Lesson 8.1 Properties of Tangents to a Circle Practice (pages 388–391)

Check

3. **a)** Since line MN intersects the circle at 2 points, M and N, MN is not a tangent to the circle.
Since line QR intersects the circle at 1 point, P, QR is a tangent to the circle.

b) Since line AB intersects the circle at 2 points, A and B, AB is not a tangent to the circle.
Since line CE intersects the circle at 1 point, D, CE is a tangent to the circle.

4. Use the Tangent-Radius Property.

a) Since the tangent is perpendicular to the radius OQ at the point of tangency Q, $d° = 90°$.

b) Since the tangent is perpendicular to the radius OQ at the point of tangency Q, $d° = 90°$.

5. Use the Tangent-Radius Property and the angle sum of a triangle.

a) Since P is a point of tangency and OP is a radius, $x° = 90°$

b) Since P is a point of tangency and OP is a radius, $\angle OPQ = 90°$.
The sum of the angles in ΔOPQ is 180°.
So, $x° + 90° + 23° = 180°$
$x° = 180° – 90° – 23°$
$x° = 67°$

c) Since P is a point of tangency and OP is a radius, $\angle OPQ = 90°$.
The sum of the angles in ΔOPQ is 180°.
$x° + 90° + 47° = 180°$
$x° = 180° – 90° – 47°$
$x° = 43°$

6. **a)** Since PQ is a tangent to the circle, $\angle OPQ = 90°$.
Use the Pythagorean Theorem in right ΔOPQ to calculate *a*.
$OQ^2 = OP^2 + PQ^2$
$a^2 = 3^2 + 4^2$
$a = \sqrt{3^2 + 4^2}$
$a = \sqrt{9 + 16}$
$a = \sqrt{25}$
$a = 5$

b) Since PQ is a tangent to the circle, $\angle OPQ = 90°$.
Use the Pythagorean Theorem in right ΔOPQ to calculate *a*.
$OP^2 + PQ^2 = OQ^2$
$a^2 + 5^2 = 13^2$
$a^2 = 13^2 – 5^2$
$a = \sqrt{13^2 - 5^2}$
$a = \sqrt{169 - 25}$
$a = \sqrt{144}$
$a = 12$

c) Since PQ is a tangent to the circle, $\angle OPQ = 90°$.
Use the Pythagorean Theorem in right ΔOPQ to calculate *a*.

$$a^2 + 15^2 = 25^2$$
$$a^2 = 25^2 - 15^2$$
$$a = \sqrt{25^2 - 15^2}$$
$$a = \sqrt{625 - 225}$$
$$a = \sqrt{400}$$
$$a = 20$$

Apply

7. a) Since T is a point of tangency, $\angle OTQ = \angle OTR = 90°$
Use the sum of the angles in ΔQOT.

$$\angle QOT + \angle OTQ + \angle OQT = 180°$$
$$d° + 90° + 28° = 180°$$
$$d° = 180° - 90° - 28°$$
$$d° = 62°$$

Use the sum of the angles in ΔROT.

$$\angle ROT + \angle OTR + \angle ORT = 180°$$
$$e° + 90° + 35° = 180°$$
$$e° = 180° - 90° - 35°$$
$$e° = 55°$$

b) Since T is a point of tangency, $\angle OTQ = \angle OTR = 90°$
Use the sum of the angles in ΔQOT.

$$\angle OQT + \angle OTQ + \angle QOT = 180°$$
$$d° + 90° + 33° = 180°$$
$$d° = 180° - 90° - 33°$$
$$d° = 57°$$

Use the sum of the angles in ΔROT.

$$\angle ORT + \angle RTO + \angle TOR = 180°$$
$$e° + 90° + 69° = 180°$$
$$e° = 180° - 90° - 69°$$
$$e° = 21°$$

8. a) Since S is a point of tangency, $\angle OSB = 90°$.
Use the Pythagorean Theorem in right ΔOSB to calculate *a*.

$$OB^2 = OS^2 + BS^2$$
$$a^2 = 3^2 + 8^2$$
$$a = \sqrt{3^2 + 8^2}$$
$$a = \sqrt{9 + 64}$$
$$a = \sqrt{73}$$
$$a \doteq 8.5$$

b) Since S is a point of tangency, ∠OSR = 90°.
Use the Pythagorean Theorem in right ΔOSR to calculate *a*.

$OS^2 + SR^2 = OR^2$
$a^2 + 9^2 = 12^2$
$a^2 = 12^2 - 9^2$
$a = \sqrt{12^2 - 9^2}$
$a = \sqrt{144 - 81}$
$a = \sqrt{63}$
$a \doteq 7.9$

9. Since S is a point of tangency, ∠OSR = ∠OSQ = 90°.
Use the Pythagorean Theorem in right ΔOSR to determine *a*.

$RS^2 + OS^2 = OR^2$
$a^2 + 6^2 = 13^2$
$a^2 = 13^2 - 6^2$
$a = \sqrt{13^2 - 6^2}$
$a = \sqrt{169 - 36}$
$a = \sqrt{133}$
$a \doteq 11.5$

Use the Pythagorean Theorem in right ΔOSQ to determine *b*.

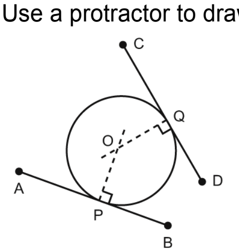
$SQ^2 + OS^2 = OQ^2$
$b^2 + 6^2 = 8^2$
$b^2 = 8^2 - 6^2$
$b = \sqrt{8^2 - 6^2}$
$b = \sqrt{64 - 36}$
$b = \sqrt{28}$
$b \doteq 5.3$

10. Answers may vary. For example:
A glue stick lying horizontally on the desk; the radius of its circular base drawn to the point where the base meets the desk is perpendicular to the horizontal line representing the desk.

The classroom round clock, with a square frame that touches the circle at 4 points; when the minute hand points to each of numbers 3, 6, 9, and 12, it represents a radius that is perpendicular to a tangent that is the frame.

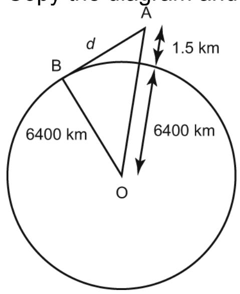
A car wheel rolling on a straight line on a flat surface; the wheel touches the ground at only one point. The radius of the wheel drawn from its centre to the point where the wheel touches the ground is perpendicular to the horizontal line representing the ground.

11. Answers may vary. P and Q are points of tangency.
Use a protractor to draw a perpendicular line at Q and a perpendicular line at P.



Since the perpendicular lines OQ and OP from the points of tangency are radii of the circle, and radii have one endpoint at the centre, these lines intersect at the centre of the circle.

12. Copy the diagram and label line segment AB as *d*.



The line segment AB from the plane to the horizon is a tangent to the circle at B.
Since the tangent AB is perpendicular to the radius OB at the point of tangency B, ΔOBA is a right triangle with $\angle$OBA = 90°.
In kilometres, AO = 1.5 + 6400
= 6401.5
Use the Pythagorean Theorem in right ΔOBA.

$AB^2 + OB^2 = AO^2$

$d^2 + 6400^2 = 6401.5^2$

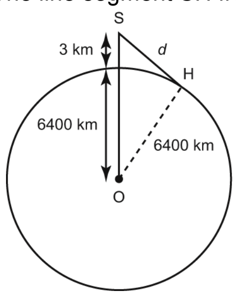
$d^2 = 6401.5^2 - 6400^2$

$d = \sqrt{6401.5^2 - 6400^2}$

$d \doteq 138.6$

The plane is about 139 km from the horizon.

13. Copy the diagram and label line segment SH as *d*.
The line segment SH from the skydiver to the horizon is a tangent at H. Join OH. OH is a radius.



Since the tangent SH is perpendicular to the radius OH at the point of tangency H, ΔOSH is a right triangle with $\angle$OHS = 90°.
In kilometres, OS = 3 + 6400
= 6403
Use the Pythagorean Theorem in right ΔOSH.

$SH^2 + OH^2 = OS^2$

$d^2 + 6400^2 = 6403^2$

$d^2 = 6403^2 - 6400^2$

$d = \sqrt{6403^2 - 6400^2}$

$d \doteq 195.9$

The horizon is about 196 km from the skydiver.

14. OB is a radius and AC is a tangent, so ΔOBA and ΔOBC are right triangles with $\angle$OBA = $\angle$OBC = 90°.
Use the Pythagorean Theorem in right ΔOBA to determine *x*.

$OA^2 = OB^2 + AB^2$

$x^2 = 6^2 + 9^2$

$x = \sqrt{6^2 + 9^2}$

$x = \sqrt{36 + 81}$

$x \doteq 10.8$

Use the Pythagorean Theorem in right ΔOBC to determine y.

$BC^2 + OB^2 = OC^2$

$y^2 + 6^2 = 12^2$

$y^2 = 12^2 - 6^2$

$y = \sqrt{12^2 - 6^2}$

$y = \sqrt{144 - 36}$

$y \doteq 10.4$

To determine the measure of the angle labelled $z°$, use the angle sum of a triangle.
The sum of the angles in ΔOBC is 180°.

$z° + \angle OCB + \angle OBC = 180°$

$z° + 30° + 90° = 180°$

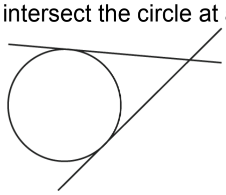
$z° = 180° - 30° - 90°$

$z° = 60°$

15. Explanations may vary. For example:

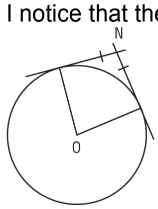
a) I can draw two tangents to a circle from any one point outside it. One tangent will go to one side and one will go to the other. If I visualize a ruler sweeping across a circle with one end fixed at my point, one tangent would come from sweeping clockwise and the other from sweeping counterclockwise.

b) I cannot draw any other tangents because any other lines I draw will either intersect twice or not intersect the circle at all.



c) I used a plastic triangle to make sure the lines I drew are perpendicular to the radius at the points of tangency. I placed the plastic triangle so the centre of the circle was on one leg and the point outside the circle was on the other leg. Then, with the right angle at the point of tangency on the circle, and I drew the tangent.

