m .
$\frac{t}{2}=\frac{8}{1.6}$
$t=\frac{2 \times 8}{1.6}$
$t=10$
The height of the tree is 10 m .

## Check

3. a) 1 line of symmetry

b) No lines of symmetry
c) 1 line of symmetry

d) 1 line of symmetry

e) 3 lines of symmetry

f) No lines of symmetry

## Apply

4. a) Three lines of symmetry through the centre of the diagram

b) Three lines of symmetry through the centre of the diagram

5. a)


| Point | Image |
| :--- | :--- |
| $P(1,4)$ | $P^{\prime}(1,-2)$ |
| $Q(5,4)$ | $Q^{\prime}(5,-2)$ |
| $R(5,1)$ | $R(5,1)$ |
| $S(2,1)$ | $S(2,1)$ |

b)


| Point | Image |
| :--- | :--- |
| $C(4,5)$ | $C^{\prime}(10,5)$ |
| $D(7,4)$ | $D(7,4)$ |
| $E(7,2)$ | $E(7,2)$ |
| $F(2,1)$ | $F^{\prime}(12,1)$ |

c)


| Point | Image |
| :--- | :--- |
| $\mathrm{T}(-1,-2)$ | $\mathrm{T}^{\prime}(-4,-5)$ |
| $\mathrm{U}(-4,-2)$ | $\mathrm{U}(-4,-2)$ |
| $\mathrm{V}(-1,-5)$ | $\mathrm{V}(-1,-5)$ |

6. a) The vertical line through the centre of the tessellation is a line of symmetry.
b) The vertical line through the centre of the blanket is a line of symmetry.
7. a) Answers will vary. For example:

b) i) and ii)

iii) $A(3,7), B(3,5), C(7,3), C^{\prime}(-1,3)$
iv) The shape has one line of symmetry: the vertical line through 3 on the $x$-axis.
c) No, I get different shapes. For one side:
i), ii)

iii) $A(3,7), B(3,5), C(7,3), B^{\prime}(5,7)$
iv) The shape has one line of symmetry: the line through side AC.

For the other side:
i), ii)

iii) $\mathrm{A}(3,7), \mathrm{B}(3,5), \mathrm{A}^{\prime}(1.4,3.8), \mathrm{C}(7,3)$
iv) The shape has one line of symmetry: the line through side $B C$.
d), e) Scalene triangles always produce a shape that is a quadrilateral with line symmetry (such as the triangle above, in part a).
A right triangle, when reflected through one of its legs, produces another triangle.


A right triangle with unequal legs will produce a rectangle when it is reflected in its hypotenuse.


An isosceles right triangle with equal legs will produce a square when it is reflected in its hypotenuse (such as the triangle in question 5 c ).
8. a)


The larger shape PSRR'S' has coordinates:
$P(4,4), S(1,1), R(8,2), R^{\prime}(8,6), S^{\prime}(1,7)$
It is a pentagon with a line of symmetry through $P Q$.
b)


| Point | Image |
| :--- | :--- |
| $P(4,4)$ | $P^{\prime}(12,4)$ |
| $Q(8,4)$ | $Q^{(8,4)}$ |
| $R(8,2)$ | $R(8,2)$ |
| $S(1,1)$ | $S^{\prime}(15,1)$ |

The larger shape PSRS'P' has coordinates:
$P(4,4), S(1,1), R(8,2), S^{\prime}(15,1), P^{\prime}(12,4)$
It is a pentagon with a line of symmetry through QR.
c)


| Point | Image |
| :--- | :--- |
| $\mathrm{P}(4,4)$ | $\mathrm{P}(4,4)$ |
| $\mathrm{Q}(8,4)$ | $\mathrm{Q}^{\prime}(4,8)$ |
| $\mathrm{R}(8,2)$ | $\mathrm{R}^{\prime}(2,8)$ |
| $\mathrm{S}(1,1)$ | $\mathrm{S}(1,1)$ |

The larger shape $P Q R S R^{\prime} Q^{\prime}$ has coordinates:
$P(4,4), Q(8,4), R(8,2), S(1,1), R^{\prime}(2,8), Q^{\prime}(4,8)$
It is a hexagon with a line of symmetry through PS.
9. a)

b)

c) $A(-3,0), B(-1,1), C(0,3), D(1,1), E(3,0), D^{\prime}(1,-1), C^{\prime}(0,-3), B^{\prime}(-1,-1)$
d) The shape has 4 lines of symmetry: $x$-axis, $y$-axis, the line through points $B^{\prime}$ and $D$, the line through points $B$ and $D^{\prime}$. Each image point is the same distance from the line of symmetry as the corresponding original point.
10. Pentagon $A$ is the reflection image in the horizontal line through 7 on the $y$-axis.