16. a) I notice that the lengths of the tangents from N to the points of tangency are equal.



b) My classmates had the same answer as I did for part a, so it seems that the two tangents drawn from the same point outside the circle are equal.

c) x and y represent the lengths of tangents from point B.
In ΔAOB, $\angle OAB = 90°$, so use the Pythagorean Theorem to determine x.

$AB^2 + AO^2 = BO^2$

$x^2 + 5^2 = 20^2$

$x^2 = 20^2 - 5^2$

$x = \sqrt{20^2 - 5^2}$

$x = \sqrt{400 - 25}$

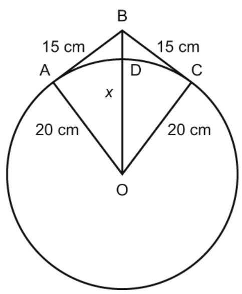
$x \doteq 19.4$

Since both OC and OA are radii, OC = OA = 5
In ΔCOB, $\angle$OCB = 90°, so use the Pythagorean Theorem to determine y.

$BC^2 + OC^2 = OB^2$
$y^2 + 5^2 = 20^2$
$y^2 = 20^2 - 5^2$
$y = \sqrt{20^2 - 5^2}$
$y = \sqrt{400 - 25}$
$y \doteq 19.4$

Since $x = y \doteq 19.4$, the answers do confirm my conclusions in part b that two tangents to a circle from a point outside the circle are equal.

17.

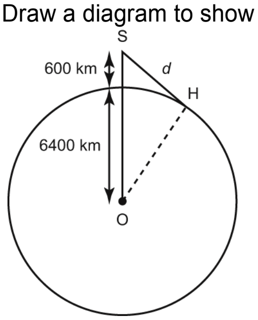


Let x represent the length of OB.
Since AB is a tangent and AO is a radius, $\angle$BAO = 90°.
Use the Pythagorean Theorem in right ΔBAO to determine x.

$OB^2 = AB^2 + OA^2$
$x^2 = 15^2 + 20^2$
$x = \sqrt{15^2 + 20^2}$
$x = \sqrt{225 + 400}$
$x = \sqrt{625}$
$x = 25$

From the diagram, BD = OB – OD
The height of the hook above the top of the mirror is the length of BD.
OD is a radius; so, OD = 20 cm
Then,
BD = 25 cm – 20 cm
= 5 cm
The hook is 5 cm above the top of the mirror.

18. The farthest point on Earth's surface that could receive a satellite signal is a point on the horizon. Draw a diagram to show the satellite, S, a point on the horizon, H, and the centre of Earth, O.

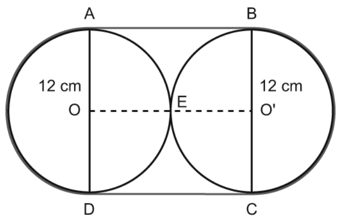


Since SH is a tangent, ΔSHO is a right triangle with $\angle$SHO = 90°.
The distance between the satellite and the farthest point on Earth's surface is SH.
Let SH be represented by d. The radius of Earth is 6400 km.
The length of SO is: 600 km + 6400 km = 7000 km

Use the Pythagorean Theorem in right $\triangle$SHO to determine the value of *d*.
$SH^2 + OH^2 = SO^2$
$d^2 + 6400^2 = 7000^2$
$d^2 = 7000^2 - 6400^2$
$d = \sqrt{7000^2 - 6400^2}$
$d \doteq 2835.49$
The distance is about 2835 km.

Take It Further

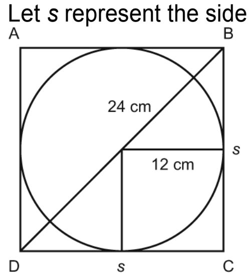
19.



The two straight sections of the strap AB and CD are tangents to the circles.
Since the circles touch, the distance between the centres, OO', is equal to OE + EO', or twice their radius:
OO' = 12 cm
So, AB = CD = 12 cm

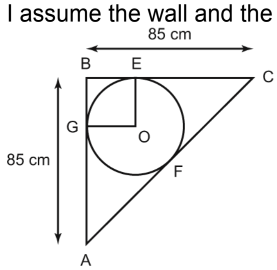
Each of the two curved sections of strap is a semicircle with diameter 12 cm, so together they are the circumference, *C*, of one rod.
$C = \pi d$
$C = \pi(12)$
$\doteq 37.7$
The length of the strap is AB + CD + *C* $\doteq$ 12 cm + 12 cm + 37.7 cm, or about 61.7 cm.

20. Let *s* represent the side length of the square.



The sides of the square are tangents to the circle. The points of tangency are at the midpoints of the sides.
Use the Pythagorean Theorem in the right $\triangle$BCD to determine *s*.
$CD^2 + BC^2 = BD^2$
$s^2 + s^2 = 24^2$
$2s^2 = 576$ Divide each side by 2.
$s^2 = 288$
$s = \sqrt{288}$
$s \doteq 16.97$
Since the circle just fits inside the square, its diameter is equal to the side length of the square.
Then the radius is one-half the length of the diameter, or one-half the side length *s*.
So, the radius is about $\frac{16.97}{2}$ cm $\doteq$ 8.5 cm.

21. I assume the wall and the ceiling are perpendicular. So, $\angle ABC = 90°$.



Let x represent the length of AC.
Use the Pythagorean Theorem in right ΔABC to determine x.

$AC^2 = AB^2 + BC^2$
$x^2 = 85^2 + 85^2$
$x = \sqrt{85^2 + 85^2}$
$x \doteq 120.2$

Since ΔABC is an isosceles triangle, the point of tangency F is at the midpoint of side AC.
So, CF in centimetres is one-half the length of AC.

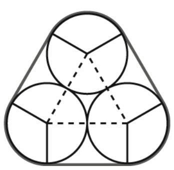
$CF \doteq \frac{1}{2} \times 120.2$
$\doteq 60.1$

CE and CF are tangents drawn from the same point. So, CE = CF.

From the diagram,
BE = BC – CE
= 85 cm – 60.1 cm
= 24.9 cm

Quadrilateral BGOE is a square. Its side length equals the radius.
So, the length of radius is 24.9 cm.
The diameter of the pipe is twice the length of the radius: 2 × 24.9 cm $\doteq$ 49.8 cm
The diameter of the pipe is approximately 50 cm.

22. a) Join each centre with its corresponding points of tangency.



Each straight section of the strap is a tangent to two circles and represents a side of a rectangle equal to the diameter of a log (1 m).

Each of the curved sections of the strap is $\frac{1}{3}$ the circumference of a log, so together the 3 curved pieces have a length equal to the circumference of one log, which is:
$\pi \times 1$ m $\doteq$ 3.14 m
The minimum length of strap is:
3 × (1 m) + 3.14 m = 6.14, or about 6 m.

b) Answers may vary.
For example: The actual strap needed would be longer because some overlap is required to fasten the strap.

Lesson 8.2 Properties of Chords in a Circle Practice (page 397–399)

Check

3. a) The line segment that joins the centre of the circle and the midpoint of the chord is perpendicular to the chord.
So, $d° = 90°$

b) The perpendicular from the centre of the circle to the chord bisects the chord.
So, $e = 5$

c) $f = \frac{14}{2}$, or 7

4. a) Since OC bisects the chord and passes through the centre of the circle,
OC is perpendicular to AB: $\angle ACO = \angle BCO = 90°$
So, $y° = 90°$

In ΔOCA, use the sum of the angles in a triangle.
$x° + 90° + 40° = 180°$
$x° = 180° - 90° - 40°$
$x° = 50°$

b) Since the radii are equal, DO = OE, ΔODE is isosceles.
Then, $\angle OED = \angle ODE = 22°$
So, $x° = 22°$

In ΔODE, use the sum of the angles in a triangle.
$y° + 22° + 22° = 180°$
$y° = 180° - 22° - 22°$
$y° = 136°$

c) In ΔFOG, use the sum of the angles in a triangle.
$x° + y° + 110° = 180°$
Since the radii are equal, FO = OG, ΔFOG is isosceles and $\angle OFG = \angle OGF$, or $x° = y°$.
Then,
$x° + x° + 110° = 180°$
$2x° = 180° - 110°$
$2x° = 70°$
$x° = 35°$
So, $x° = y° = 35°$

5. a) Since OK is perpendicular to chord HJ, then OK bisects HJ, and HK = KJ, or $a = b$
Use the Pythagorean Theorem in right ΔOKJ to calculate a.
$OH^2 = HK^2 + OK^2$
$10^2 = a^2 + 3^2$
$10^2 - 3^2 = a^2$
$a = \sqrt{10^2 - 3^2}$
$a = \sqrt{100 - 9}$
$a = \sqrt{91}$
$a \doteq 9.5$
Since $a = b$, then $b \doteq 9.5$

b) Since OP is perpendicular to chord MN, then OP bisects MN, and NP = PM.

Then, PM = $\frac{\text{MN}}{2}$ or $a = \frac{b}{2}$

Use the Pythagorean Theorem in right ΔOPM to calculate *a*.

$\text{OM}^2 = \text{OP}^2 + \text{PM}^2$

$7^2 = 4^2 + a^2$

$a^2 = 7^2 - 4^2$

$a = \sqrt{7^2 - 4^2}$

$a = \sqrt{49 - 16}$

$a = \sqrt{33}$

$a \doteq 5.745$, or about 5.7

Since $a = \frac{b}{2}$, then $b = 2a$

$b \doteq 2 \times 5.745$

$b \doteq 11.490$, or about 11.5

Apply

6. PO is a radius, so PO is one-half of the length of diameter PQ: $\frac{1}{2} \times 18 = 9$

Use the Pythagorean Theorem in right ΔOSP to calculate PS.

$\text{OP}^2 = \text{PS}^2 + \text{OS}^2$

$9^2 = \text{PS}^2 + 5^2$

$\text{PS}^2 = 9^2 - 5^2$

$\text{PS} = \sqrt{9^2 - 5^2}$

$\text{PS} = \sqrt{81 - 25}$

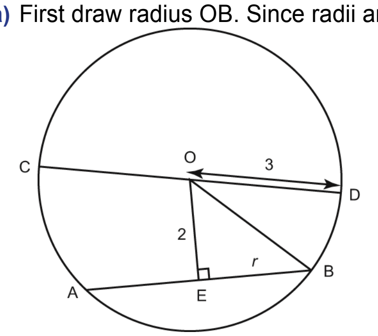
$\text{PS} = \sqrt{56}$

$\text{PS} \doteq 7.5$

Use Perpendicular to Chord Property 1: The perpendicular from the centre of a circle to a chord bisects the chord. Since OS is perpendicular to chord PR, then OS bisects PR, and PS = SR or PS = *b*.

So, $b \doteq 7.5$

7. a) First draw radius OB. Since radii are equal, OD = OB = 3.



Use the Pythagorean Theorem in right ΔOEB.

$\text{OB}^2 = \text{OE}^2 + \text{EB}^2$

$3^2 = 2^2 + r^2$

$r^2 = 3^2 - 2^2$

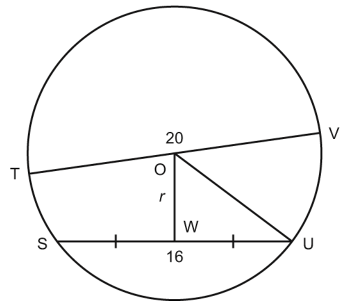
$r = \sqrt{3^2 - 2^2}$

$r = \sqrt{9 - 4}$

$r = \sqrt{5}$

$r \doteq 2.24$, or about 2.2

b) First draw radius OU. Radii are equal, so OU = OV = $\frac{1}{2} \times 20 = 10$.



Since OW joins the centre O and the midpoint of chord SU, OW is perpendicular to chord SU.

OW bisects SU, and SW = WU = $\frac{1}{2} \times 16 = 8$

Use the Pythagorean Theorem in right ΔOWU.

$OU^2 = OW^2 + WU^2$

$10^2 = r^2 + 8^2$

$r^2 = 10^2 - 8^2$

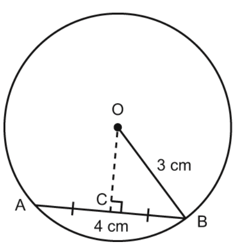
$r = \sqrt{10^2 - 8^2}$

$r = \sqrt{100 - 64}$

$r = \sqrt{36}$

$r = 6$

8. a) I drew a circle with radius 3 cm. Chord AB measures 4 cm. Then, the distance from the centre O to the chord AB is the perpendicular distance from the centre to the chord OC



OC bisects the chord AB, and AC = CB = 2 cm.

Use the Pythagorean Theorem in right ΔOCB to calculate OC.

$OB^2 = OC^2 + CB^2$

$3^2 = OC^2 + 2^2$

$OC^2 = 3^2 - 2^2$

$OC = \sqrt{3^2 - 2^2}$

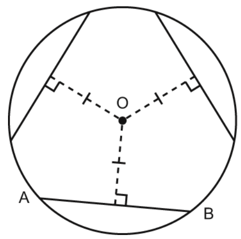
$OC = \sqrt{9 - 4}$

$OC = \sqrt{5}$

$OC \doteq 2.24$

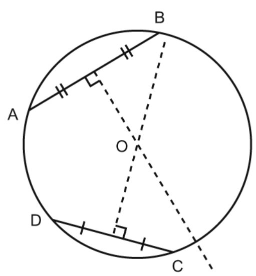
Measure to check: The measure of line segment OC is about 2.2 cm.

b) The distance measured from the centre to each 4-cm chord is about 2.2 cm.

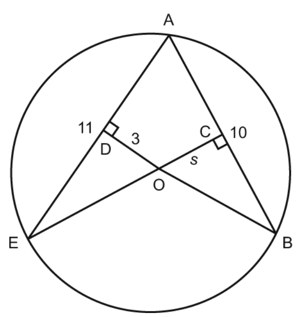


c) All congruent chords are the same distance from the centre of the circle.

9. Use Perpendicular to Chord Property 2: The perpendicular bisector of a chord in a circle passes through the centre of the circle.
I measure to find the midpoints of the chords; then, I draw the perpendicular bisector for each chord.
Both bisectors pass through the centre of the circle, so where they intersect is the centre, O.



10. a) Copy the diagram and draw radius OB.



Since OD is perpendicular to chord AE, OD bisects the chord and ED = DA

$= \frac{1}{2} \times 11$

$= 5.5$

Use the Pythagorean Theorem in right $\triangle ODE$ to calculate radius OE.

$OE^2 = OD^2 + DE^2$

$OE^2 = 3^2 + 5.5^2$

$OE = \sqrt{3^2 + 5.5^2}$

$OE = \sqrt{9 + 30.25}$

$OE = \sqrt{39.25}$

$OE \doteq 6.26$

Since radii are equal, then $OE = OB \doteq 6.26$

Then, OC is perpendicular to chord AB. So, OC bisects AB and $AC = CB = \frac{1}{2} \times 10 = 5$

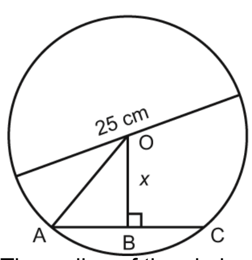
Use the Pythagorean Theorem in right ΔOCB to calculate s.
$OB^2 = OC^2 + CB^2$
$6.26^2 = s^2 + 5^2$
$s^2 = 6.26^2 - 5^2$
$s = \sqrt{6.26^2 - 5^2}$
$s = \sqrt{39.1876 - 25}$
$s = \sqrt{14.1876}$
$s \doteq 3.77$, or about 3.8

b) Since FG is perpendicular to chord HJ, FG bisects the chord and HG = GJ = $\frac{1}{2} \times 6 = 3$

Radii are equal; so, FO = OJ = 4
Use the Pythagorean Theorem in right ΔOGJ to calculate OG.
$OJ^2 = GJ^2 + OG^2$
$4^2 = 3^2 + OG^2$
$OG^2 = 4^2 - 3^2$
$OG = \sqrt{4^2 - 3^2}$
$OG = \sqrt{16 - 9}$
$OG = \sqrt{7}$
$OG \doteq 2.65$

Then, FG = FO + OG $\doteq$ 4 + 2.65 = 6.65
Use the Pythagorean Theorem in right ΔFGJ to calculate s.
$FJ^2 = FG^2 + GJ^2$
$s^2 = 6.65^2 + 3^2$
$s = \sqrt{6.65^2 + 3^2}$
$s = \sqrt{44.2225 + 9}$
$s = \sqrt{53.2225}$
$s \doteq 7.30$, or about 7.3

11.



The radius of the circle, OA, is one-half the length of the diameter, or $\frac{1}{2}$ of 25 cm = $\frac{1}{2} \times 25 = 12.5$ cm.

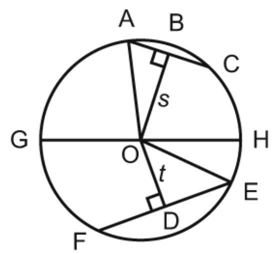
The distance x to chord AC is the perpendicular distance from the centre of the circle to the chord, OB.
This perpendicular line bisects the chord, so AB is $\frac{1}{2}$ of 16 cm = $\frac{1}{2} \times 16 = 8$ cm.

Use the Pythagorean Theorem in right ΔAOB.
$OB^2 + AB^2 = OA^2$
$x^2 + 8^2 = 12.5^2$
$x^2 = 12.5^2 - 8^2$
$x = \sqrt{12.5^2 - 8^2}$
$x = \sqrt{156.25 - 64}$
$x = \sqrt{92.25}$
$x \doteq 9.6$
The chord is about 9.6 cm from the centre of the circle.

12. a) The diameter is the longest chord in a circle.
So, in a circle with a diameter of 14 cm, no chord has a length greater than 14 cm.

Chords with lengths 5 cm, 9 cm, and 14 cm are the only possibilities.
I could check my answers by drawing a circle with diameter 14 cm and using a ruler to check if I can locate a chord with each length.

b) The radius of the circle is 7 cm.
Draw an accurate diagram.



i) For chord AC with length 5 cm: Let s represent the length of OB.

Since the perpendicular OB bisects the chord, AB is $\frac{1}{2}$ of 5 cm = $\frac{1}{2} \times 5$ cm, or 2.5 cm.

Use the Pythagorean Theorem in right ΔOBA:

$$OB^2 + AB^2 = OA^2$$
$$s^2 + 2.5^2 = 7^2$$
$$s^2 = 7^2 - 2.5^2$$
$$s = \sqrt{7^2 - 2.5^2}$$
$$s = \sqrt{49 - 6.25}$$
$$s = \sqrt{42.75}$$
$$s \doteq 6.5$$

The 5-cm chord is about 6.5 cm from the centre of the circle.

ii) For chord FE with length 9 cm: Let t represent the length of OD.

Since the perpendicular OD bisects the chord, ED is $\frac{1}{2}$ of 9 cm = $\frac{1}{2} \times 9$ cm, or 4.5 cm.

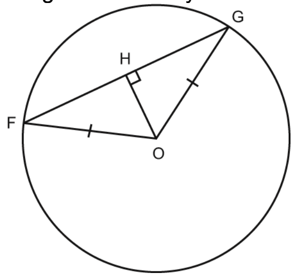
Use the Pythagorean Theorem in right ΔOED.

$$OD^2 + DE^2 = OE^2$$
$$t^2 + 4.5^2 = 7^2$$
$$t^2 = 7^2 - 4.5^2$$
$$t = \sqrt{7^2 - 4.5^2}$$
$$t = \sqrt{49 - 20.25}$$
$$t \doteq 5.4$$

The 9-cm chord is about 5.4 cm from the centre of the circle.