The line of symmetry is the horizontal line through 7 on the $y$-axis.
Pentagon C is the reflection image in the vertical line through 5 on the $x$-axis.
The line of symmetry is the vertical line through 5 on the $x$-axis.
Pentagon D is the reflection image in the horizontal line through 3 on the $y$-axis.
The line of symmetry is the horizontal line through 3 on the $y$-axis.

## Take It Further

11. a)

b)

d)


e) The final shape has 4 lines of symmetry: $x$-axis, the vertical line through 4 on the $x$-axis, the line through the points $(2,2)$ and $(6,-2)$, and the line through the points $(6,2)$ and $(2,-2)$.

## Lesson 7.6

Rotations and Rotational Symmetry

## Check

4. a) The angle of rotation symmetry $=\frac{360^{\circ}}{\text { the order of rotation }}$

$$
\begin{aligned}
& =\frac{360^{\circ}}{3} \\
& =120^{\circ}
\end{aligned}
$$

b) $\frac{360^{\circ}}{\text { the order of rotation }}=\frac{360^{\circ}}{5}$

$$
=72^{\circ}
$$

c) $\frac{360^{\circ}}{\text { the order of rotation }}=\frac{360^{\circ}}{9}$

$$
=40^{\circ}
$$

d)


$$
=30^{\circ}
$$

5. a) The order of rotational symmetry $=\frac{360^{\circ}}{\text { the angle of rotation symmetry }}$

$$
\begin{aligned}
& =\frac{360^{\circ}}{60^{\circ}} \\
& =6
\end{aligned}
$$

b) $\frac{360^{\circ}}{\text { the angle of rotation symmetry }}=\frac{360^{\circ}}{20^{\circ}}$

$$
=18
$$

c) $\frac{360^{\circ}}{\text { the angle of rotation symmetry }}=\frac{360^{\circ}}{45^{\circ}}$

$$
=8
$$

d) $\frac{360^{\circ}}{\text { the angle of rotation symmetry }}=\frac{360^{\circ}}{36^{\circ}}$

$$
=10
$$

6. a) An equilateral triangle has rotational symmetry of order 3 .

$$
\begin{aligned}
\text { The angle of rotation symmetry } & =\frac{360^{\circ}}{\text { the order of rotation }} \\
& =\frac{360^{\circ}}{3} \\
& =120^{\circ}
\end{aligned}
$$

b) A regular pentagon has rotational symmetry of order 5 .

$$
\begin{aligned}
\text { The angle of rotation symmetry } & =\frac{360^{\circ}}{\text { the order of rotation }} \\
& =\frac{360^{\circ}}{5} \\
& =72^{\circ}
\end{aligned}
$$

c) A square has rotational symmetry of order 4 .

$$
\begin{aligned}
\text { The angle of rotation symmetry } & =\frac{360^{\circ}}{\text { the order of rotation }} \\
& =\frac{360^{\circ}}{4} \\
& =90^{\circ}
\end{aligned}
$$

d) A regular octagon has rotational symmetry of order 8 .

$$
\begin{aligned}
\text { The angle of rotation symmetry } & =\frac{360^{\circ}}{\text { the order of rotation }} \\
& =\frac{360^{\circ}}{8} \\
& =45^{\circ}
\end{aligned}
$$

## Apply

7. a) Yes; the snowflake has rotational symmetry of order 6 , and the angle of rotation symmetry is:

$$
\frac{360^{\circ}}{6}=60^{\circ}
$$

b) No; the picture of the Ferris wheel does not have rotational symmetry, because the picture does not coincide with itself after a rotation of less than $360^{\circ}$ about its centre.
8. a) Yes; the shape has rotational symmetry of order 4 and the angle of rotation symmetry is $90^{\circ}$.
b) Yes; the shape has rotational symmetry of order 6 and the angle of rotation symmetry is $60^{\circ}$.
9. a)

b)

c)

10. a)

b)

11. a) The tessellation has rotational symmetry of order 4 about a point where the heads of 4 lizards meet. The tessellation has rotational symmetry of order 4 about a point where the tails of 4 lizards meet. The tessellation has rotational symmetry of order 2 about a point where the elbows of 2 lizards meet.
b) The design has rotational symmetry of order 15 about the centre.
12. a)

b) The shape is a dodecagon (12-sided shape) that has rotational symmetry of order 2 about the origin. I think the rotational symmetry occurred because the original design was rotated $180^{\circ}$ about the origin to get the new design.
13. a) i)

ii)

b) i)

ii)

c) i)

ii)

d) In each of parts $a, b$, and $c$, the image in part $i$ is always 2 copies of the original shape joined at a vertex, while the image in part ii is always the same as the original shape. Both images were created using rotations. A different shape is used in each part, and the shapes were rotated through different angles. The images in parts $a, b$, and c ii have rotational symmetry, but the image in part ci does not.
14. a) i)

ii)

iii)

b) The shape formed has rotational symmetry of order 4 about $P$. The shape and its 3 images are all squares that combine to form a larger square.