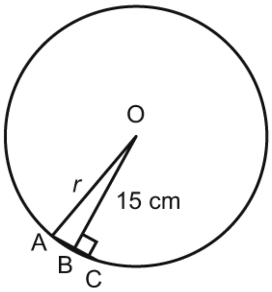
iii) The 14-cm chord is diameter GH.
Since GH passes through the centre, it is 0 cm from the centre of the circle.

13. Diagrams will vary.



Point O is the centre of the circle. Since radii are equal, FO = OG, and ΔFOG is isosceles.
ΔOHF is congruent to ΔOHG. So, FH = HG.

14.



Chord AC is 6 cm long.

OB is the perpendicular bisector of chord AC, so AB = BC = $\frac{1}{2} \times$ 6 cm, or 3 cm.

Use the Pythagorean Theorem in right ΔOAB to calculate the radius *r*.

$OA^2 = AB^2 + OB^2$

$r^2 = 3^2 + 15^2$

$r = \sqrt{3^2 + 15^2}$

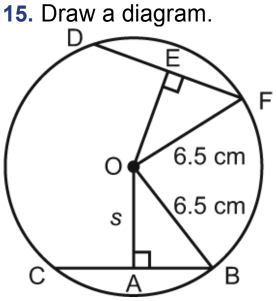
$r = \sqrt{9 + 225}$

$r = \sqrt{234}$

$r \doteq 15.3$

The radius is about 15.3 cm.

15. Draw a diagram.



a) DF = CB = 8 cm

The perpendicular, OA, bisects chord CB, so AB = $\frac{1}{2} \times$ 8 cm, or 4 cm.

The radius of the circle is $\frac{1}{2}$ of 13 cm = $\frac{1}{2} \times$ 13 cm = 6.5 cm.

Let *s* represent the shortest distance from CB to O.

Use the Pythagorean Theorem in right ΔOAB.

$OB^2 + AB^2 = OB^2$

$s^2 + 4^2 = 6.5^2$

$s^2 = 6.5^2 - 4^2$

$s = \sqrt{6.5^2 - 4^2}$

$s = \sqrt{42.25 - 16}$

$s = \sqrt{26.25}$

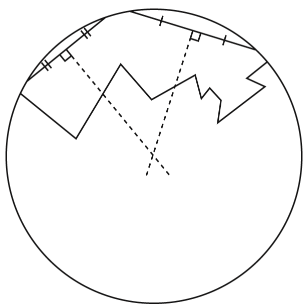
$s \doteq 5.1$

In right ΔOEF, EF = 4 cm and OF = 6.5 cm, so ΔOEF is congruent to ΔOAB.

So, OE = OA $\doteq$ 5.1 cm

b) The congruent chords are the same distance from the centre of the circle.

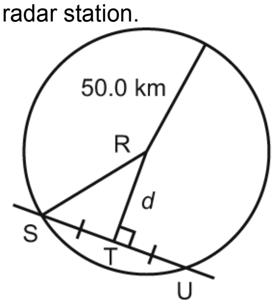
16.



I draw two chords and their perpendicular bisectors. I locate the centre of the plate by finding the intersection point of the perpendicular bisectors of the two chords. Then I use a compass centred at this point to complete a sketch of the plate.

17. The ship's path is a chord in the circle representing the radar zone.

The ship's closest distance, *d*, to the radar station, R, is the perpendicular distance from the chord to the radar station.



Since RT is perpendicular to chord SU, then RT bisects the chord.

So, ST = TU = $\frac{1}{2} \times 62.5$ km, or 31.25 km

RS is a radius; so, RS = 50.0 km

Use the Pythagorean Theorem in right ΔRST to determine *d*.

$RT^2 + ST^2 = SR^2$

$d^2 + 31.25^2 = 50^2$

$d^2 = 50^2 - 31.25^2$

$d = \sqrt{50^2 - 31.25^2}$

$d = \sqrt{2500 - 976.5625}$

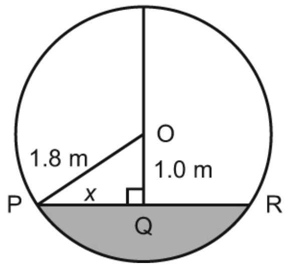
$d = \sqrt{1523.4375}$

$d \doteq 39.03$

The closest distance is about 39.0 km.

18. The width of the path PR is a chord in the circle.
Its distance from the centre of the circle is:
OQ = 2.8 m – 1.8 m = 1 m.

Since OQ is perpendicular to chord PR, OQ bisects chord PR.
Then, PQ = QR = $\frac{1}{2}$ of PR.

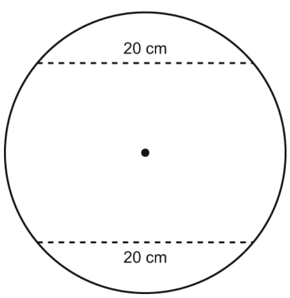


Let the length of PQ be represented by x.

Use the Pythagorean Theorem in right ΔOPQ to determine x.
$PQ^2 + OQ^2 = PO^2$
$x^2 + 1^2 = 1.8^2$
$x^2 = 1.8^2 - 1^2$
$x = \sqrt{1.8^2 - 1^2}$
$x = \sqrt{3.24 - 1}$
$x = \sqrt{2.24}$
$x \doteq 1.4967$
Then, the width of the path is: 2 × 1.4967 = 2.9934
The path is about 3.0 m wide.

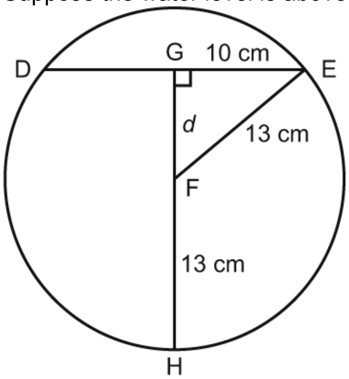
Take It Further

19. a) Draw a circle to represent the spherical fish bowl.
The surface of the water forms a chord.
The 20-cm chord could be above or below the centre of the circle:



The maximum depth of the water is the depth measured at the centre of the chord.

Suppose the water level is above the centre of the bowl:



The radius of the circle is one-half the diameter, which is 13 cm.
Let the depth of water from the chord to the centre of the circle be represented by *d*.
The maximum depth of the water, GH, is: GF + FH = *d* + 13 cm

Since FG is the perpendicular bisector of DE, then GE = $\frac{1}{2}$ of DE, or $\frac{1}{2} \times 20$ cm = 10 cm.

Use the Pythagorean Theorem in right ΔGEF to calculate *d*.

$GF^2 + GE^2 = FE^2$

$d^2 + 10^2 = 13^2$

$d^2 = 13^2 - 10^2$

$d = \sqrt{13^2 - 10^2}$

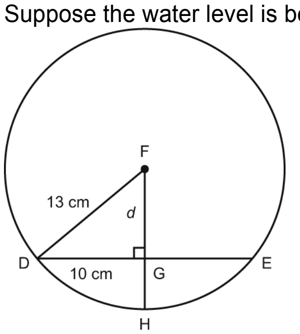
$d = \sqrt{169 - 100}$

$d = \sqrt{69}$

$d \doteq 8.307$

So, the maximum depth of the water, GH, is about 8.307 cm + 13 cm $\doteq$ 21.3 cm.

Suppose the water level is below the centre of the bowl:



Then, the maximum depth, GH, is: FH – FG = 13 cm – *d*
Use the Pythagorean Theorem in right ΔFGD to find *d*.

$d^2 + DG^2 = FD^2$

$d^2 + 10^2 = 13^2$

$d^2 = 13^2 - 10^2$

$d = \sqrt{13^2 - 10^2}$

$d = \sqrt{169 - 100}$

$d = \sqrt{69}$

$d \doteq 8.307$

So, the maximum depth of the water, GH, is about 13 cm – 8.307 cm $\doteq$ 4.7 cm

b) There are 2 answers for part a: about 21.3 cm or about 4.7 cm

Mid-Unit Review (page 403)

Lesson 8.1

1. Use the Tangent-Radius Property.

a) Since the tangent QP is perpendicular to the radius PO at the point of tangency P, ΔQPO is a right triangle, with $\angle P = y° = 90°$
The sum of the angles in ΔQPO is 180°.
So, $x° + y° + 68° = 180°$
$x° + 90° + 68° = 180°$
$x° = 180° - 90° - 68°$
$x° = 22°$

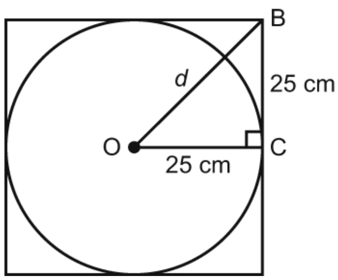
b) Since the tangent ST is perpendicular to the radius OP at the point of tangency P, $\angle OPT = \angle OPS = 90°$
The sum of the angles in ΔSPO is 180°.
So, $90° + y° + 57° = 180°$
$y° = 180° - 90° - 57°$
$y° = 33°$
The sum of the angles in ΔSPT is 180°.
So, $90° + x° + 44° = 180°$
$x° = 180° - 90° - 44°$
$x° = 46°$

2. Since PQ is a tangent, $\angle QPO = 90°$.
Use the Pythagorean Theorem in right ΔOPQ to determine *a.*
$PQ^2 + OP^2 = OQ^2$
$a^2 + 6^2 = 12^2$
$a^2 = 12^2 - 6^2$
$a = \sqrt{12^2 - 6^2}$
$a = \sqrt{144 - 36}$
$a = \sqrt{108}$
$a \doteq 10.4$

3. Copy the diagram.

Since the disc just fits inside the square, its diameter is equal to the side length of the square, which is 50 cm. So, the radius is 25 cm. Each side of the square is a tangent to the circle with the midpoint of the side as the point of tangency. So, the length of BC is $\frac{1}{2}$ of 50 cm, or 25 cm.

Let *d* represent the distance between the corner of the sheet and the centre of the disc.
Since OC is a radius and BC is a tangent, then $\angle OCB = 90°$.