15. a) i)

ii)

iii)

b) $C(0,2), A(0,4), B(2,3), A^{\prime}(2,2), B^{\prime}(1,0), A^{\prime \prime}(0,0), B^{\prime \prime}(-2,1), A^{\prime \prime \prime}(-2,2), B^{\prime \prime \prime}(-1,4)$ The shape formed has rotational symmetry of order 4 about $C$.


## Take It Further

16. a) For example, draw a rectangle with coordinates $A(2,2), B(2,4), C(6,4), D(6,2)$
i) Rotate the rectangle $180^{\circ}$ about $D(6,2)$.
$A(2,2), B(2,4), C(6,4), D(6,2), E=C^{\prime}(6,0), F=B^{\prime}(10,0), G=A^{\prime}(10,2)$

ii) Rotate the rectangle $180^{\circ}$ about $(4,2)$.
$H=C^{\prime}(2,0), I=B(2,4), J=C(6,4), K=B^{\prime}(6,0)$

b) I drew a rhombus.
i) I rotated the rhombus about its bottom vertex through $120^{\circ}$ to get a polygon that has rotational symmetry of order 3.
ii) I rotated the rhombus about its bottom vertex through $60^{\circ}$ to get a polygon that has rotational symmetry of order 6.


## Lesson $7.7 \quad$ Types of Symmetry on the Cartesian Plane

## Check

3. a) No line symmetry; rotational symmetry of order 2
b) No line symmetry; rotational symmetry of order 2
c) Line symmetry: the horizontal line through the centre is a line of reflection; no rotational symmetry
d) Line symmetry: the horizontal line through the centre is a line of reflection; no rotational symmetry
4. a) 8 lines of symmetry through the centre; rotational symmetry of order 8 about the centre

b) 5 lines of symmetry through the centre; rotational symmetry of order 5 about the centre

c) No line symmetry; no rotational symmetry

d) No line symmetry; rotational symmetry of order 5 about the centre


## PEARSON MMS 9 UNIT 7

Similarity and Transformations
5. This face has 4 lines of symmetry and rotational symmetry of order 4 about its centre:


This face has 2 lines of symmetry and rotational symmetry of order 2 about its centre.


This face has 2 lines of symmetry and rotational symmetry of order 2 about its centre.


This face has 4 lines of symmetry and rotational symmetry of order 4 about its centre.


This face has 4 lines of symmetry and rotational symmetry of order 4 about its centre.


This face has 2 lines of symmetry and rotational symmetry of order 2 about its centre.


## Apply

6. a) Square D and the red square are related by rotational symmetry of order 2 about the origin.
b) Square $B$ and the red square are related by line symmetry; the vertical line through 5 on the $x$-axis is the line of symmetry.
Square $C$ and the red square are related by line symmetry; the $x$-axis is the line of symmetry.
7. a) Corresponding vertices on each polygon are equidistant from the $y$-axis.

So, the two polygons are related by line symmetry; the $y$-axis is the line of symmetry.
When one polygon is rotated about the origin, it does not coincide with the other polygon.
So, there is no rotational symmetry about the origin.
b) Corresponding vertices on each polygon are equidistant from the line through ( $1,-1$ ) and ( $-1,1$ ). So, the two polygons are related by line symmetry; the line through $(1,-1)$ and $(-1,1)$ is the line of symmetry.
When a tracing of one polygon is rotated $180^{\circ}$ about the origin, it coincides with the other polygon. So, the two polygons are related by rotational symmetry of order 2 about the origin.
c) Corresponding vertices on each polygon are equidistant from the $x$-axis.

So, the two polygons are related by line symmetry; the $x$-axis is the line of symmetry.
When one polygon is rotated about the origin, it does not coincide with the other polygon.
So, there is no rotational symmetry about the origin.
d) Corresponding vertices on each polygon are equidistant from the line through ( $-1,-1$ ) and (1, 1). So, the two polygons are related by line symmetry; the line through $(-1,-1)$ and $(1,1)$ is the line of symmetry.
When a tracing of one polygon is rotated $180^{\circ}$ about the origin, it coincides with the other polygon. So, the two polygons are related by rotational symmetry of order 2 about the origin.
8. a) The two octagons are related by line symmetry in the $x$-axis, and by rotational symmetry of order 2 about the point $(-2.5,0)$.
b) When the top octagon is rotated by $90^{\circ}$ clockwise about the point $(2,3)$, it coincides with the other octagon. So, the two octagons are related by rotational symmetry.
9. a) The diagram has rotational symmetry of order 2 about the origin.
10. a) The diagram has 1 line of symmetry, which is the vertical line through the centre of the diagram.
b) The diagram has rotational symmetry of order 2 about the centre of the diagram.
11. a) When one triangle is rotated, it does not coincide with the other. There is no line symmetry relating the triangles.