Use the Pythagorean Theorem in right ΔOBC.
$OB^2 = OC^2 + BC^2$
$d^2 = 25^2 + 25^2$
$d = \sqrt{25^2 + 25^2}$
$d = \sqrt{625 + 625}$
$d = \sqrt{1250}$
$d \doteq 35.355$
The distance is about 35.4 cm.

Lesson 8.2

4. Since OE bisects chord FG, OE is perpendicular to FG and $\angle$OEG = 90°.
Use the angle sum in ΔEOG.
$m° + 90° + 71° = 180°$
$m° = 180° - 90° - 71°$
$m° = 19°$

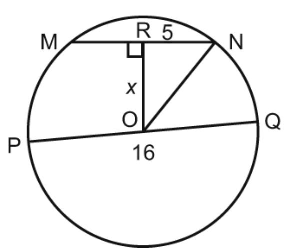
5. a) Since OJ is perpendicular to chord HK, OJ bisects HK, and HJ = $\frac{1}{2}$ of x, or $\frac{x}{2}$.

Let y represent the length of HJ.
Use the Pythagorean Theorem in right ΔOHJ.
$HJ^2 + OJ^2 = OH^2$
$y^2 + 5^2 = 11^2$
$y^2 = 11^2 - 5^2$
$y = \sqrt{11^2 - 5^2}$
$y = \sqrt{121 - 25}$
$y = \sqrt{96}$
$y \doteq 9.798$

$x = 2y$
$x \doteq 2 \times 9.798$
$\doteq 19.6$

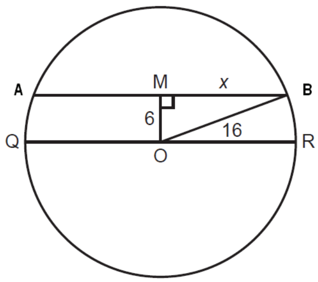
b) Join ON; this is a radius with length $\frac{1}{2} \times 16$, or 8.

The perpendicular distance OR bisects chord MN, so NR = $\frac{1}{2} \times 10 = 5$.



Use the Pythagorean Theorem in right ΔONR.
$OR^2 + NR^2 = ON^2$
$x^2 + 5^2 = 8^2$
$x^2 = 8^2 - 5^2$
$x = \sqrt{8^2 - 5^2}$
$x = \sqrt{64 - 25}$
$x = \sqrt{39}$
$x \doteq 6.2$

6. a)



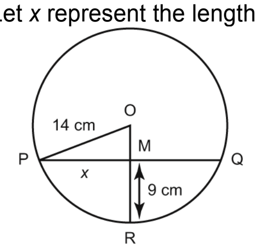
b) OB is a radius; it is one-half the length of the diameter: OB = $\frac{1}{2} \times 32$ cm = 16 cm

OM is the distance to chord AB from the centre of the circle. So, OM is perpendicular to AB.
I can use the property that states that the perpendicular to a chord bisects the chord, then I know that MB is one-half the length of AB.
Let x represent the length of MB.
Use the Pythagorean Theorem in right ΔOMB.

$OB^2 + MO^2 = OB^2$
$x^2 + 6^2 = 16^2$
$x^2 = 16^2 - 6^2$
$x = \sqrt{16^2 - 6^2}$
$x = \sqrt{256 - 36}$
$x = \sqrt{220}$
$x \doteq 14.832$

The length of the chord AB is $2x$.
The chord is about: 2×14.832 cm $\doteq$ 29.7 cm

7. Let x represent the length of PM.



OP and OR are radii; OP = OR = 14 cm.
Then, OM = OR – MR = 14 cm – 9 cm = 5 cm

Use the property that states that the perpendicular to a chord bisects the chord. Since OR is vertical and PQ is horizontal, $\angle$OMP = 90°, and PM = MQ = $\frac{1}{2}$ the length of PQ.

Use the Pythagorean Theorem in right ΔOMP.

$MP^2 + MO^2 = OP^2$
$x^2 + 5^2 = 14^2$
$x^2 = 14^2 - 5^2$
$x = \sqrt{14^2 - 5^2}$
$x = \sqrt{196 - 25}$
$x = \sqrt{171}$
$x \doteq 13.077$
$PQ = 2x$
$\doteq 2 \times 13.077$
$\doteq 26.2$

So, surface of the water is about 26.2 cm wide.

Lesson 8.3 Properties of Angles in a Circle Practice (page 410–412)

Check

3. **a)** Minor arc DE subtends inscribed ∠DFE and central ∠DOE.

b) Minor arc PQ subtends inscribed ∠PRQ and central ∠POQ.

c) Minor arc NM subtends inscribed ∠NJM and ∠NKM, and central ∠NOM.

4. **a)** Both inscribed ∠BAC and central ∠BOC are subtended by minor arc BC.
So, the inscribed angle is one-half the central angle.

$x° = \frac{1}{2} \times 130°$

$x° = 65°$

b) ∠DEF is inscribed in a semicircle.
So, $x° = 90°$

c) Since ∠JGK and ∠JHK are inscribed angles subtended by the same arc JK, they are equal.
So, $x° = 40°$

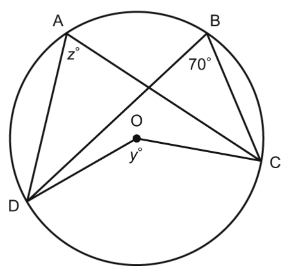
d) Both inscribed ∠NMR and central ∠NOR are subtended by minor arc NR.
So, the central angle is twice the inscribed angle.

$x° = 2 \times 29°$

$x° = 58°$

Apply

5. **a)**



Since ∠DAC and ∠DBC are inscribed angles subtended by the same arc DC, they are equal.
So, $z° = 70°$

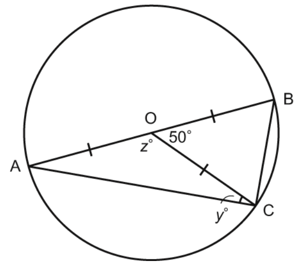
Inscribed ∠DBC and ∠DAC and central ∠DOC are subtended by minor arc DC.
So, the central angle is twice the inscribed angle.

$y° = 2 \times 70°$

$y° = 140°$

I used the Central Angle Property and the Inscribed Angle Property.

b)



The sum of the angles in a triangle is 180°. In isosceles ΔOBC:
∠BOC + ∠OCB + ∠OBC = 180° and ∠OCB = ∠OBC
50° + 2 × ∠OCB = 180°
2 × ∠OCB = 180° – 50°

$$\angle OCB = \frac{130^\circ}{2}$$

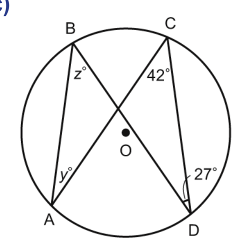
∠OCB = 65°

∠ACB is inscribed in a semicircle. So, ∠ACB = 90°
Then, *y*° + ∠OCB = 90°
y° + 65° = 90°
y° = 90° – 65°
y° = 25°

AB is a diameter. ∠AOB is a straight angle, or ∠AOB = 180°
Then, ∠AOC + ∠COB = 180°
z° + 50° = 180°
z° = 180° – 50°
z° = 130°

I used the Angle in a Semicircle Property.

c)

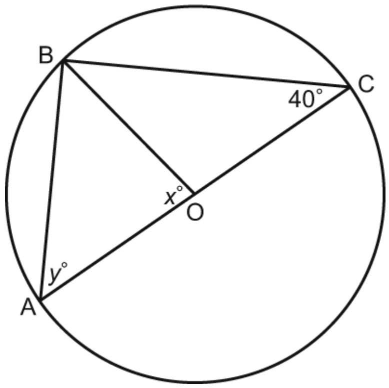


Since ∠ABD and ∠ACD are inscribed angles subtended by the same arc AD, they are equal.
So, *z*° = 42°

∠BAC and ∠BDC are inscribed angles subtended by the same arc BC. So, they are equal.
y° = 27°

I used the Inscribed Angles Property.

6. a)



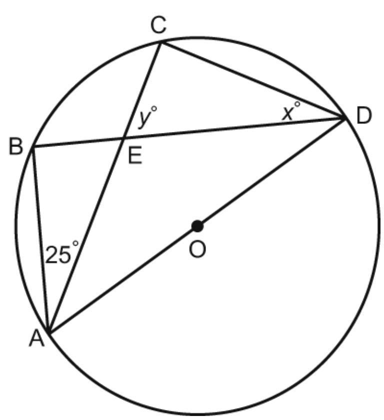
Central ∠AOB and inscribed ∠ACB are subtended by the same arc AB.
So, the central angle is twice the inscribed angle.

$x° = 2 \times 40°$
$x° = 80°$

ΔAOB is isosceles because OB and OA are equal radii.
So, ∠OBA = ∠BAO = $y°$
Use the angle sum in ΔAOB.
$y° + y° + 80° = 180°$
$2y° + 80° = 180°$
$2y° = 100°$
$y° = 50°$

I used the Central Angle Property and the Inscribed Angle Property.

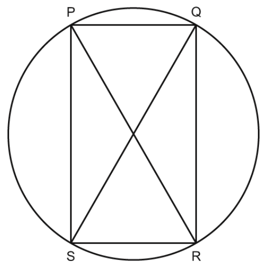
b)



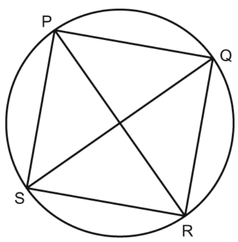
∠BAC and ∠BDC are inscribed angles subtended by minor arc BC.
So, they are equal: $x° = 25°$

∠DCA is inscribed in a semicircle; so, ∠DCA = 90°.
Use the angle sum in ΔCDE.
$y° + 25° + 90° = 180°$
$y° = 65°$

I used the Inscribed Angles Property and the Angles in a Semicircle Property.

7. 