| Point | Image |
| :--- | :--- |
| $\mathrm{A}(-2,5)$ | $\mathrm{A}^{\prime}(-2,11)$ |
| $\mathrm{B}(-2,1)$ | $\mathrm{B}^{\prime}(-2,7)$ |
| $\mathrm{C}(-4,4)$ | $\mathrm{C}^{\prime}(-4,10)$ |

b)

| $\boldsymbol{y}^{2}$ | 2 | $y$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Point | Image |
| :--- | :--- |
| $\mathrm{D}(2,-1)$ | $\mathrm{D}^{\prime}(6,-1)$ |
| $\mathrm{E}(2,-3)$ | $\mathrm{E}^{\prime}(6,-3)$ |
| $\mathrm{F}(6,-3)$ | $\mathrm{F}^{\prime}(10,-3)$ |
| $\mathrm{G}(6,-1)$ | $\mathrm{G}^{\prime}(10,-1)$ |

The diagram has line symmetry and rotational symmetry. The line of symmetry is the vertical line through 6 on the $x$-axis and the 2 rectangles are related by rotational symmetry of order 2 about $(6,-2)$.
12. a) - c)


| Point | Image |
| :--- | :--- |
| $C(2,3)$ | $C^{\prime}(3,6)$ |
| $D(-2,-1)$ | $D^{\prime}(-1,2)$ |
| $E(3,-2)$ | $E^{\prime}(4,1)$ |

d) The translation does not result in any symmetry because there is no line in which the triangle can be reflected to coincide with its image, and there is no point about which the triangle can be rotated to coincide with its image in less than one complete turn.
e) The translation R2, U2 results in a line of symmetry.

Each point on CDE has a corresponding point on C'D'E'. These points are equidistant from the line through $(3,0)$ and $(0,3)$.

13. a), b) i) The diagram has rotational symmetry of order 4 about (4, 2).

ii) The diagram has line symmetry, because the horizontal line through 1 on the $y$-axis is a line of reflection.

iii) The diagram does not have line symmetry because there is no line on which a mirror can be placed so that one parallelogram is the reflection image of the other.
The diagram does not have rotational symmetry because there is no point about which it can be rotated so that it coincides with itself.

14. a) Sketches of the digit 7 may vary. For example:


Digits 1 and 3 have a horizontal line of symmetry.
Digits 1, 2, and 5 have rotational symmetry of order 2.
Digits 4, 6, 7, and 9 have no line or rotational symmetry.
Digits 8 and 0 have both horizontal and vertical lines of symmetry and rotational symmetry of order 2 .
b) Digits $1,3,8$, and 0 can be completed by the reflection of these halves of the digits in the dotted line in this diagram.

c) Digits $1,2,5,8$, and 0 can be completed by a rotation of part of the digit about each dot shown.

d) 2 and 5 are related by line symmetry in the vertical line through 3 on the $x$-axis.

15. a)

b)

| Point | Image |
| :--- | :--- |
| $\mathrm{G}(-1,3)$ | $\mathrm{G}^{\prime}(2,-3)$ |
| $\mathrm{H}(1,3)$ | $\mathrm{H}^{\prime}(-1,-3)$ |
| $\mathrm{J}(2,2)$ | $\mathrm{J}^{\prime}(-2,-2)$ |
| $\mathrm{K}(2,-1)$ | $\mathrm{K}^{\prime}(-2,1)$ |
| $\mathrm{M}(-2,-1)$ | $\mathrm{M}^{\prime}(2,1)$ |
| $\mathrm{N}(-2,2)$ | $\mathrm{N}^{\prime}(2,-2)$ |

The coordinates of the vertices of the larger shape are:
$G(-1,3), H(1,3), J(2,2), N^{\prime}(2,-2), G^{\prime}(1,-3), H^{\prime}(-1,-3), J^{\prime}(-2,-2), N(-2,2)$
c) The larger shape has 2 lines of symmetry: the $x$-axis and the $y$-axis.

The larger shape also has rotational symmetry of order 2 about the origin.

## Take It Further

16. The solution presented here assumes that 1 s on a digital clock display have line symmetry with horizontal and vertical lines of reflection and rotational symmetry.
Some students may show that a time such as $01: 10$ has line symmetry with a horizontal line of reflection, but no other symmetry, since the 1 in 01 is closer to : than the 1 in 10 . Solutions based on this reasoning are also acceptable.

00:00, 01:10, 10:01, 11:11 have line symmetry with a horizontal and vertical line of reflection and rotational symmetry of order 2.
$00: 01,00: 03,00: 08,00: 10,00: 11,00: 13,00: 18,00: 30,00: 31,00: 33,00: 38,01: 00,01: 01,01: 03,01: 08$, 01:11, 01:13, 01:18, 01:30, 01:31, 01:33, 01:38, 03:00, 03:01, 03:03, 03:08, 03:10, 03:11, 03:13, 03:18, 03:30, 03:31, 03:33, 03:38, 08:00, 08:01, 08:03, 08:08, 08:10, 08:11, 08:13, 08:18, 08:30, 08:31, 08:33, $08: 38,10: 00,10: 03,10: 08,10: 10,10: 11,10: 13,10: 18,10: 30,10: 31,10: 33,10: 38,11: 00,11: 01,11: 03$, $11: 08,11: 10,11: 13,11: 18,11: 30,11: 31,11: 33,11: 38,13: 00,13: 01,13: 03,13: 08,13: 10,13: 11,13: 13$, $13: 18,13: 30,13: 31,13: 33,13: 38,18: 00,18: 01,18: 03,18: 08,18: 10,18: 11,18: 13,18: 18,18: 30,18: 31$, $18: 33,18: 38$ have line symmetry with a horizontal line of reflection.

02:50, 05:20, 12:51, 15:21, 20:05, 21:15, 22:55 have line symmetry with a vertical line of reflection.
02:20, 05:50, 12:21, 15:51, 20:02, 21:12, 22:22 have rotational symmetry of order 2.

Review

## Lesson 7.1

1. I measure the photo to be 3 cm by 5 cm . To determine the dimensions of the enlargement, multiply the dimensions of the photo by the scale factor.
a) Length: $3 \mathrm{~cm} \times 3=9 \mathrm{~cm}$

Width: $5 \mathrm{~cm} \times 3=15 \mathrm{~cm}$
The dimensions of the enlargement are 9 cm by 15 cm .
b) Length: $3 \mathrm{~cm} \times 2.5=7.5 \mathrm{~cm}$

Width: $5 \mathrm{~cm} \times 2.5=12.5 \mathrm{~cm}$
The dimensions of the enlargement are 7.5 cm by 12.5 cm .
c) Length: $3 \mathrm{~cm} \times \frac{3}{2}=\frac{9}{2} \mathrm{~cm}$, or 4.5 cm

Width: $5 \mathrm{~cm} \times \frac{3}{2}=\frac{15}{2} \mathrm{~cm}$, or 7.5 cm
The dimensions are 4.5 cm by 7.5 cm .
d) Length: $3 \mathrm{~cm} \times \frac{21}{5}=\frac{63}{5} \mathrm{~cm}$, or 12.6 cm

Width: $5 \mathrm{~cm} \times \frac{21}{5}=21 \mathrm{~cm}$
The dimensions are 12.6 cm by 21 cm .
2. Multiply each side length of the original pentagon by the scale factor 2.5 .


## Lesson 7.2

3. a) The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}$

$$
\begin{aligned}
& \frac{180 \mathrm{~cm}}{270 \mathrm{~cm}}=\frac{2}{3} \\
& \frac{92 \mathrm{~cm}}{138 \mathrm{~cm}}=\frac{2}{3}
\end{aligned}
$$

The scale factor for this reduction is $\frac{2}{3}$.
b) $144 \mathrm{~cm} \times \frac{2}{3}=\frac{144 \mathrm{~cm} \times 2}{3}$

$$
=96 \mathrm{~cm}
$$

The length of a model of this pool cue is 96 cm .
4. Lengths on the scale ramp:

Base: 5.6 cm
Height: 1 cm
The scale factor is: $\frac{\text { Length on scale diagram }}{\text { Length on original diagram }}$
$\frac{1 \mathrm{~cm}}{180 \mathrm{~cm}}=\frac{1}{180}$
This means that 1 cm on the diagram represents 180 cm on the ramp.
Each actual measure is 180 times as great as the measure on the scale diagram.
So, the length of the ramp is: $5.6 \mathrm{~cm} \times 180=1008 \mathrm{~cm}$, or about 10 m .
5. Wall of dog house on scale diagram $=4.6 \mathrm{~cm}$

Wall of dog house $=2 \mathrm{~m}$, or 200 cm
The scale factor is: $\frac{4.6 \mathrm{~cm}}{200 \mathrm{~cm}}=0.023$
The other two sides are: $\frac{5.5 \mathrm{~cm}}{0.023} \doteq 239.1 \mathrm{~cm}$, or about 2.4 m

$$
\frac{7 \mathrm{~cm}}{0.023} \doteq 304.3 \mathrm{~cm}, \text { or about } 3 \mathrm{~m}
$$

The other two sides of the dog run are about 2.4 m and about 3 m .

## Lesson 7.3

6. Pentagon $Z$ is similar to the red pentagon. Their corresponding angles are equal, and their corresponding sides are proportional.
$\frac{1.5}{1.35}=\frac{3.0}{2.70}=\frac{1.0}{0.90}=\frac{3.5}{3.15}=\frac{2.5}{2.25}=1 . \overline{1}$
The ratios of the corresponding sides are all equal to $\frac{10}{9}$, or $1 . \overline{1}$.
7. a) On courtyard $A B C D E F$, side $A F$ is 12 m , which corresponds to 6 units on the grid paper.