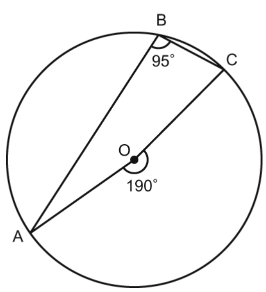
a) Since diagonals SQ and PR are diameters of the circle, each angle of the quadrilateral PQRS is inscribed in a semicircle. Each angle measure is 90°.
I know a rectangle is a quadrilateral with four right angles. So, quadrilateral PQRS is a rectangle.

b) 

PR and QS are diameters; so, PR = QS.
I know the perpendicular from the centre of a circle to a chord bisects the chord (Perpendicular to Chord Property 1). The diagonals are perpendicular, so they bisect each other, forming congruent right triangles.
So, PQ = QR = RS = SP
With all sides equal and all angles measures 90°, quadrilateral PQRS is a square.

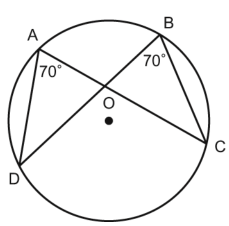
8. Diagrams may vary. For example:

a)



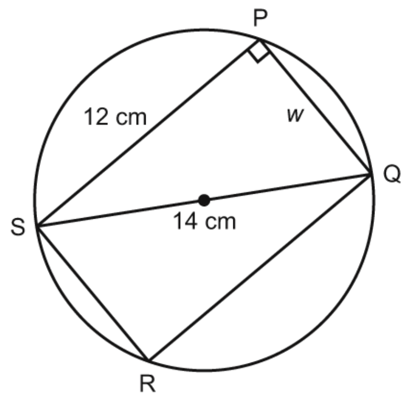
The central reflex ∠AOC and the inscribed ∠ABC are subtended by the major arc AC.
∠AOC = 190°, ∠ABC = 95°
The central angle is twice the measure of the inscribed angle.

b)



Both inscribed ∠DAC and ∠DBC are subtended by the same arc, DC.
∠DAC = ∠DBC = 70°

9. **a)** All the angles in a rectangle are right angles; so, each angle is inscribed in a semicircle.

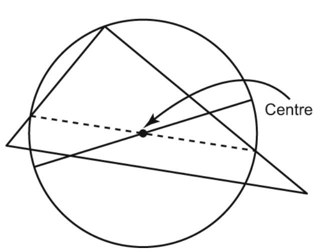


b) SQ is a diameter of the circle; SQ = 2 × 7 cm = 14 cm.
Let the width of the rectangle be represented by *w*.
Use the Pythagorean Theorem in right ΔSPQ.
$PQ^2 + PS^2 = SQ^2$
$w^2 + 12^2 = 14^2$
$w^2 = 14^2 - 12^2$
$w = \sqrt{14^2 - 12^2}$
$w = \sqrt{196 - 144}$
$w = \sqrt{52}$
$w \doteq 7.211$
The rectangle is about 7.2 cm wide.

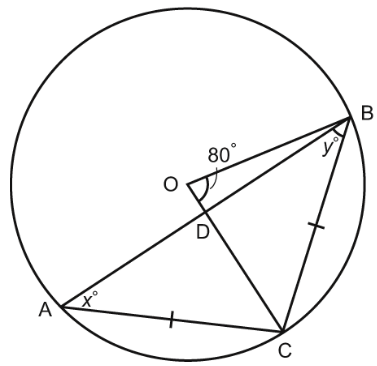
10. The angle inscribed in a semicircle is a right angle. So, the endpoints of the arms of the right angle are endpoints of a diameter.

I placed the set square with the vertex of the right angle on the circle, then marked the two points where the legs of the set square intersected the circle. These points are the endpoints of a diameter.

I repeated this construction with the vertex at a different point on the circle to draw a different diameter. The centre of the circle is the point where the two diameters intersect.



11. a)



Inscribed ∠BAC and central ∠BOC are subtended by the same arc BC.
So, inscribed ∠BAC is one-half the measure of central ∠BOC.

$\angle BAC = \frac{1}{2}\angle BOC$

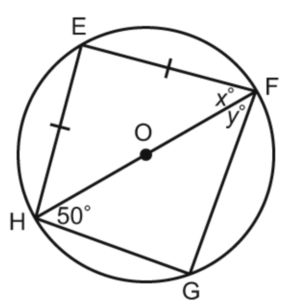
$x° = \frac{1}{2} \times 80°$

$x° = 40°$

∠BAC and ∠CBA are equal angles in isosceles ΔABC.
So, $y° = x° = 40°$

This question illustrates the Central Angle Property and Inscribed Angle Property.

b)



Both ∠HEF and ∠FGH are inscribed in semicircles, so ∠HEF = ∠FGH = 90°
Since EH = EF, ΔHEF is right isosceles, with ∠EHF = ∠EFH = $x°$

Use the angle sum in right isosceles ΔHEF.

$x° + x° + 90° = 180°$

$2x° = 90°$

$x° = 45°$

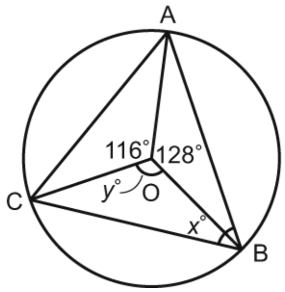
Then, use the angle sum in right ΔHGF.

$y° + 50° + 90° = 180°$

$y° = 40°$

This question illustrates the Angles in a Semicircle Property.

c)



Inscribed ∠ABC and central ∠AOC are subtended by the same arc, AC.
So, inscribed ∠ABC is one-half the measure of central ∠AOC.

$$\angle ABC = \frac{1}{2}\angle AOC$$

$$x° = \frac{1}{2} \times 116°$$

$$x° = 58°$$

The sum of the central angles of a circle is 360°.
So, $y° + 116° + 128° = 360°$

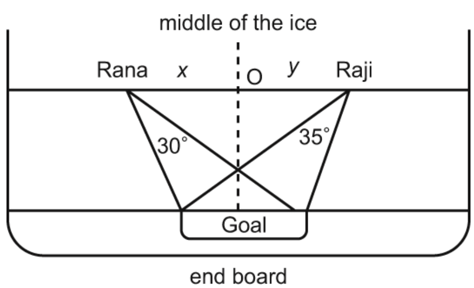
$$y° = 360° - 116° - 128°$$

$$y° = 116°$$

This question illustrates the Central Angle Property and Inscribed Angle Property.

12. The major arc AB subtends reflex central ∠AOB and obtuse inscribed ∠ACB.
The property that relates inscribed angles and central angles subtended by the same arc still applies.
The measure of reflex central angle is twice the measure of the obtuse inscribed angle.

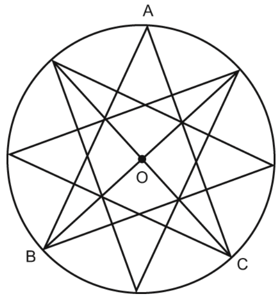
13. a) Diagrams may vary. For example:



b) Raji is closer to the middle of the ice because her shooting angle is greater.
Because Rana's shooting angle is less than Raji's, but both girls are the same distance from the goal line, Rana is farther to one side than Raji.
In terms of circles, Raji is standing on a circle, centre O, with a radius that is less than that of Rana's circle: $y < x$

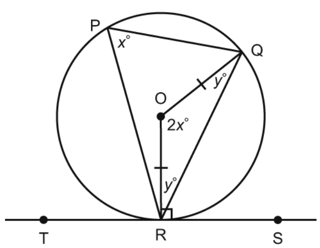
Take It Further

14.



Each vertex of the star is an inscribed angle subtended by an arc with endpoints that are alternate points on the circle.
Each of these arcs also subtends a central angle that is one-quarter of the central angle:
$\frac{1}{4} \times 360° = 90°$
For example, inscribed ∠A is subtended by the same arc BC as central ∠BOC.
Since central ∠BOC = 90°, inscribed ∠A is one-half the measure of central ∠BOC: $\frac{1}{2}$ of 90° = 45°.
So, each inscribed angle measure is 45°.

15. a)



Angle QRS appears to be equal to ∠QPR.

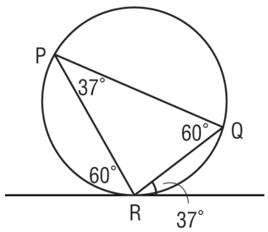
To check, draw the radius OR from the centre, O, to the tangent TS at R. Then ∠ORS = 90°
Inscribed ∠QPR and central ∠QOR are subtended by the same arc QR.
Let ∠QPR be represented as x°; then, central ∠QOR = $2x$°
OQ and OR are radii; so, ΔOQR is isosceles with OQ = OR
Let ∠ORQ = ∠OQR = y°.

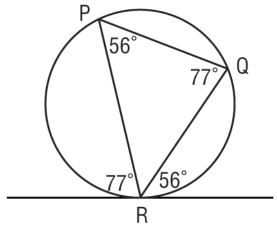
Use the angle sum in ΔOQR.