Side $B C$ is 3 units on the grid paper.
$\frac{12}{6}=\frac{B C}{3}$
$B C=\frac{12 \times 3}{6}$
$B C=6$
$B C$ is 6 m .
b) Using the properties of similar polygons, $\frac{A^{\prime} F^{\prime}}{A F}=\frac{B^{\prime} C^{\prime}}{B C}$.

Substitute $A^{\prime} F^{\prime}=8, A F=12$, and $B C=6$.
$\frac{8}{12}=\frac{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}{6}$
$B^{\prime} C^{\prime}=\frac{8 \times 6}{12}$
$\mathrm{B}^{\prime} \mathrm{C}^{\prime}=4$
$B^{\prime} C^{\prime}$ is 4 m .
c) Find $A B$, which corresponds to 4 units on the grid paper.

$$
\begin{aligned}
& \frac{12}{6}=\frac{A B}{4} \\
& 2=\frac{A B}{4} \\
& A B=8
\end{aligned}
$$

Using the properties of similar polygons,
$\frac{A^{\prime} F^{\prime}}{A F}=\frac{A^{\prime} B^{\prime}}{A B}$
Substitute $A^{\prime} F^{\prime}=8, A F=12$, and $A B=8$
$\frac{8}{12}=\frac{A^{\prime} B^{\prime}}{8}$
$A^{\prime} \mathrm{B}^{\prime}=\frac{8 \times 8}{12}$
$A^{\prime} B^{\prime}=5 . \overline{3}$
$A^{\prime} B^{\prime}$ is about 5.3 m .
8. a) $\frac{\mathrm{QP}}{\mathrm{VU}}=\frac{\mathrm{PN}}{\mathrm{UT}}$

Substitute QP $=3.0 \mathrm{~cm}, \mathrm{VU}=2.4 \mathrm{~cm}$, and $\mathrm{UT}=1.6 \mathrm{~cm}$.
$\frac{3.0 \mathrm{~cm}}{2.4 \mathrm{~cm}}=\frac{\mathrm{PN}}{1.6 \mathrm{~cm}}$
$\mathrm{PN}=\frac{3.0 \mathrm{~cm} \times 1.6 \mathrm{~cm}}{2.4 \mathrm{~cm}}$
$\mathrm{PN}=2 \mathrm{~cm}$
b) $\frac{V U}{Q P}=\frac{T S}{N M}$

Substitute QP $=3.0 \mathrm{~cm}, \mathrm{VU}=2.4 \mathrm{~cm}$, and $\mathrm{NM}=3.5 \mathrm{~cm}$.

$$
\begin{aligned}
& \frac{2.4 \mathrm{~cm}}{3.0 \mathrm{~cm}}=\frac{\mathrm{TS}}{3.5 \mathrm{~cm}} \\
& \mathrm{TS}=\frac{2.4 \mathrm{~cm} \times 3.5 \mathrm{~cm}}{3.0 \mathrm{~cm}} \\
& \quad=2.8 \mathrm{~cm}
\end{aligned}
$$

## Lesson 7.4

9. Since 3 pairs of corresponding angles are equal, $\triangle \mathrm{PQS} \sim \Delta \mathrm{TRS}$.
$\frac{P Q}{T R}=\frac{P S}{T S}$
Substitute $\mathrm{PQ}=d, \mathrm{TR}=28 \mathrm{~m}, \mathrm{PS}=23 \mathrm{~m}+35 \mathrm{~m}=58 \mathrm{~m}$, and $\mathrm{TS}=35 \mathrm{~m}$.

$$
\begin{gathered}
\frac{d}{28}=\frac{58}{35} \\
d=\frac{58 \times 28}{35} \\
=46.4
\end{gathered}
$$

The distance across the pond is 46.4 m .
10. The right triangle with leg lengths 16 m and 18 m is similar to the right triangle with leg lengths $d$ and 40 m .

$$
\begin{aligned}
& \frac{d}{16}=\frac{40}{18} \\
& \begin{aligned}
d & =\frac{40 \times 16}{18} \\
& =35.5
\end{aligned}
\end{aligned}
$$

The distance across the river is about 35.6 m .
11. The right triangle with leg lengths 22.5 m and $12.5 \mathrm{~m}+25 \mathrm{~m}=37.5 \mathrm{~m}$ is similar to the right triangle with leg lengths $x$ and 25 m .
$\frac{x}{22.5}=\frac{25}{37.5}$
$x=\frac{25 \times 22.5}{37.5}$
$=15$
The distance $x$ is 15 m .