$2x° + y° + y° = 180°$
$2x° + 2y° = 180°$ Divide each side by 2.
$x° + y° = 90°$

From the Tangent-Radius Property, ∠SRO = 90°
But ∠SRO = ∠QRS + ∠ORQ
So, 90° = ∠QRS + y°
Since $x° + y° = 90°$ and 90° = ∠QRS + y°, then ∠QRS = x° and ∠QRS = ∠QPR
Similarly, ∠PRT = ∠PQR.

b) Diagrams may vary. For example:





Review (pages 425–427)

Lesson 8.1

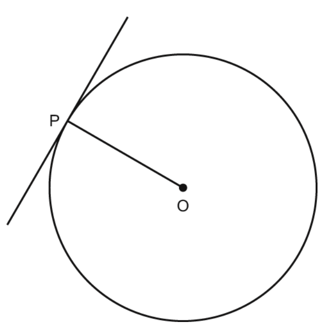
1. a) The radius OP is perpendicular to the tangent PT, so $x° = 90°$.
Use the angle sum in ΔOPT to determine $y°$.
$y° + 25° + 90° = 180°$
$y° = 65°$

b) The radius OP is perpendicular to the tangent PT, so $\angle OPT = 90°$.
Use the angle sum in ΔOPT to determine $y°$.
$y° + 54° + 90° = 180°$
$y° = 36°$
Use the Pythagorean Theorem in right ΔOPT to calculate a.
$PT^2 + OP^2 = OT^2$
$a^2 + 7^2 = 12^2$
$a^2 = 12^2 - 7^2$
$a = \sqrt{12^2 - 7^2}$
$a = \sqrt{144 - 49}$
$a = \sqrt{95}$
$a \doteq 9.7$

c) Since P is a point of tangency, ΔOPT is a right triangle, with $\angle OPT = 90°$.
Use the Pythagorean Theorem in right ΔOPT.
$PT^2 + OP^2 = OT^2$
$a^2 + 9^2 = 20^2$
$a^2 = 20^2 - 9^2$
$a = \sqrt{20^2 - 9^2}$
$a = \sqrt{400 - 81}$
$a = \sqrt{319}$
$a \doteq 17.9$
Tangents to a circle from the same point are equal, so $b = a \doteq 17.9$

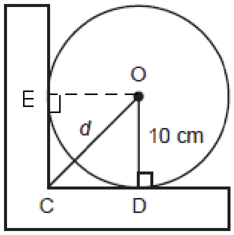
2. Explanations may vary. For example:
If the wire was a tangent to the circle at P and at Q, then each angle between the wire and the radius would be 90°. The lengths of the sides of ΔPOH would satisfy the Pythagorean Theorem.
$7^2 + 13^2 = 49 + 169 = 218$ and $16^2 = 256$, so $7^2 + 13^2 \neq 16^2$
So, the angle between the wire and the radius at P and Q is not a right angle, and the wire is not a tangent to the mirror at P and at Q.

3.



Use a protractor to construct a line perpendicular to line segment OP through P. This line is a tangent.
I used the Tangent-Radius Property.

4. I made the assumption that the two sides of the shelf are perpendicular.



Where the shelf touches the plate, the shelf is a tangent to the plate.
So, the radius OD is perpendicular to CD.

The radius is one-half the diameter, or $\frac{1}{2} \times 20$ cm = 10 cm

OE and OD are radii; so, OE = OD
Since CD = OE, $\triangle$OCD is isosceles, with CD = OD = 10 cm

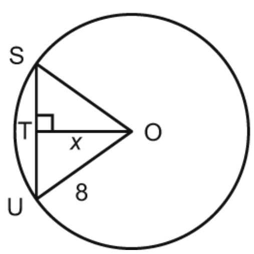
Let d represent the length of OC.
Use the Pythagorean Theorem in right isosceles $\triangle$OCD to calculate d.

$OC^2 = OD^2 + CD^2$
$d^2 = 10^2 + 10^2$
$d = \sqrt{10^2 + 10^2}$
$d = \sqrt{100 + 100}$
$d = \sqrt{200}$
$d \doteq 14.1$

The centre of the plate is about 14.1 cm from the inside corner of the shelf.
I used the Tangent-Radius Property.

Lesson 8.2

5. a)

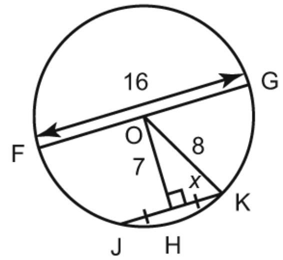


Since OT is perpendicular to chord SU, then OT bisects SU and TS = TU = 5.
Use the Pythagorean Theorem in right $\triangle$OTU.

$OT^2 + TU^2 = OU^2$
$x^2 + 5^2 = 8^2$
$x^2 = 8^2 - 5^2$
$x = \sqrt{8^2 - 5^2}$
$x = \sqrt{64 - 25}$
$x = \sqrt{39}$
$x \doteq 6.2$

b) Since OH bisects chord JK, then OH is perpendicular to JK and ∠OHK = 90°.

Draw radius KO; then, KO is one-half the diameter; that is, $\frac{1}{2}$ of 16 = $\frac{1}{2} \times 16 = 8$



Use the Pythagorean Theorem in right ΔOHK.

$HK^2 + OH^2 = OK^2$

$x^2 + 7^2 = 8^2$

$x^2 = 8^2 - 7^2$

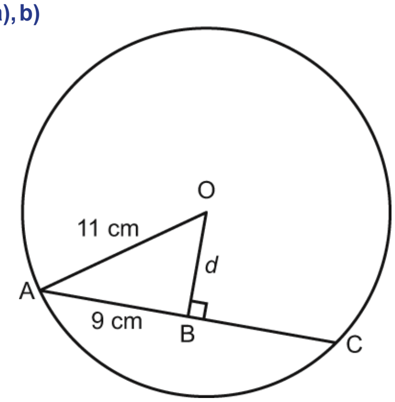
$x = \sqrt{8^2 - 7^2}$

$x = \sqrt{64 - 49}$

$x = \sqrt{15}$

$x \doteq 3.9$

6. a), b)



The radius is one-half the length of the diameter: $\frac{1}{2} \times 22$ cm = 11 cm

Let the distance between the chord and the centre of the circle be represented by *d*.
Line segment OB is the perpendicular bisector of chord AC.

So, AB = $\frac{1}{2} \times 18$ cm = 9 cm

Use the Pythagorean Theorem in right ΔOAB.

$OB^2 + AB^2 = OA^2$

$d^2 + 9^2 = 11^2$

$d^2 = 11^2 - 9^2$

$d = \sqrt{11^2 - 9^2}$

$d = \sqrt{121 - 81}$

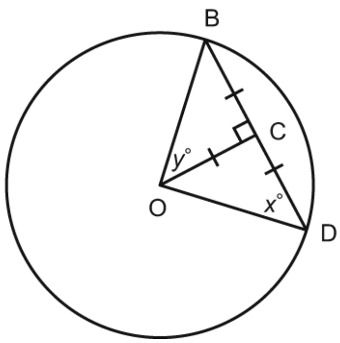
$d = \sqrt{40}$

$d \doteq 6.3$

The chord is about 6.3 cm from the centre of the circle.

7. a) The triangle is isosceles because two of its sides are radii. So, $x° = 35°$
Use the angle sum in the triangle.
$y° + 35° + 35° = 180°$
$y° = 180° – 35° – 35°$
$y° = 110°$
I used the property that the radii of a circle are equal.

b)



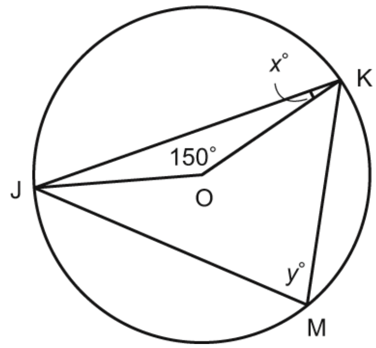
Since OC bisects chord BD, OC is perpendicular to BD and $\angle BCO = \angle DCO = 90°$.
ΔDCO is an isosceles right triangle, so the equal acute angles have a sum of 90°.
So each acute angle is 45°, and $x° = 45°$
ΔBCO is an isosceles right triangle, so the equal acute angles have a sum of 90°.
So each acute angle is 45°, and $y° = 45°$
I used the property that the line segment that joins the centre of a circle to the midpoint of a chord is perpendicular to the chord.

8. Since the square is inscribed in a circle, its right angles are inscribed in semicircles. The diagonals are diameters.
Let d represent the length of a diameter.
Use the Pythagorean Theorem in the isosceles right triangle in one semicircle.
$5^2 + 5^2 = d^2$
$d = \sqrt{5^2 + 5^2}$
$d = \sqrt{25 + 25}$
$d = \sqrt{50}$
$d \doteq 7.071$
The diameter is about 7.071 cm.
The radius, in centimetres, is one-half the diameter: $\frac{1}{2} \times 7.071 = 3.5355$, or about 3.5 cm

9. a) Since O is the centre of the circle, the line segment through O is a diameter.
Both angles with measures of $x°$ and $y°$ are inscribed in semicircles. So, $x° = y° = 90°$

b) The inscribed angle with measure $x°$ and the central angle with measure 120° are subtended by the same arc. Then, $x°$ is $\frac{1}{2} \times 120° = 60°$
Since the inscribed angles with measures $x°$ and $y°$ are subtended by the same arc, the measures are equal, so $y° = 60°$.

c)



OJ and OK are radii. So, ΔKOJ is an isosceles triangle, and $\angle$KJO = $\angle$JKO = $x°$
Use the angle sum in ΔKOJ.

$x° + x° + 150° = 180°$

$x° + x° = 180° - 150°$

$2x° = 30°$

$x° = 15°$

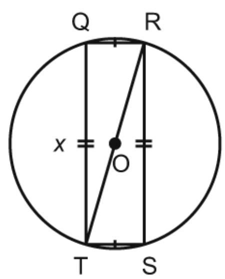
The measure of the inscribed angle is one-half the measure of the central angle subtended by the same arc.

So, $y° = \frac{1}{2} \times 150°$

$y° = 75°$

Lesson 8.3

10. Since the rectangle is inscribed in a circle, the right angles of the rectangle are inscribed in semicircles, and the diagonals are diameters.



Let x represent the length of the longer sides of the rectangle.
A shorter side QR = 10.0 cm
The diagonal RT = 36.0 cm

Use the Pythagorean Theorem in right ΔQRT.

$QT^2 + QR^2 = RT^2$

$x^2 + 10^2 = 36^2$

$x^2 = 36^2 - 10^2$

$x = \sqrt{36^2 - 10^2}$

$x = \sqrt{1296 - 100}$

$x = \sqrt{1196}$

$x \doteq 34.6$

Each of the two longer sides is about 34.6 cm long.

Practice Test (page 428)

1. Since P is a point of tangency, ΔOPQ is a right triangle, with $\angle$OPQ = 90°.
Use the Pythagorean Theorem in right ΔOPQ to determine x.

$$OP^2 + PQ^2 = OQ^2$$
$$x^2 + 10^2 = 12^2$$
$$x^2 = 12^2 - 10^2$$
$$x = \sqrt{12^2 - 10^2}$$
$$x = \sqrt{144 - 100}$$
$$x = \sqrt{44}$$
$$x \doteq 6.6$$

So, x is about 6.6 cm.

Use the angle sum in ΔOPQ to calculate y°.

$$y° + 90° + 56° = 180°$$
$$y° = 180° - 90° - 56°$$
$$y° = 34°$$

2. Inscribed $\angle$DAE and central $\angle$DOE are subtended by the same arc, DE. The inscribed angle is one-half the measure of the central angle subtended by the same arc.

So, $x° = \frac{1}{2} \times 122°$

$x° = 61°$

Line segment BE is a diameter. The angle inscribed in a semicircle is a right angle. So, $y°$ = 90°
The sum of the central angles in a circle is 360°. Then, reflex $\angle$DOE = 360° – 122° = 238°
Central reflex $\angle$DOE and inscribed $\angle$EFD are subtended by the same arc, major arc DE. So, $\angle$EFD is one-half the measure of central reflex $\angle$DOE:

$\angle EFD = \frac{1}{2} \times 238° = 119°$

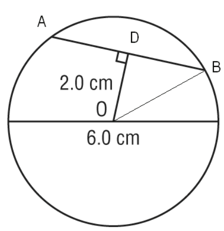
ΔDEF is isosceles because DF = FE; so $\angle$DEF = $\angle$FDE = z°

Use the angle sum in isosceles ΔDEF.

$$z° + z° + 119° = 180°$$
$$2z° = 61°$$
$$z° = 30.5°$$

I used the Central Angle Property and the Angles in a Semicircle Property.

3. a)



b) Segment OD is the perpendicular bisector of chord AB, so AD = DB.

The radius OB is one-half the diameter: $\frac{1}{2} \times 6$ cm = 3 cm

Let the length of DB be represented by x.

Use the Pythagorean Theorem in right ΔBDO.

$DB^2 + DO^2 = OB^2$

$x^2 + 2^2 = 3^2$

$x^2 = 3^2 - 2^2$

$x = \sqrt{3^2 - 2^2}$

$x = \sqrt{9 - 4}$

$x = \sqrt{5}$

$x \doteq 2.236$

Chord AB is twice the length *x*.

AB $\doteq$ 2 × 2.236

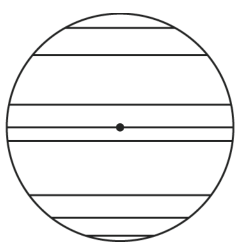
AB $\doteq$ 4.472

AB is about 4.5 cm.

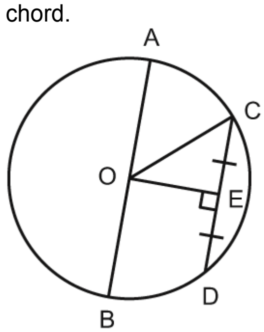
c) The longest chord in a circle is a diameter. A chord that is closer to the diameter is longer than a chord that is farther away. Since chord CD is farther from the centre, CD is shorter than AB.

4. Explanations may vary. For example:
An angle inscribed in a semicircle is subtended by a semicircle.
The central angle subtended by a semicircle is a straight angle, and its measure is 180°.
The Central Angle Property states that the inscribed angle is equal to one-half the measure of the central angle subtended by the same arc.
So, the angle inscribed in a semicircle is one-half of 180° = $\frac{1}{2} \times 180° = 90°$.

5. Explanations may vary. For example:
I think the longest chord in any circle is a diameter.



Any chord that is not a diameter will be bisected by the perpendicular from the centre of the circle to the chord.

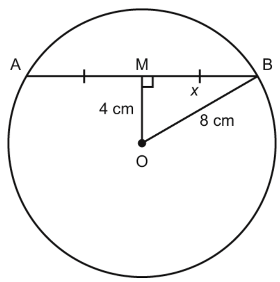


From the Pythagorean Theorem in right ΔOCE, the line segment CE is shorter than the hypotenuse OC, which is the radius of the circle. Chord CE is one-half the length of chord CD. The diameter is the only chord whose length is equal to twice the length of the radius. So, the diameter is the longest chord in a circle.

6. a) The radius is one-half the length of the diameter: $\frac{1}{2} \times 16$ cm, or 8 cm

That means that the chords can be 8 cm or closer from the centre of the circle.
Only the measures in parts i and ii could be distances of chords from the centre of the circle.
I could check my answers by drawing a circle with radius 8 cm and then draw chords that are 4 cm and 6 cm from the centre.

b) i)

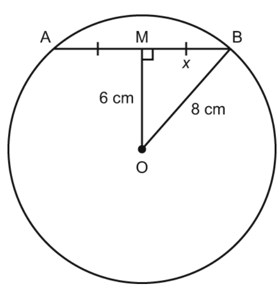


Let the length of MB be represented by x.
Use the Pythagorean Theorem in right ΔBMO.

$$MB^2 + MO^2 = OB^2$$
$$x^2 + 4^2 = 8^2$$
$$x^2 = 8^2 - 4^2$$
$$x = \sqrt{8^2 - 4^2}$$
$$x = \sqrt{64 - 16}$$
$$x = \sqrt{48}$$
$$x \doteq 6.928$$

AB is twice the length of MB.
AB = $2x$
AB $\doteq 2 \times 6.928 = 13.856$
So, the length of chord AB is about 13.9 cm.

ii)

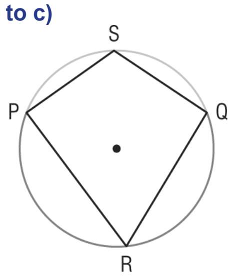


Use the Pythagorean Theorem in right ΔBMO.

$$MB^2 + MO^2 = OB^2$$
$$x^2 + 6^2 = 8^2$$
$$x^2 = 8^2 - 6^2$$
$$x = \sqrt{8^2 - 6^2}$$
$$x = \sqrt{64 - 36}$$
$$x = \sqrt{28}$$
$$x \doteq 5.291$$

AB is twice the length of MB.
AB = $2x$
AB $\doteq 2 \times 5.291 = 10.582$
So, the length of chord AB is about 10.6 cm.

7. a) to c)

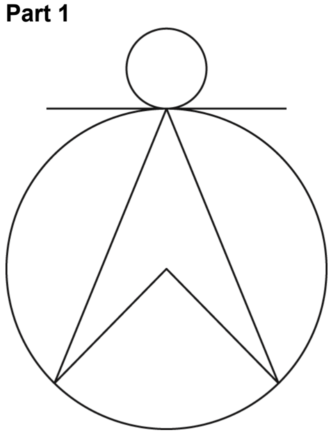


d) Inscribed ∠PRQ is subtended by the minor arc PQ.
Inscribed ∠PSQ is subtended by the major arc PQ.
From the Central Angle Property, each inscribed angle is one-half measure of the central angle subtended by the same arc. So, the sum of the inscribed angles subtended by major and minor arcs PQ is one-half the measure of the central angles.
I know that the sum of the central angles in a circle is 360°.
So, the inscribed angles have a sum that is one-half of 360°: $\frac{1}{2} \times 360° = 180°$
The measures of ∠PRQ and ∠PSQ have a sum of 180°; that is, the angles are supplementary.

Unit Problem **Circle Designs** **(page 421)**

My partner and I decided to design a fitness club logo. We used what we learned about circles to generate a design that suggests an active, healthy lifestyle accessible to everyone.

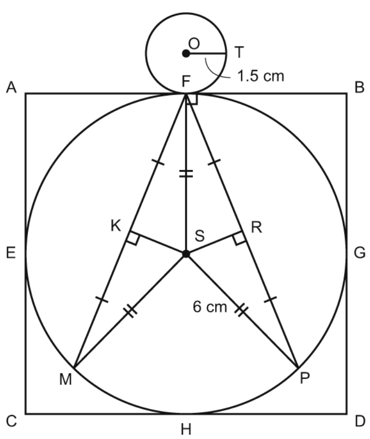
Part 1



First, we sketched two circles of different diameters.
The circles intersect at one point, which is a point of tangency.
Then, we drew a tangent through this point.
From the point of tangency, we drew two chords.
The two chords form an inscribed angle.
We then drew a central angle subtended by the same arc as the inscribed angle.

Part 2

For accuracy, we used a ruler, a compass, a protractor, and a set square to do all the constructions specified in the sketch.

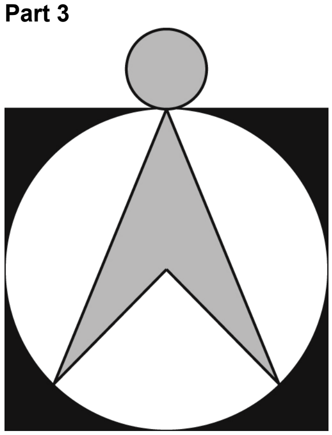


The greater circle, centre S, has a diameter of 12 cm.
Line segments FS, MS, and PS are radii of the same circle, centre S.
The length of the radii is one-half the length of the diameter: $\frac{1}{2} \times 12\text{ cm} = 6\text{ cm}$

The smaller circle, centre O, has a diameter of 3 cm.
Its radius OT is one-half the length of the diameter: OT = $\frac{1}{2} \times 3\text{ cm} = 1.5\text{ cm}$

Line segment AB intersects both circles at only one point, F.
Point F is a point of tangency for both circles.
From the Tangent-Radius Property, I know that tangent AB is perpendicular to radius SF at the point of tangency, F.
From point F, I drew chords MF and FP, 11.5 cm long.
They form inscribed ∠MFP = 30°, subtended by the minor arc MP.
I know from the Central Angle Property that the central angle is twice the measure of the inscribed angle subtended by the same arc.
I used a protractor to check if the measure of central ∠MSP is 60°, twice the measure of inscribed ∠MFP.
I know from the Perpendicular-to-Chord Property 2 that the perpendicular bisector of a chord passes through the centre of the circle.
The perpendicular bisectors of chords FM and FP intersect at the centre of the circle, S.
Tangents to a circle drawn from the same point are equal.
So, AF = AE, CE = CH, DH = DG, and BG = BF. ABDC is a square.

Part 3



The logo reflects the philosophy of the Body and Soul fitness club, which is the commitment to educate people toward a more balanced, healthier lifestyle through exercise and fitness.