## Lesson 7.5

12. a) There is 1 line of symmetry: a horizontal line through the centre of the sign.
b) There are no lines of symmetry.
c) There are 2 lines of symmetry: a vertical line and a horizontal line through the centre of the sign.
d) There are 3 lines of symmetry: 3 lines, each starting from a corner of the sign.
13. a) i)

ii)

iii)

b)

c) i) $A(-1,2), A^{\prime}(1,2), E(1,0), F(-1,0)$
ii) $A(-1,2), B(0,2), C(0,1), D(1,1), D^{\prime}(1,-1), C^{\prime}(0,-1), B^{\prime}(0,-2), A^{\prime}(-1,-2)$
iii) $A(-1,2), B(0,2), C(0,1), D(1,1), E(1,0), F(-1,0)$
b) $A(-1,2), B(0,2), C(0,1), D(1,1), A^{\prime}(1,2), B^{\prime}(2,2), C^{\prime}(2,1), D^{\prime}(3,1), E^{\prime}(3,0), F(-1,0)$
d) i) 4 lines of symmetry: the $y$-axis, the horizontal line through $(-1,1)$ and $(1,1)$, the diagonal line through $(-1,2)$ and $(1,0)$, and the diagonal line through $(-1,0)$ and $(1,2)$.
ii) 1 line of symmetry: the $x$-axis.
iii) 1 line of symmetry: the diagonal line through $(-2,-1)$ and $(2,3)$.
b) No line symmetry

## Lesson 7.6

14. a) The shape has rotational symmetry of order 3 ; it coincides with itself 3 times during a rotation of $360^{\circ}$ about its centre.
b) The shape has rotational symmetry of order 2 ; it coincides with itself 2 times during a rotation of $360^{\circ}$ about its centre.
c) The shape has rotational symmetry of order 6 ; it coincides with itself 6 times during a rotation of $360^{\circ}$ about its centre.
d) The shape has rotational symmetry of order 8 ; it coincides with itself 8 times during a rotation of $360^{\circ}$ about its centre.
15. a) i)

ii)

iii)

b) i) The diagram has no rotational symmetry, because there is no point about which the shape can be rotated so that it coincides with itself after a rotation of less than $360^{\circ}$.
ii) The diagram has rotational symmetry of order 2 about $\mathrm{B}(0,3)$, because it coincides with itself 2 times during a rotation of $360^{\circ}$ about the point $(0,3)$.
iii) The diagram has rotational symmetry of order 4 about ( $-2,2$ ), because it coincides with itself 4 times during a rotation of $360^{\circ}$ about the point $(-2,2)$.

## Lesson 7.7

16. i) 1 line of symmetry: the line through the points $(-6,0)$ and $(0,6)$ is a line of reflection.
ii) The diagram has no line symmetry because there is no line on which a mirror can be placed so that one rectangle is the reflection image of the other.
iii) 4 lines of symmetry: the vertical line through -2 on the $x$-axis, the horizontal line through 2 on the $y$-axis, the line through the points $(-3,1)$ and $(-1,3)$, and the line through the points $(-3,3)$ and $(-1,1)$ are all lines of reflection.
17. a) 1 line of symmetry: the line through the points $(-2,2)$ and $(2,-2)$ is a line of reflection. When one pentagon is rotated $180^{\circ}$ about the origin, it coincides with the other pentagon, so there is rotational symmetry of order 2 about the origin.
b) 1 line of symmetry: the vertical line through 0.5 on the $x$-axis is a line of reflection.
18. a) Rotational symmetry of order 3 about the centre; 3 lines of symmetry.
b) 1 line of symmetry: the vertical line through the centre of the artwork is a line of reflection.
19. a) i)

|  |  | 4 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  | $D^{\prime}$ | 2 | G |  |  |  |  |

ii)

b) i) Yes; the translation results in rotational symmetry of order 2 about $\mathrm{G}(1,1)$.
ii) Yes; the translation results in rotational symmetry of order 2 about (3.5, 3).

## PEARSON MMS 9 UNIT 7

Similarity and Transformations

## Practice Test

1. a) Use the property of similar polygons.

$$
\begin{aligned}
\frac{B A}{X W} & =\frac{B C}{X Y} \\
\frac{4.5}{3.0} & =\frac{B C}{5.4} \\
B C & =\frac{4.5 \times 5.4}{3.0} \\
& =8.1
\end{aligned}
$$

The length of $B C$ is 8.1 m .
b) $\frac{W Z}{A D}=\frac{Y Z}{C D}$

$$
\begin{aligned}
\frac{W Z}{7.8} & =\frac{4.4}{6.6} \\
W Z & =\frac{4.4 \times 7.8}{6.6} \\
& =5.2
\end{aligned}
$$

The length of $W Z$ is 5.2 m .
c) Multiply each side length of WXYZ by the scale factor, 2 .

d) Multiply each side length of $\operatorname{ABCD}$ by the scale factor, $\frac{1}{3}$.

|  | 2.7 m |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 1.5 m | 2.2 m |  |  |
|  | 2.6 m |  |  |

2. a)

b) The corresponding angles in the triangles are equal.
c) Let $y$ represent the height of the tree.

The right triangle with leg lengths $y$ and 6.3 m is similar to the right triangle with leg lengths 1.7 m and 3.15 m .

$$
\begin{gathered}
\frac{y}{1.7}=\frac{6.3}{3.15} \\
y=\frac{6.3 \times 1.7}{3.15} \\
=3.4
\end{gathered}
$$

The height of the tree is 3.4 m .
3. a)

b) i)

ii) Equilateral triangle: rotational symmetry of order 3; angle of rotation symmetry $120^{\circ}$ Square: rotational symmetry of order 4; angle of rotation symmetry $90^{\circ}$ Rectangle: rotational symmetry of order 2; angle of rotation symmetry $180^{\circ}$ Parallelogram: rotational symmetry of order 2; angle of rotation symmetry $180^{\circ}$
Regular hexagon: rotational symmetry of order 6; angle of rotation symmetry $60^{\circ}$
c) For example: this shape has line symmetry (the vertical line down the middle of the shape is a line of reflection), but no rotational symmetry.

d) For example: this shape has rotational symmetry of order 2 about the centre, but no line symmetry.

4. a) i)

ii) $A^{\prime}(0,3), B^{\prime}(1,4), C^{\prime}(3,4), D^{\prime}(4,3), E^{\prime}(3,2), F^{\prime}(1,2)$
iii) There are 4 lines of symmetry: the vertical line through 2 on the $x$-axis, the horizontal line through 3 on the $y$-axis, the line through the points $(0,1)$ and $(4,5)$, and the line through the points $(0,5)$ and $(5,0)$; there is rotational symmetry of order 4 about $(2,3)$.
b) i)

ii) $A^{\prime}(4,1), B^{\prime}(3,2), C^{\prime}(3,4), D^{\prime}(4,5), E^{\prime}(5,4), F^{\prime}(5,2)$
iii) There are 2 lines of symmetry: the vertical line through 3 on the $x$-axis, and the horizontal line through 3 on the $y$-axis; there is rotational symmetry of order 2 about $(3,3)$.
c) i)

ii) $A^{\prime}(2,3), B^{\prime}(1,2), C^{\prime}(1,0), D^{\prime}(2,-1), E^{\prime}(3,0), F^{\prime}(3,2)$
iii) There are 2 lines of symmetry: the vertical line through 2 on the $x$-axis, and the horizontal line through 2 on the $y$-axis; there is rotational symmetry of order 2 about $(2,2)$.
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## Part 1

1 line of symmetry:
Each of the following flags has a horizontal line through the middle of the flag that is a line of reflection:

| alpha | hotel |
| :--- | :--- |
| bravo | kilo |
| golf | tango |

The echo flag has a vertical line through the middle of the flag that is a line of reflection

## 2 lines of symmetry:

Each of the following flags has a horizontal and a vertical line through the middle of the flag that are a lines of reflection:
charlie quebec
delta romeo
foxtrot sierra
india victor
juliet whiskey
mike xray
papa

No lines of symmetry:
The following flags have no line symmetry:

| lima | uniform |
| :--- | :--- |
| november | yankee |
| oscar | zulu |

Rotational symmetry:
The following flags have rotational symmetry of order 2 about the centre of the flag:

| charlie | papa |
| :--- | :--- |
| delta | quebec |
| foxtrot | romeo |
| india | sierra |
| juliet | uniform |
| lima | victor |
| mike | whiskey |
| november | xray |

No rotational symmetry:
The following flags have no rotational symmetry:

| alpha | kilo |
| :--- | :--- |
| bravo | oscar |
| echo | tango |
| golf | yankee |
| hotel | zulu |

## Part 2



My flag has 2 lines of symmetry: a horizontal line and a vertical line of symmetry through the centre of the flag are lines of reflection. It also has rotational symmetry of order 2 about the centre of the flag: if I rotate a tracing of my flag about the centre of the flag, the tracing will coincide with the original flag 2 times in one full turn. The angle of rotation symmetry is: $\frac{360^{\circ}}{2}=180^{\circ}$

The actual flag is 3 m by 2 m , or 300 cm by 200 cm .
I used the scale factor $\frac{1}{25}$.
The dimensions of my scale diagram are:
$300 \mathrm{~cm} \times \frac{1}{25}=12 \mathrm{~cm}$
$200 \mathrm{~cm} \times \frac{1}{25}=8 \mathrm{~cm}$
The dimensions of the scale diagram of my flag are 12 cm by 8 cm .
The diameter of the circle on the original flag is 170 cm .
The diameter of the circle on my scale diagram is:
$170 \mathrm{~cm} \times \frac{1}{25}=6.8 \mathrm{~cm}$
The diameter of the circle on my scale diagram is 6.8 cm .
My flag will be used for the Tree Planters' Union.
The drawing in the middle of the flag represents a tree branch.

